Part IV

Metagrafs

Metagrafs are relationships between anigraf models. The simplest such relationship is the identity, when one graph is the same as another. Typically, the nodes or edges will have different meanings in different contexts. More complex relationships include how one anigraf model may be transformed into another, or observing a set of models whose members share one or more parameters. Such mappings can serve as a basis for assigning classes to anigraf models. Relations between model classes form a metagraf, that places the class members in a much broader perspective. Explorations of such relationships is relevant to understanding intuitive and creative processes, where models become reconfigured or augmented. The higher-level abstractions underlie notions of beauty and self.

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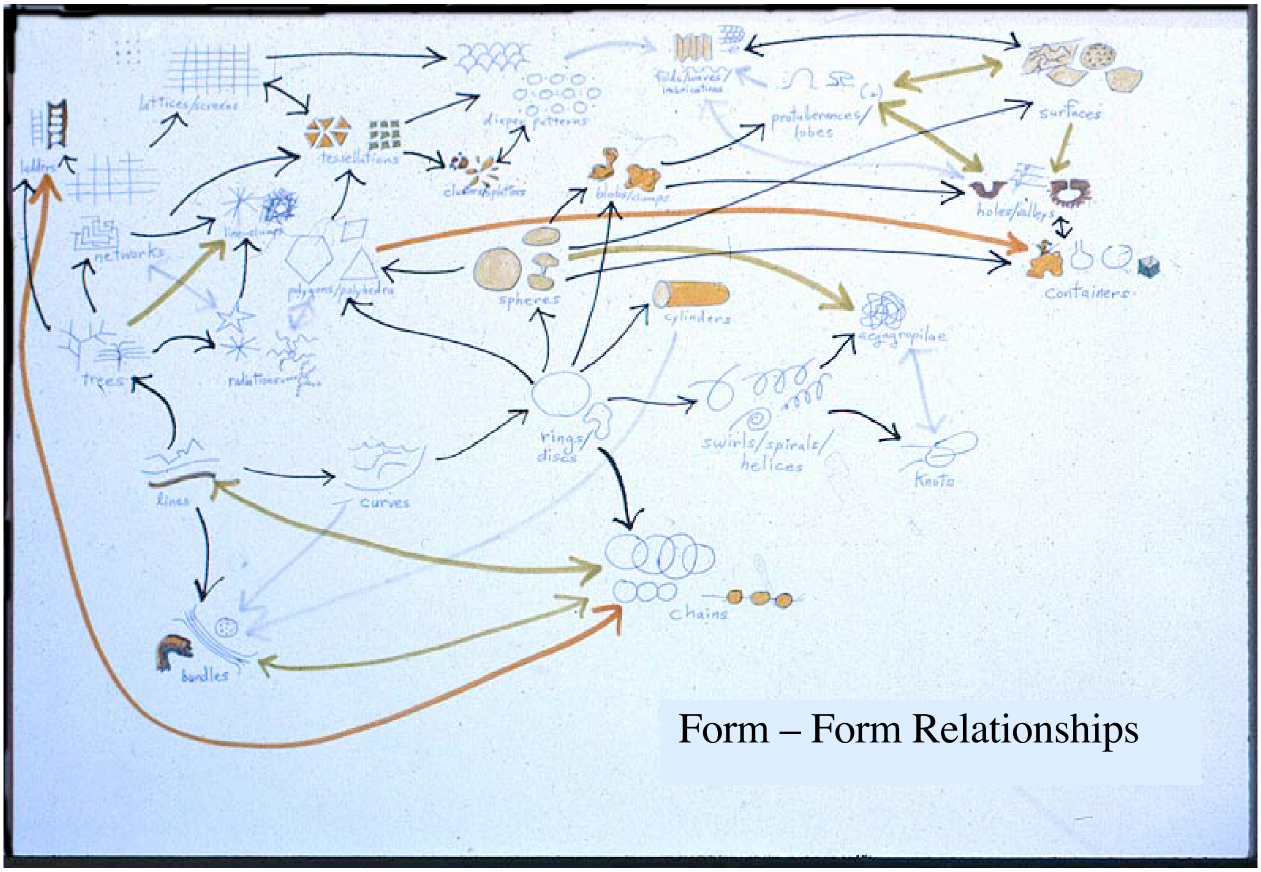


Depiction of a Metagraf relating classes of forms (from P. Gunkel, *Ideonomy Project*).

Metagraf1: Morphologies

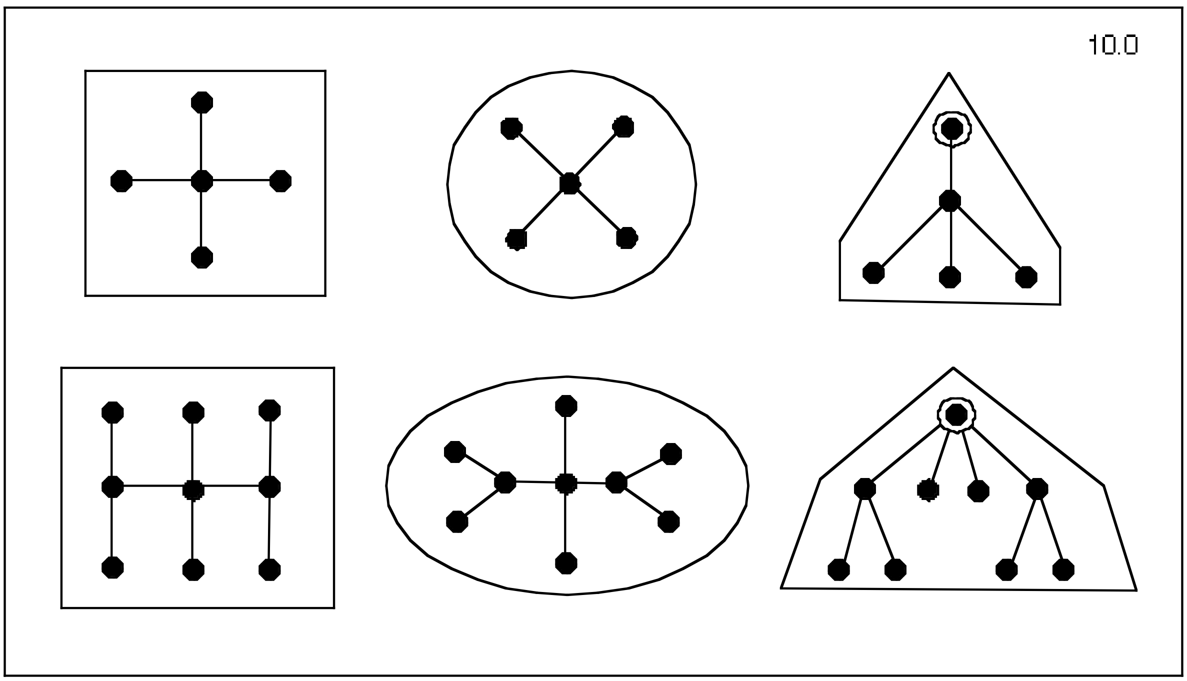


Fig. 10.0: Contextual effects on form induced by the framing of two graphs. On the right, the bull's-eyes identify roots of the trees.

10.0 Preliminaries

Graphs have been used to depict relationships between different mental organisms. For only eight types of mental organisms, there are over 10,000 possible relational forms; for twelve the number exceeds ten billion. If such relationships are constrained in part by Nature's designs, then certain graphical forms will be favored over others. At the human scale of observation, the tree-form is one very common class, that includes inheritance hierarchies, phylogenetic depictions, models for drainage systems, arterial and capillary designs, grammars, etc. If an anigraf is known to be a member of a class such as a tree, or ring, or bipartite, etc., then strong inferences can be made not only about its form, but also its evolution. Class membership is thus very informative. At yet another, more abstract level, one may also see analogies in the sequences and development of form in different contexts, or between different classes. Such correspondences can spark creative thought, especially if collections of free agents have mimicked a class membership, but are used in another context. These kinds of abstractions of relations between different classes of graphs can also be depicted as graphs. These are the Metagrafs. Such models of models support intuitive and inductive thought.

To create classes of graphs, attributes must be given to unlabeled vertices or edges. Without such attributes, graphs are like numbers, which in isolation are meaningless. A series of positive numbers becomes real when beads or coins are counted, or when sizes are compared. Pi is simply one of an infinity of transcendental numbers, but takes on a special meaning when we idealize "the circle" and wish to calculate some of its properties. Graphs are no different. We can discover abstract relations between graphs, or define a class (e.g. regular, bipartite, cubic, etc.), just as we do for numbers (e.g. integers, imaginary, complex, irrational, etc.) But this step imposed attributes on the nature of edges or vertices. The class "regular" required that all vertices in the graph have the same degree; the class "tree" requires no loops. There are an infinity of such classes.

Within each class, there will be rules that transform one member into another. For the class Ring (R_n) we can map all rings into the set of integers with n > 2. The Metagraf for this set will be a chain C_n (See Fig. 10.1)

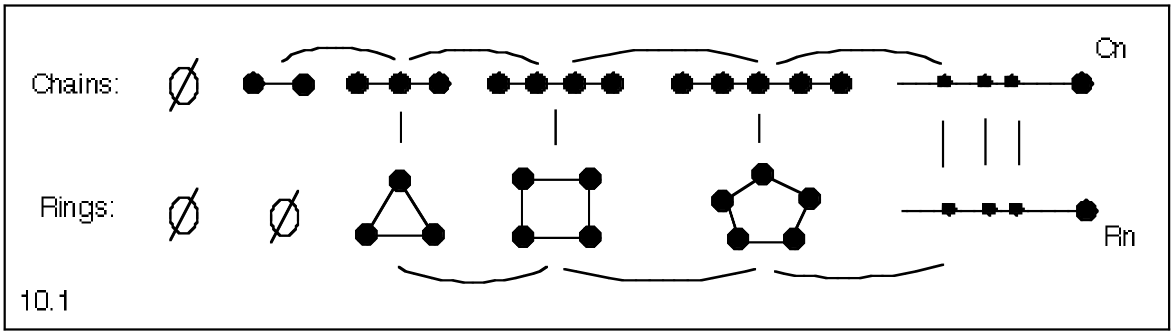


Fig. 10.1: The mapping between a chain metagraf and a ring metagraf creates a higher order, ladder- like Metagraf.

At the next level of abstraction, we can also create a ladder – like Metagraf that relates chain graphs and rings by the simple operation of adding (or deleting) an edge. Any evolutionary sequence of forms is a metagraf. (See Appendix Fig. 4.1 for the evolution of a set of anigrafs that is slightly more complex than the typical phylogenetic tree.)

10.1 Context

Two other kinds of attributes are important in a simple treatment of Metagrafs. These are the origin or root of a graph, and its frame. In Fig. 10.1 the root of the chain metagraf is "2" (or "1" if there is the notion of a zero), and "3" (or 2) for the ring Metagraf. The root will depend in part on the transformation rules.

The second contextual factor is how a graph is presented. This property is analogous to imposing a coordinate frame on an object, where the coordinates suggest dimensions along which the object will change. Figs. 10.0 and 10.2 clarify. In Fig. 10.0, two tree graphs T_5 and T_9 appear at the left. Their edges are parallel to sides of a rectangular frame. This framing suggests that edges share an attribute with other edges similarly aligned. In the middle, the same graph is redisplayed in an ellipse, and on the right, as a rooted tree. At first glance, the graphs are seen as quite different – and indeed they should be regarded as different because different constraints have been imposed on the edge attributes. For each depiction, the transformation rules relating members of the class can be expected to differ.

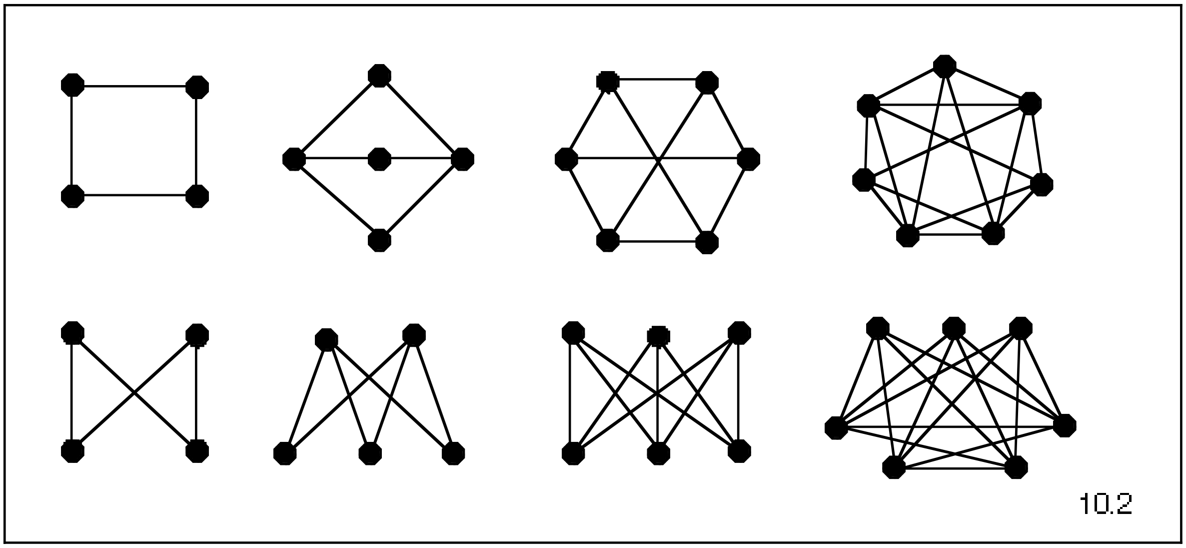


Fig 10.2: Four pairs of connected graphs that appear different, but are the same. Associated with each depiction is a different interpretation of the graph.

In Fig. 10.2, we see another example of framing. The upper and lower members of each column are the same. Again the graphs look quite different. In each case, the left to right sequence is a Metagraf of regular graphs. On the top, the number of vertices (i.e. graph size) follows the integer sequence: 4,5,6,7….. In the bottom row, the graphs are all bipartite, with the sequence (2,2), (2,3), (3,3), (3,4)….The origin for each metagraf is the same (R_4), but the framing led to a different transformation rule.

10.2 Metagraf Basis Functions

Although many evolutionary processes resemble trees (or ladders if class relations are noted), Metagrafs can have much more complex structures. Consider Fig. 10.3. The upper row is a chain metagraf of ordered stars S_n: s3, s4, s5 …… sn, with s4 illustrated at the right. Below this metagraf is another similar metagraf of rings. The elements of each metagraf are related to the other by equating the number of vertices. In addition, the two sets of elements may be combined to create a wheel graph W_n. If more and more edges are now added to the wheel (in some specified order), the final result will be the complete graph K_n. Thus, for each n, there will be an elemental metagraf of a triangle plus a tail, with the length of the tail increasing by $O({}_nC_2)$. One could regard these planar sections through the entire Metagraf as its basis set. If such

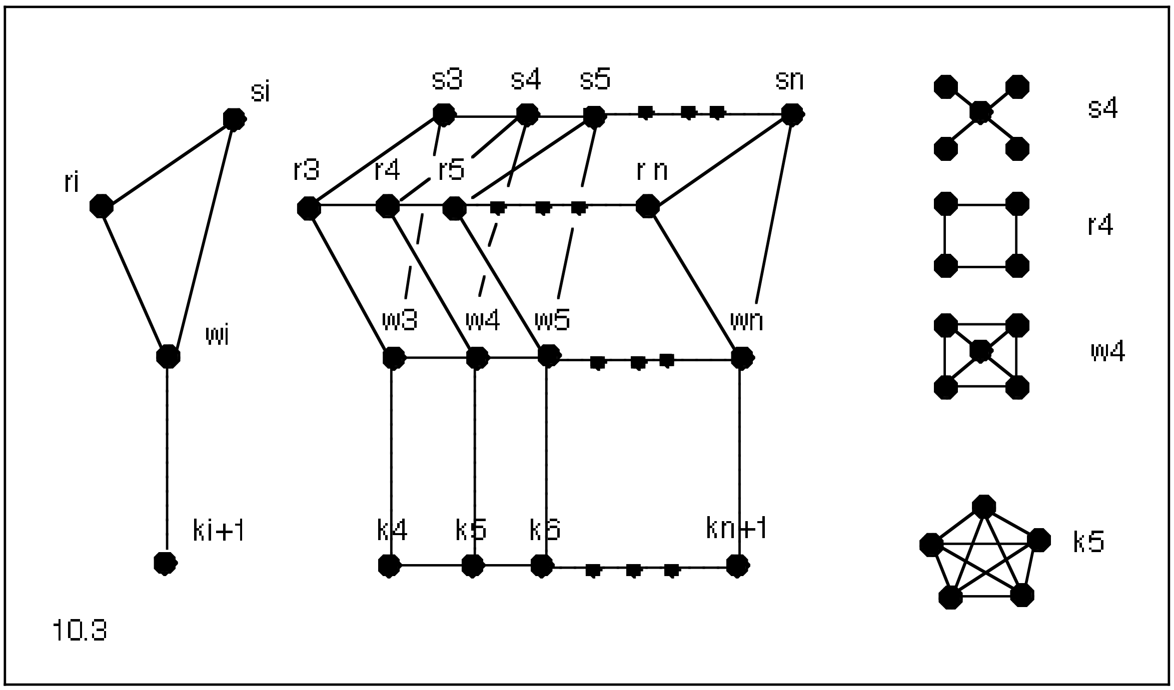


Fig. 10.3. Two chain metagraphs Sn and Rn are linked and combined to form wheels Wn. As edges are added to the wheels, the final state will be another chain of complete graphs Kn. The triangle plus tail graph at the left illustrates the form of the basic building block.

basis elements can be identified for any Metagraf, its structure can be further simplified by another abstraction and a more elegant characterization of the domain. Alternately, such as with trees, the graph might be characterized by its branching rules.

From Paths to Byways

Unlike sterile, unlabeled graphs, all anigrafs assign attributes to their vertices and edges, regardless whether or not these are explicitly defined. This property is especially clear for edges of a metagraf, where a transformation rule or parameterization links one vertex to another. A path (or trail) characterized by a parameterization or attribute is called a by-way. The chain-metagraf of rings is a simple example, with increments in size as the shared attribute of all edges. The smallest element of a byway has two edges and three vertices. Extensions from this basic element to additional vertices of the graph cannot be arbitrary. Rather, the additional edge (or edges) in the byway must contain the attribute or parameterization associated with previous edges. A subway map with various routes marked by color is a common example of a set of byways. If there is a major byway that defines a frame, then perhaps these may be labeled as highways.

Several properties of graphs having edge attributes or parameterizations are obvious: (1) when two or more byways cross at a vertex, there is a special vertex or landmark that is quite different from the intersections of paths (or trails) in classical graphs; (2) similarly, if one or more byways meet and terminate at a vertex, we have a junction (e.g. where a “tail” joins a “body” vertex); (3) branch points (or forks) are another obvious consequence when three or more edges at a vertex all share a common attribute; and finally, (4) because byways make explicit the parameterizations of a network, they can be used to frame a graph, as well as providing grounds for a minimal metrics.”

10.3 Complementarity

Models of models are highly cognitive artifacts. Like trees, certain parameters and transformations tend to re-occur, thereby specifying identical or similar metagrafs. The spectrum of love to hate, friend to foe, good to evil are examples. Each element lies at the opposite extreme of a signed parameterization. With most terminal paths through relational spaces, comes the notion of opposites, or, more generally, of complements. These also may include pairs about any neutral origin that are not necessarily at the extremes, but are in some sense are objects located at symmetric positions in the space. These mappings to complements are metagrafs.

For connected graphs, complements can be specified precisely, using a slight revision of the graph-theoretic version:

Definition: *The (connected)* ***complement*** $\underline{\mathbf{G}}_n$ *of the graph* G_n *is obtained by removing the edges of* G_n *from* K_n *subject to both* G_n *and* $\underline{G}_n$ *being connected.*

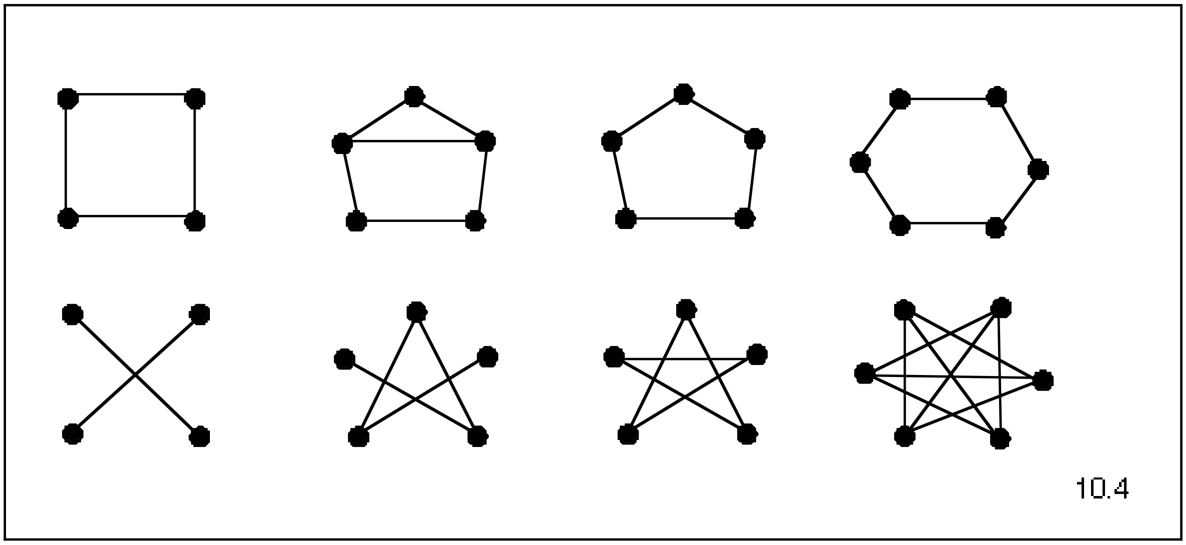


Fig. 10.4. The lower graphs are the complements of those immediately above, and vice versa. For anigrafs, however, only connected graphs will be considered valid complements.

Figure 10.4 illustrates. The upper and lower rows are complements of each other. In the third case, $\underline{G}_n$ and G_n are self-complements. For these ring graphs note that their complements (without unfolding) appear to belong to a class of "star-like" graphs. The complementing operation on rings in this context has produced a new set of graphs that belong to the metagraf complement of the ring metagraf. (Note that the metagrafs for each set of complements intersects at the pentagon.) This metagrafical transformation is easily understood; hence the new (context-sensitive) patterns are readily assimilated and categorized. Through use of the metagraf, the anigraf has a cognitive tool that supports creative innovation.

10.4 Link or Line Graphs

Mental organisms are associated with the vertices of a graph. Each mental organism has its own preferred goal for the anigraf system. The edges between mental organisms reflect a similarity in goals. As has been emphasized in metagraf constructions, each edge thus has a distinctive attribute, just as does a vertex. For example, in Fig. 1.4 there are five choices for actions: attack, inspect, freeze, flee, hide. Both attack and inspect are related by a forward movement; flee and hide require backward movements. The attributes for the edges on each side of freeze are thus opposite, but are linked through the freeze. Like the choices associated with vertices, we can construct a graph that shows the relations among such attributes of edges. A graph of these relationships is a line or link graph L(G). In some sense it is the dual of G, just as points and lines are duals in Euclidean geometry.

Definition: *The* ***link (or line) graph*** $\boldsymbol{L(G_n)}$ ***of*** $\boldsymbol{G_n}$ *is constructed such that the vertices of L(G) are the edges of G, with two points of L(G) adjacent whenever corresponding edges of G are adjacent.*

To illustrate the potential power of the link graph representation, consider the graphs in Fig 10.5. We start with a square with two limb segments (upper left.) G is then transformed into L(G). Then the complement of L(G) is found, to obtain the “opposite” to L(G), now labeled as L(H). The graph H is then seen as a diamond with a segmented tail (far right.) In anigraf world, this “species” H might be considered quite different from “G”.

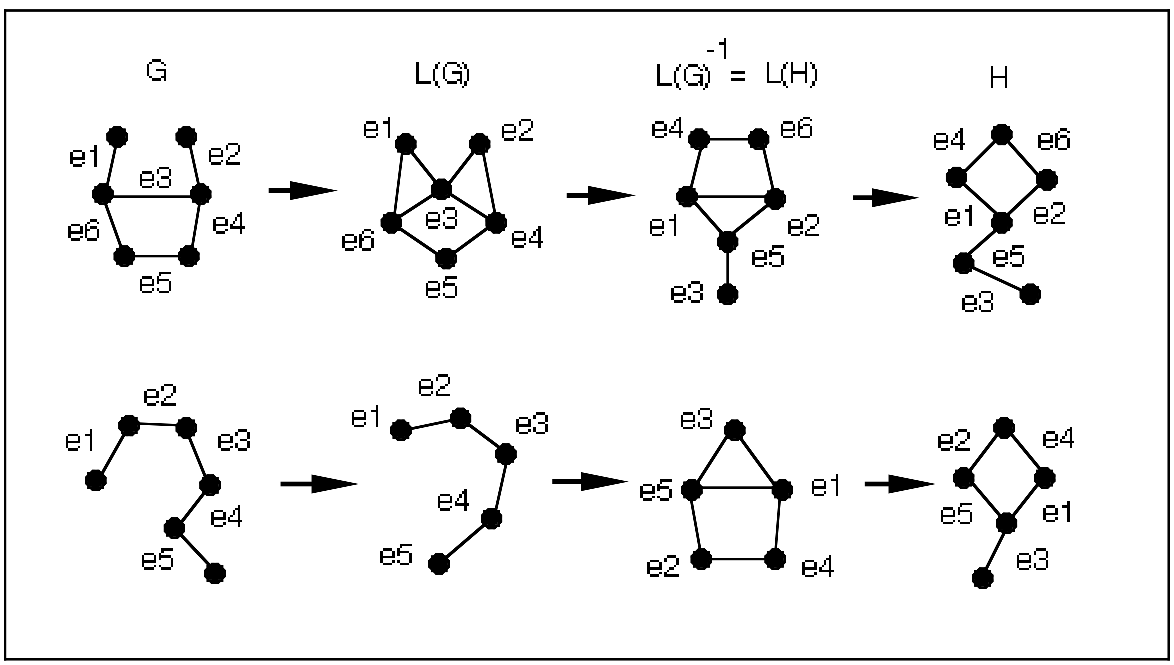


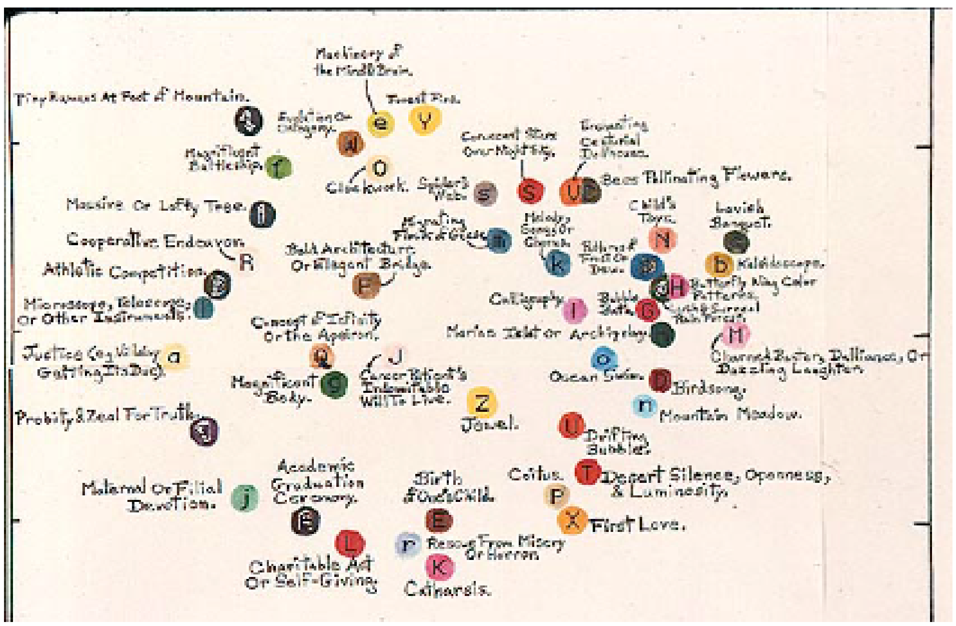
Fig. 10.5. Two examples of a transformation to a line graph L(G), followed by the complement of L(G), which is then converted into the graph H from which L(H) would be derived. In the first step, note that the edges ei become the vertices of L(G). See text.

The lower series in Fig. 10.5 shows another set of transformations starting with a 6-chain. The result is again a diamond, but with only a single segment tail. (Note that the transformation also led to another graph in the same class as G (i.e. a chain.) Whether or not these types of transformations lead to metagrafs of predictive power is not the issue. Rather, such transformations illustrate creative potential that would not follow from classical split and merge operations.

10.5 From Intuition to Intellect

Metagrafs make explicit the relationships between elements of model classes, as well as how one class relates to another. Metagrafs provide the anigraf with a potential for self-induced intuitive insights. They offer cognitive tools for intuition and creative thoughts, especially through analogies and induction. Observations and evidence can be seen from novel perspectives, such as recognizing that a different kind of model also explains "the facts" and indeed might have greater predictive power. The gain is a greater intelligence. An anigraf's ability to manipulate its own concept space is just a first step.

Metagraf2: a Gestalt



Multi-dimensional scaling map for aspects of beauty by P. Gunkel
(see http//ideonomy.mit.edu)

18 Aug 07

11.0 Ideonomy

Just like the world has modal structures, so must anigraf models that support the sharing of knowledge. Although attempts to discover laws underlying modes of thought date from at least Plato, Francis Bacon (1652) was the first to suggest that there could be a science of concept creation. Was number theory simply the product of bartering and exchange? Are the concepts like i, e, pi, root 2 just the result of building "tidy" models? Ideonomy is the term coined by Patrick Gunkel (1960) to describe the study of the laws underlying a host of conceptual spaces. Metagrafs are examples of ideonomic constructs. At the highest level, they elaborate the analysis of biological forms and allometry (Thompson, 1917; Huxley, 1932), and reveal similarities between many conceptual models, such as emotion and movement, classical atomic structures and galactic form, or even quantum potentials and black holes. The goal is not simply to discover new, unexpected relationships, but more importantly, to recognize the inherent simplicity of the relation between the old and the new. Although a new Gestalt may be associated with a set of relationships seen as a whole, there is simultaneously the recognition of its distinct subpatterns, and how the relations among the subpatterns relate to the whole. In other words, there is an "understanding" that encompasses several levels of relationships at several scales (or anigraf groupings.) The parts all blend into the whole to create the Gestalt. At present, no compelling, formal models have been discovered for this metagraf Gestalt. Gunkel's ideonomy (ideonomy.mit.edu) is a preliminary step; yet the tools and conceptual capabilities are still very impoverished. One area of continued interest is beauty. Cerillo (1965) emphasized group structures for music and art. Here, we take a related tack, more closely aligned with anigraf design.

11.1 Beauty

Although aspects of beauty can be shared in a society, fundamentally it is a personal experience. Hence revelations or insights are often experienced as "beautiful", demonstrating that one's own perspective and intrinsic structures play an important role. The breadth of what may be considered beautiful is enormous. A creative act, or a simple mathematical expression is more than just elegant. There is the beauty of true love, or the beauty of a sunset, a forest, a seashore. Music, design, poetry all may evoke feelings of beauty, depending upon the context and the beholder. What, then, might be considered the core of beauty in anigraf world? Is there any set of relationships that might be common to all forms of beauty as entertained by anigrafs?

Definition: *Anigraf* ***Beauty*** *is when many different relationships are seen not only in an entirety, but are also appreciated as a variety of interrelated decompositions of the whole.*

More simply put, using the grafical representation, at one level there is a unification of all relationships, such as depicted by K_n, whereas at other levels, there are sets of subgraphs, each of which fit together seamlessly (to the anigraf's view), yet can be reintegrated into the whole, which is then broken down into yet another set of subgraphs. In the human mind, a prevalent example would be the beauty of the conviction that there is one overarching "God" possessing an all encompassing love for all creatures, who are seen in his image striving for loving relationships with each other. Another example would be how a scientist might view (or believe in) explanations of the world about us, from its very atomic or galactic core to life itself. Yet another, on a more mundane and less awesome plane, would be the interconnectedness of the pieces of a musical composition, a poem, or a story. In all these cases, the significant events are tightly interwoven and complex, yet still are felt to be an integrated, coherent whole. In anigraf world, we can explore this "model" for beauty a bit more precisely.

As before, any event or property is represented as a node in a graph, with edges indicating close relationships. Of course, these relationships may change with time as graphs evolve into new structures. However, at the abstract level of metagrafs, these evolutionary steps should result in new graphical forms (or subgraphs) that are in the metagraf set, otherwise they will be unfathomable. So our challenge for anigraf beauty is to understand how accepted metagraf forms (ie models for events and relationships) first can be seen as subgraphs of K_n, and then re-evolve into yet another set of decompositions, or augmentations, if $K_n \rightarrow K_{n+1}$.

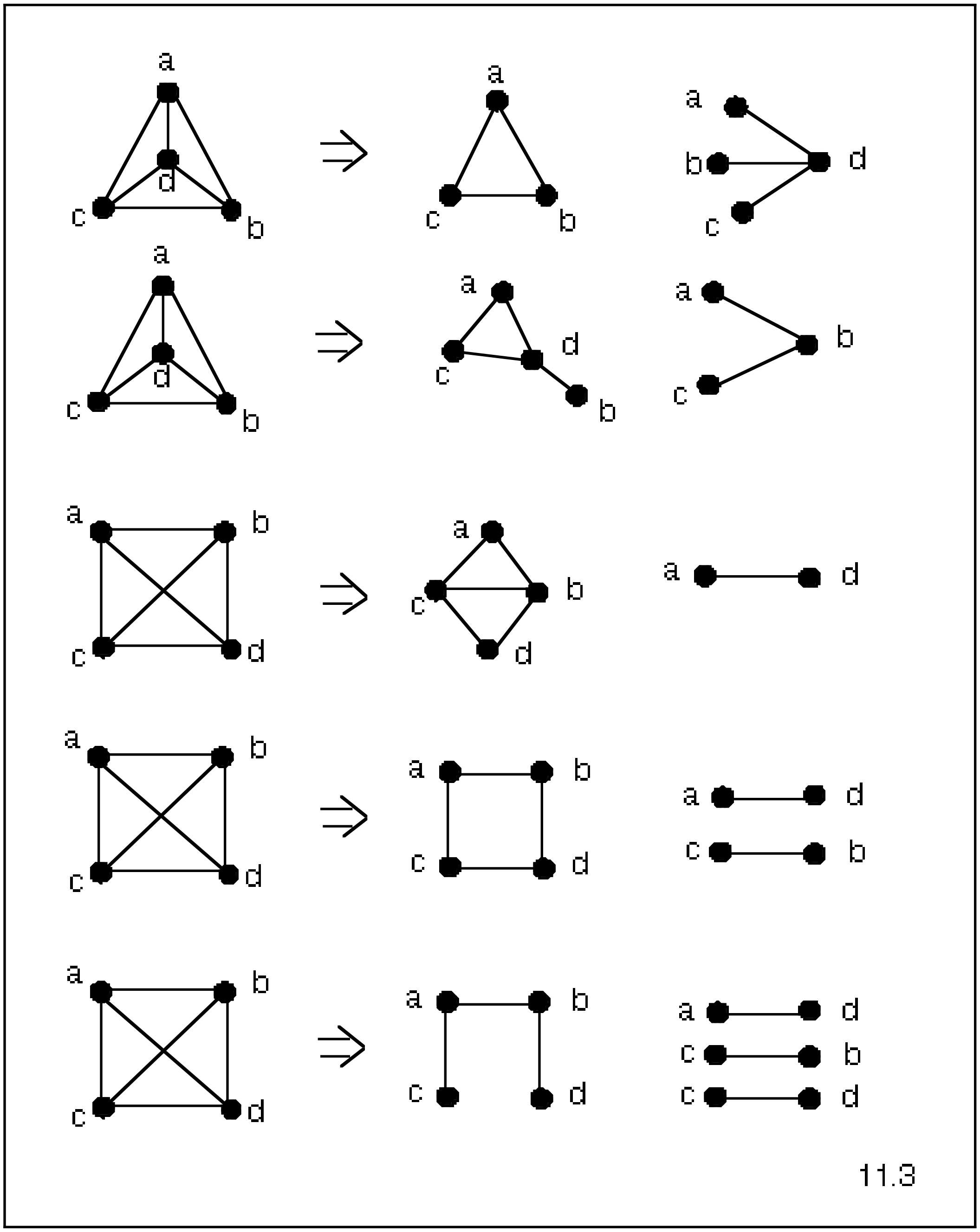


Fig.11.1. The left column of graphs are the complete graph K4. To their right are shown five pairs of subgraphs that result with edge removals, with all members to the right of the arrow being different.

11.2 Simple example.

The key here is that beauty is about relationships, not the form of the objects themselves. Rather, the nature of the objects is assumed to set the context, thereby influencing the observed relationships. The basic idea is best illustrated with K_4. There are five different connected subgraphs that result when edges are removed from K_4. These, and their residuals, are shown at the right in Fig. 11.1. For clarity, two depictions of K_4 are used, one having a triangular outline, the other a square. At the top, we have K4 = K3 + t3. Indeed, for any Kn, it is obvious that Kn = Kn-1 + tn-1. Hence any Kn can be decomposed into a (non-unique) set of trees. In the third row of the figure, K4= R4 + {gi}, where gi in this case are two versions of K2. Still more possibilities are given in the remaining rows.

From the point of view of these graphs seen as patterns, there is nothing especially striking about these decompositions. For example a ring and two edge segments with nodes seem quite distinctively different and hard to "relate" to one another. Perhaps the strongest relationships are the top two triangular forms in the second column, and the two square rings below. Ideally, we would like the subgraph decompositions of K_n to be have more compelling relationships. Such a metagraf that organizes the objects in Fig 11.1 in a meaningful way is obscure.

11.3 Complementarity

A different tack would be to break down K_n into relationships that are already part of the anigraf's repertoire. Graphical complements are an option. Let Kn be divided initially into complementary pairs having as nearly as possible the same number of edges thus creating an "equal" partitioning. Then proceed to manipulate these pairs by adding an edge to one, and removing an edge to the other. Figure 11.2 shows this result for K_5. Note that for the

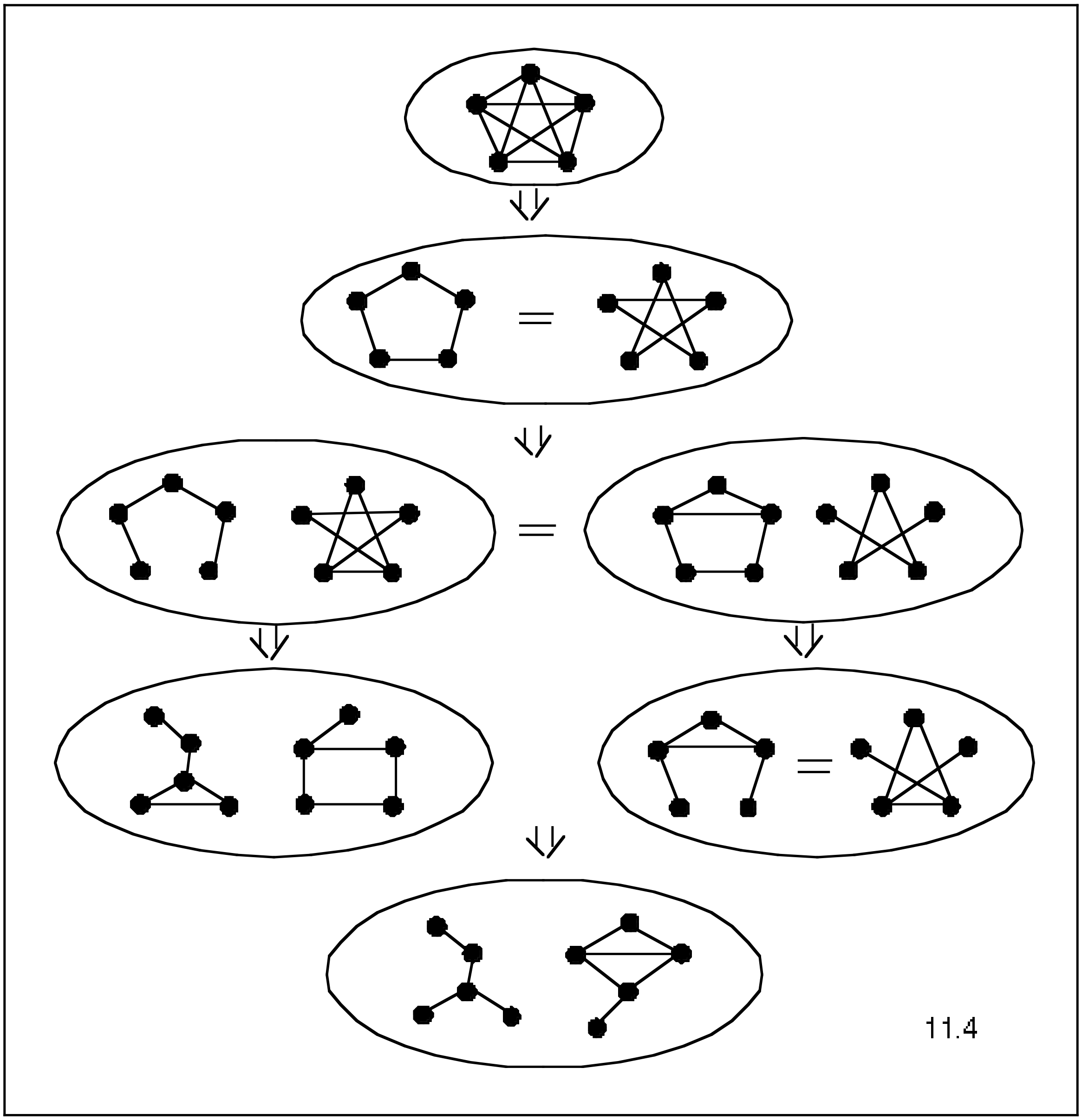


Fig. 11.2: A decomposition of K5 into pairs of complementary subgraphs.

anigraf, the initial split is very compelling in this depiction. The transitions seem to follow rather well. The entire structure is a candidate for the metagrafical organization of K_5, seen as a decomposition of connected subgraphs of order 5. (The lower orders follow trivially.) Perhaps one kind of "beauty" associated with K_5 are these subgraph relationships. Of interest is that all of these decompositions contain the chain C_4 as a subgraph. Seeing this common element offers further insights. Perhaps at the root of beauty is the appreciation of such a common element to any decomposition of the whole.

11.4. Ramsey Decompositions

Given n objects under consideration, let the edges of Kn represent all possible bidirectional (non-metric) relationships. We now ask, given any decomposition of these edges into two sets, is there a connected subgraph always guaranteed to be present in at least one of the decompositions ? If so, then those subgraphs present for Kn but not appearing in decompositions of Kn-1 will be designated the "atoms" for Kn. Clearly one element of a chain, namely C2, is the simplest atom (seed?),but does not qualify as an atom for Kn, n > 2. Hence the atoms of special interest are the "largest" for any Kn -- those which appear first for Kn but not for Kn-1.

Definition: *The **"atoms" $å_n$ for K_n** are the connected subgraphs that are present in at least one member of all binary decompositions of the edge sets of K_n, but are not present in all such decompositions of K_{n-1}.*

By definition, the atoms $å_n$ are thus those connected graphs associated with the diagonal Ramsey numbers r(G,G) where r = n. Part of this set is shown in Figure 11.3.

Not surprisingly, most of these atoms have appeared throughout the text. Simple trees and rings were the building blocks for primitive anigrafs. The impact is that for ***any*** system of n "objects" built upon relationships, then these must be the expected prevalent forms, because they survive any decomposition of the complete set of relationships. Hence we can now place a new, more meaningful ordering on the trees and graphs resembling rings. For example, ring shapes are now not ordered by the number of edges, but by their Ramsey

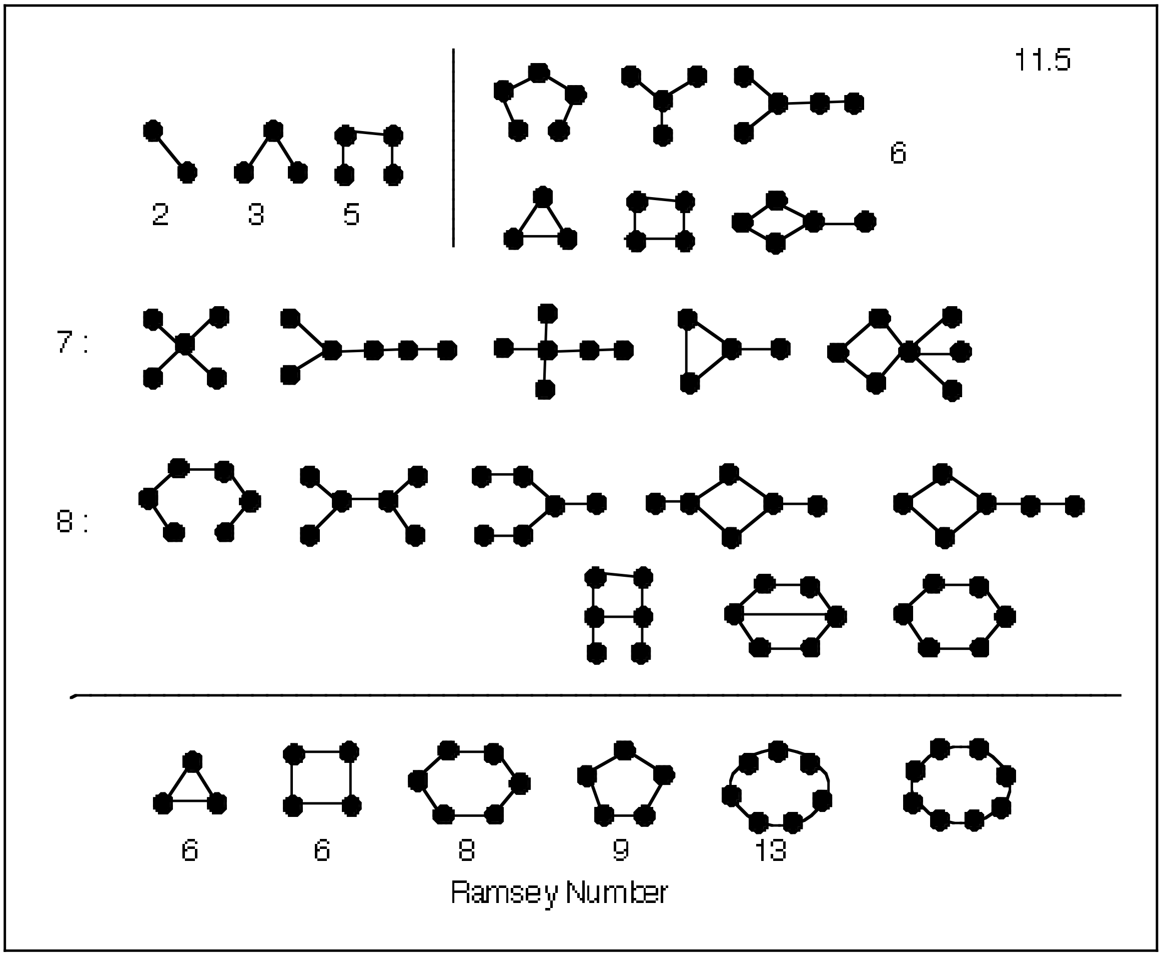


Fig. 11.3: Upper panel: the connected graphs associated with the diagonal Ramsay numbers 2 to 8. Lower panel: ring graphs ordered by their Ramsay numbers. Note position of pentagon.

numbers. As shown in the lower portion of Figure 11.3, the hexagon is "simpler" than the pentagon . Furthermore, in the upper panel, there is an appealing progression of triangles or square subgraphs with tails -- very reminiscent of how cellular animate anigrafs "evolved." As Eddington noted: "We have been walking down a freshly washed beach and have come upon a set of footprints, which, upon more careful inspection, we recognize as our own."

11.5 The Unattainable Self

The transcendent nature of beauty provides a model for a being's sense of "wholeness". The anigraf argument is based on a mental organisms having preferences for the next state of the system, rather than simply choosing a local action. The sharing of such preferences among other organisms creates a host of mini-models for various aspects of the system's activities. When all models are integrated together, one obtains a large, complex model not accessible to any one individual. Any agent-based self-reflection has no capability of seeing "the whole." Nevertheless, for successful communications, each agent or mental organisms needs to understand to some degree the variations on its own model that are held by neighboring agents. If these variations "make sense" as an extrapolation of or a revision of an agent's own model, then this knowledge can provide glimpses of a more complete whole that will transcend one's own immediate knowledge. The "whole' is there, but unattainable. The situation is quite analogous to "beauty."

A natural question in this era of computation-based modeling is whether a sense of conscious self can be created in a machine. Certainly all that has preceded in anigraf design can be embodied in an information sharing device built from non-biological components. Obviously, there will be enormous problems as to how each agent acquires the goals of the system of which it is a part, as contrasted with the much easier task of learning simple, local, vehicle-like actions. But assume this formidable learning task is mastered. Then would such a machine experience self-awareness? Again, if components had adequate models of agents with which they interacted, then we can envision programs that would compare model similarities and be capable of extrapolating further beyond these immediate models, just as was done for metagrafs and beauties. This ability would have to reside within each (or many) agent systems, giving each agent system a sense of a whole (by definition) transcending their immediate knowledge. Thus, a sense of self has to be tied closely to collective consciousness as embodied in any anigraf-like entity.

The rub is that for complete self-consciousness, each agent needs knowledge not just about its neighbors, but also about all participants. In other words, each agent needs a complete model of the system as a whole. As systems become more complex, clearly the capacity to integrate all agent viewpoints diminishes. Unless agents themselves become exponentially more powerful knowledge structures, the sense of "self wholeness" must diminish. Except for very limited or simple systems, a complete self-awareness is unattainable. Beauty presents a similar dilemma.

A Chart of a Morphodynamic Trajectory Manifold
(adapted from P. Gunkel)

12.0 Epilogue

Anigrafs explores the consequences of viewing living creatures as a society of agents, whose beliefs and preferences have relationships captured by a graph. The intent is of this abstraction is to open new windows to understanding mind, revealing computational complexities, relationships and structures not previously considered. Although many anigraf behaviors may resemble or mimic earlier servo-mechanisms, this similarity should not obscure the fact that anigraf designs are not *vehicles*. Rather, the latter are at best, simply an intrinsic, less conscious part of the anigraf's behavior. Simply put, Anigrafs should be regarded as reflective, not reflexive.

To sharpen this distinction, consider that: " mind is to a brain as time is to a clock" (Harrison, 1998.) Clocks have many different forms such as atomic, spring-wound, pendular, etc. Yet all serve the same function, namely to keep time. Similarly, just as we have many forms for clocks, so brains may differ widely -- especially as silicon, carbon-based, and perhaps even quantum-based machines are further developed. Despite these differences, all are seen as "controlling" behaviors of encapsulated entities. It is the observation of this behavior that gives brains meaning. Just as clocks are meaningless without a notion of time, so brains need a notion of mental activities that observers (including the creature itself) can recognize and typically utilize.

The reality of "mind" in a society is no different. Observations of behaviors are related by models and customs that shared by the community. The evidence is that useful communications and cooperativities result, together with a sense that one's actions have one final, common cause, and not simply a hodgepodge of reflexive activities. This implies an underlying set of constraints that enforce a non-random structure to mental activities. Just as in physics, there are constraints on atomic structures that can be used to organize a "periodic table", so one should seek a similar (context-sensitive) structure for mental models, both of ourselves and of our societies. Anigrafs is an attempt to take some steps in this direction. The goal is the development of more powerful natural and social models. Here, the highest abstract form of these models is, of course, a Metagraf. Like any model or theory, the validity of this approach will rest in the useful artifacts or new thoughts that result.