

Glossary(10Jun05)

adjacency matrix: The rows of $A=[e_{ij}]$ correspond to the n vertices v_i of the unlabeled graph G_n , and the n columns are all the possible neighbors v_j . If there is an edge from v_i to v_j , then $e_{ij}=1$, otherwise $e_{ij}=0$.

agent: a decision-making entity having specific goals and preferences.

algebra: a formal system for relating and operating on elements of abstract structures, such as number systems or groups.

anigraf: a social network of mental organisms or agents.
See Part I, section 4.

atoms: connected graphs associated with the diagonal Ramsay numbers.
See sec. 11.4.

automorphism (of G): a one to one mapping f of the vertex set $V(G)$ onto itself with the property that $f(v)$ and $f(w)$ are adjacent if and only if v and w are.

bipartite; $K(m,n)$ is a graph G with mn vertices where the vertices can be partitioned into two unconnected subsets such that every edge of G joins vertices in one set with vertices in the other

Borda Count: Preferences are weighted inversely to their rank to determine the voting power of an individual's choices. Thus, if there are three choices, the Borda weight vector will be $\{3,2,1\}$. This vector will be applied respectively to the first, second and third choices of each voter, and then these weights will be summed for all voters to give the Borda Count. See Appendix 1.

broker: a high-level mental organism of an anigraf that controls sequences of actions by agents.

causal network: a directed acyclic graph that represents the history of the graf's evolution.

centroid: that vertex with the highest degree; but see 10.2.

chain: C_n is a chain if all vertices have degree two, excepting two (ends) which have degree one.

chaos: an irregular and unpredictable sequence of states of a system such that past behaviors are not repeated. Typical examples include non-linear mechanical oscillators, fluids, and some chemical reactions. However, chaos can also occur in social decision-making.

circumference: the length of the longest graph cycle.

clique: a subgroup of an anigraf population with similar goals.

co-evolution: the evolution of different anigraf species whose

development depends critically on the interactions among the species. See Appendix 4 for an example.

Coelenterate: invertebrate animals such as sea anemones, jellyfish, and hydroids.

Cognitive Modes: frameworks for knowledge, thought and especially Reasoning by analogy. More generally, constraints on theories of natural modes. See also Ideonomy.

complement: the complement of G_n has all the remaining edges of the complete graph K_n , and none of the edges of G_n .

conditional probability: $p(A/B)$: the probability of event A, given that event B has occurred.

Condorcet winner: the alternative (goal, choice) that beats all other alternatives in a pair-wise contest.

coordinate frame: a distinguished point (the origin of the frame) with axes representing the different variables with categories such as numbers assigned to positions along the axes.

covered graph: If at least one vertex is adjacent to all the remaining vertices, the graph will be covered. Covered graphs will always have a Condorcet winner.

cutpoint: a vertex is a cutpoint of a connected graph if the removal of this vertex creates a disconnected graph. If the removal of an edge creates a disconnected graph, the edge is called a *bridge*.

cybernetics: from the Greek work “steersman”. The field of (feedback) control and communication theory, whether in the machine or in the animal (N. Wiener, 1948.)

cycle, graph: an alternating sequence of $n \geq 3$ distinct vertices and distinct edges linking these vertices with the last member of the vertex sequence also being the first, thus creating a closed path.

cycle, social: given at least three alternatives, a sequence of winners in a pairwise (Condorcet) contest such that the last member of the sequence beats the first. See top-cycle.

daemon: a mental organism.

dance: typically, a sequence of movements, such as gaits, executed in coordination by two physically distinct entities. However, can be generalized to include the interplay between two mental organisms.

degree: the number of edges incident with a vertex.

diameter: the longest length of the shortest path between any two vertices in a graph.

digraph: D_n is a graph G with directed edges between vertices.

dimensional analysis: a method that utilizes the dimensions of variables used to describe natural phenomena in order to recover the form of the underlying model. Assumes that the phenomenon can be described by a dimensionally correct equation among the particular variables. Dimensionless products of variables and linear algebra play a key role.

directed graph: see digraph.

dispersion game: a game with an outcome that maximally disperses agents over a set of possible actions. Useful for optimizing the roles of players on team, where actions and talents of the players differ.

domain map: see knowledge structure.

dynamical system: a smooth transformation of variables that characterizes the evolution of the state of the system. For example, $x_{n+1} = f(x_n)$.

eccentricity (of a vertex): the longest length of the shortest path from the chosen vertex to any other.

Euglena: a one-celled organism up to .01 inch long with a "tail" (flagellum) that whips to provide locomotion. Often found on the surface of ponds, where chlorophyll used for photosynthesis gives the organism a greenish color.

fixed point: the terminal point of an iterated function or map. For Anigrafs, also an equilibrium or stable point.

Flagellum: a long, thin fiber that moves like a whip to propel an organism.

frame: see coordinate frame

free agent: mental organisms whose preferences are initially unassigned, but are capable of acquiring a new goal state and learning its relation to other goals already in place.

gait: a style of animal locomotion. For legged creatures, examples are a walk or gallop for a horse, or a tripod gait for a cockroach.

game theory: a mathematical theory of bargaining first outlined in detail by von Neumann and Morgenstern.

General Possibility theorem (Arrow): impossibility of universal choice rules for total preference orders, given five very plausible assumptions: non-dictatorship, collective rationality, Pareto(unanimity), independence of irrelevant alternatives, conflict resolution.

girth: the length of the shortest graph cycle.

Grammar, graph: a set of rules or productions (and possibly prototypes or motifs) that create or transform one graph into another

graph (undirected): G_n is a finite non-empty set of n vertices (points, nodes) with up to n^2 edges (lines, arcs) between the n vertices.

graph: bipartite: see bipartite

graph: chain: see chain

graph: covered: see covered graph

graph: diameter: see diameter

graph: isomorphic: G and H are isomorphic if they have a one to one correspondence between their vertex sets that preserves adjacencies.

graph: labeled: see labeled graph

graph: line: see line graph

graph: ring: see ring

groups: the permutation group of G are the automorphisms of G .

Ideonomy: a term coined by Patrick Gunkel to describe the study of laws underlying conceptual spaces, their manipulations and their extensions. See ideonomy.mit.edu.

induced subgraph: for $k < n$, if all k vertices and edges of g_k are in G_n , then g_k is a subgraph of G_n . The induced subgraph $\langle g_k \rangle$ is one where two points in g_k are adjacent if and only if they are adjacent in G_n .

information (bits): Shannon information is $[-(1/p)\log_2(p)]$, where p = event probability. For Anigrafs, bits also used to measure the informativeness of an event, namely the extent to which an Anigraf model is revised or updated, and hence a measure of meaningfulness. See sec.x.

IRM: innate releasing mechanism.

Isomorphic: two graphs are isomorphic if their vertex and edge sets are identical.

knowledge depth (kd): two less than the maximum number of levels in the partial ordering of a voter's preferences. If tallies are conducted using the ideal point and only its neighbors in G_n , with all other alternatives third-ranked, then $kd=1$.

knowledge structure: a model (graph) of the similarity relations

among alternatives. If the knowledge structure is *shared*, then all possible individual preference rankings are consistent with the model.

labeled graph: a graph with its vertices distinguished from one another by names.

landmark: the intersection of at least two causal paths generated by two independent parameters.

Laplacian matrix: for the undirected graph G_n with n vertices v_i , each assigned to one row and column, the (symmetric) entries are defined by d_i if $i=j$; -1 if $i \neq j$ and there is an edge (i,j) ; and 0 otherwise, for $i,j = 1 \dots n$ and where d_i is the degree of v_i .

likelihood: in context, C , the probability of a particular observation, o_i , given the presence of an event e_i ; expressed as $p(o_i/e_i,C)$.

line graph: the vertices of the line graph $L(G)$ of G consists of the edges of G , with two vertices in $L(G)$ adjacent whenever the corresponding edges of G are adjacent. Note that every cutpoint of $L(G)$ will be a bridge in G .

MDS: multi-dimensional scaling (Shepard, 19xx) A technique for mapping similarities among items.

mental organisms: agents or daemons in a mind or cognitive system, here represented as nodes in a graph that shows the communication links and similarities of goals of the constituents of the cognitive system.

metagraf: a relationship between a set of anigraf models. These higher order representations include transformations and dynamics, cast as graphs.

model: given a representational system, a model fills out that representation with assignments of variables or particular relations that make explicit how the representation can be used.

multi-scale graph: a graph with a fractal distribution of the shortest mean path lengths from one vertex to all others. See scale-free graph, and small world graphs. The latter have been shown to be multi-scale (Kasturirangan, 1999.)

Natural mode: a correlated set of quasi-independent rules or regularities describing a natural phenomenon. The presence of one regularity is highly predictive of the other regularities.

network: here used almost synonymously with a connected graph, with the typical network being a very large (random) graph.

Pandemonium: the name Oliver Selfridge gave in 1959 to a fanciful pattern recognition system where feature daemons shouted evidence for types of image features, which were then heard by cognitive daemons who are each listening for evidence supporting particular patterns, and these daemons in turn call out to a decision daemon who makes the final choice.

path: an alternating sequence of distinct vertices and distinct edges in which each edge is incident with the two vertices immediately preceding and following it in the sequence.

phase plot: for Anigrafs, a plot showing regions of Condorcet winners and regions of top-cycles, if present. See Appendix 5 and plates.

planar graph: when drawn in a plane, no two edges meet (or cross) except at a vertex in which both are incident.

preference order: a partial ordering of alternatives that makes explicit an individual's first, second, third etc. choices among those alternatives.

priors: the probability of events at the beginning of a causal chain of inference; for beliefs, the priors summarize information or knowledge before the beliefs are updated.

Prisoner's dilemma: a game in which players (partners in crime) may defect (confess) or cooperate with each other (don't confess) on each trial. There are payoffs to each player depending on their choices. For two players, the lowest payoff is to the one who cooperates when the other defects, who then gets the highest payoff. The next lowest payoff is when both defect, and the second highest payoff is when both cooperate.

Proxy: an agent or mental organism that resides in one anigraf, but votes the wishes of another. See 4.2.

radius: the minimum eccentricity of the vertices.

Ramsay numbers $r(G,H)$: Given two graphs, G and H , r is the smallest number such that if the edges of the complete graph K_r are colored red and green, then there is either a set of red edges forming a subgraph isomorphic to G or a set of green edges forming a subgraph isomorphic to H . (Read & Wilson, 1998.)

regular graph: all vertices have the same degree.

representation: a formal system for making explicit certain entities or types of information, together with a specification of how the system does this. (Marr, 1982.) Note that the representational form plays an important role in how variables are manipulated. A simple example is the Arabic, Roman, and binary systems for representing numbers.

ring: a closed chain; a regular graph with all vertices of degree two.

rooted tree: a tree graph with once vertex distinguished as the origin.

Sarkovskii's lemma (theorem): a cycle of period three in a finite attractor for quadratic maps implies periodic points with all periods.

scale-free graph: one with the distribution of vertex degrees, k , obeys a power law: $p(k) = ck^{-1}$.

sensory order; Hayek's view that perception should be regarded as classification of sensory data, thus establishing an order to these data. Similarly, the mental order of events in an environment would be the internal principles used to place an ordering on relations attributed to these events. Note underlying Natural Modes assumption, which related physical orders to sensory orders.

similarity: the percent of vertices in two graphs having identical adjacencies. Relevant to the similarity of Condorcet outcomes and information measures. See 8.2.

small world: the observation that there are short paths of connectivity between any two individuals, thought to be about six for the optimal iteration of mutual acquaintances (hence dubbed "six degrees of separation." A small world graph is highly clustered and yet has rather a short median distance for the means of the shortest path lengths connecting each vertex to all other vertices.

social network: a graph showing the connectivity relations or interactions among individuals or groups or nations, etc.

social order: S_n is the single partial ordering of alternatives that is the outcome of a voting procedure that aggregates a finite number of weighted partial orderings of these same alternatives. More simply, the ranking of all alternatives based on the results of a vote, with the winner ranked first, runner-up second, etc.

spectrum (of a graph): the eigen-values for the characteristic polynomial describing a graph. See Read & Wilson, 1998.

story line: a directed causal graph depicting a sequence of actions and events carried out by at least two players. Simplest story is: boy meets girl; a negative force comes into play and boy loses girl; the negative force is overcome, and boy reunited with girl.

subgraph (induced): for $k < n$, if all k vertices and edges of G_k are in G_n , then G_k is a subgraph of G_n . The induced subgraph $\langle G_k \rangle$ is one where two points in G_k are adjacent if and only if they are adjacent in G_n .

tit-for-tat: a strategy for cooperation or defection in iterated games such as the Prisoner's dilemma: namely, cooperate on the first move and then do whatever the other player did on the preceding move.

TM: a technique for mapping paths of similarity relations among items (Richards & Koenderink, 1990.)

top-cycle: a set of k alternatives a_i , with $k > 2$, such that a_1 beats a_2 beats a_3 a_{k-1} beats a_k and a_k beats a_1 ; also, every alternative not in the top-cycle is beaten by at least one

alternative in the top-cycle.

tree: a graph with no cycles.

vector: a set of objects, typically a set of scalars, with rules for addition (commutative, associative) and scalar multiplication. In \mathbb{R}^3 a vector is usually defined as a directed line between pairs of triples (x,y,z) .

vehicle: Braitenberg's creation of a robot-like machine with a very simple internal structure, and equipped with sensors and motors for interactions with its environment. Their impact is such that one is tempted to use psychological language to describe a vehicle's behavior.

wheel: (W_n) the ring graph R_{n-1} with an extra point that covers all $n-1$ vertices in the ring.

Bibliography (Jun 05)

- Abbott, E.A. (1884) *Flatland: a romance of many dimensions*.
5th Edition (1950) Basil, Blackwell, Oxford.
- Arbib, M.A. (1964) Chapt.4 Cybernetics pp 93-105 in: *Brains, Machines and Mathematics*. McGraw-Hill.
- Arrow, K. J. (1951,1963) *Social Choice and Individual Values*, 2nd ed. Yale University Press, New Haven, CT.
- Axelrod, R, and Dean, D. (1988) Further evolution of cooperation.
Science 242, 1385.
- Barbour, J. (1999) *The end of time: the next revolution in physics*.
Oxford University Press.
- Barabasi, A-L (2002) *Linked: the new science of networks*. Person Publ., Cambridge, MA.
- Beinecke, L. W. & R. J. Wilson (1997) *Graphical connections*.
Clarendon Press, Oxford.
- Berg, H. C. (1996) Touring Machines. *Current Biol.* 6, 624.
- Black, D. (1958) *The theory of Committees and Elections*. Cambridge University Press.
- Borda, J-C. (1781) Memoire sur les elections au Scrutin. Histoire de l'Academi Royale des Sciences.
- Braitenberg, V. (1984) *Vehicles: experiments in synthetic psychology*. MIT Press, Cambridge, MA.
- Campbell, J. (1949) *The Hero with a Thousand Faces*. Chapt. IV, The Keys. Princeton Univ. Press.
- Conant, R. & Ashby, R. (1970). Every good regulator of a system must be a model of that system. *Int. J. Systems Sci.*, Vol.1, No. 2, 89-97).
- Condorcet, Marquis de (1785) Essai sur l'application de l'analyse a la probabilite des decisions rendues a la probabilite des voix. De l'imprimerie royale, Paris.
- Eddington, A.S. (1920) *Space, Time, and Gravitation*. Cambridge Univ. Press. (paraphrased quotation.)
- Gunkel, P. (2000) *Ideonomy*. See: <http://Ideonomy.mit.edu>
- Harary, F. (1969) *Graph Theory*. Addison-Wesley, Reading, MA.

- Hayek, F.A. (1952) *The Sensory Order*. Univ. Chicago Press.
- Huntington, S.P. (1996) *The clash of civilizations and the remaking of world order*. Simon & Schuster.
- Koffka, K (1935) *Principles of Gestalt Psychology*. Harcourt, Brace, and Ward.
- Langhaar, H.L. (1951) *Dimensional Analysis and theory of models*. Wiley & Sons.
- Li, T-Y & J.A. Yorke (1975) Period three implies chaos. *Amer. Math Monthly* 82, 985-992.
- MacKay, D. M. (1969) *Information, Mechanism and Meaning*. MIT Press
- Marr, D. (1970) A Theory for Cerebral Neocortex. *Proc. Royal Soc. Lond. B* 176,161-234.
- McMahon, T.A. (1977) Allometry. *Yearbook of Science and Technology*. Pp 48-57. McGraw-Hill. Also (1973) Size and Shape in Biology. *Science* 179, 1201-1204.
- Minsky, M. (1986) *Society of Mind*. Simon & Schuster.
- Moon, J.W. (1968) *Topics on Tournaments*. Holt, Rinehart & Winston.
- Nauta, W and M. Feirtag (1986) *Fundamental neuroanatomy*. Freeman, NY.
- Newman, M.E.J., A-L. Barabasi & D. Watts (Eds.) (2005) *Structure and dynamics of complex networks*. Princeton Univ. Press.
- Nicod, J. (1970) *Geometry and Induction*. Univ. Calif. Press.
- Palmer, E.M. (1985) *Graphical Evolution*. Wiley & Sons.
- Paynter, H.M. (1961) *Analysis and Design of Engineering Systems*. (class notes for subject 2.751) MIT Press, Cambridge, MA
- Pedley, T.J. (ed.) (1977) *Scale Effect in Animal Locomotion*. Academic Press.
- Poston, T. & I. Stewart (1978) *Catastrophe theory and its Applications*. Pitman Publishing.
- Raibert, M. (1988) Balance and Symmetry in Running. Chapt. 36 in *Natural Computation*, W. Richards (Ed.) MIT Press.
- Read, R.C. & R.J. Wilson (1998) *An Atlas of Graphs*. Oxford University Press
- Richards, W. & A. Bobick (1988) Playing Twenty Questions with Nature. In: *Compu. Processes in Human Vision*, Z. Pylyshyn (Ed.)

Ablex, Norwood, NJ.

Richards, D., McKay, B.D. & W. Richards (1998) Collective Choice and mutual knowledge structures. *Adv. In Complex Systems* 1, 221-236.

Richards, W. , McKay, B.D. & D. Richards (2002) The Probability of Collective choice with shared knowledge structures. *J. Math. Psy ch.* 46, 338-351.

Saari, D. (1994) *Geometry of Voting*, Springer-Verlag

Searle, J.R. (1983) *Intentionality*. Cambridge Univ. Press.

Selfridge, O. (1958) Pandemonium. In: *Mechanisms of Thought Processes: Proc. 10th Symp. Nat. Phys. Lab. Vol. 1.*

Shepard, R. (1962) The analysis of proximities: multidimensional scaling with an unknown distance function. *Psychometrika* 27, 219-246.

Thompson, D'Arcy W. (1968) *On Growth & Form*, Cambridge Univ. Press, 2nd Ed.

Watts, D.J. (1999) *Small Worlds: the dynamics of networks between Order and randomness.* Princeton Univ. Press.

Weiner, N. (1948) *Cybernetics: or control and command in animals and machines.* Wiley & Sons.

Wilson, E.O. (1971) *The Insect Societies.* Harvard Univ. Press.

Wolfram, S. (2002) *A New Kind of Science.* Wolfram Media, Inc.

Young, H. P. (1998) *Individual Strategy and Social Structure.* Princeton University Press.

Young, H.P. (1988) Condorcet's Theory of Voting. *Amer. Pol. Sci. Rev.* 82, 1231-1244.