Anigraf8: Alliances
coordinating diversity
8.0 Heterogeneity

Consider a society of identical anigrafs. With such homogeneity, we can expect accurate, intelligible communications among members of the society, but at the sacrifice of an ability of the group to see the world from different vantage points. Behaviors will also be very predictable. Clearly, the flexibility and adaptability of a society depends upon its members possessing a range of talents to execute a host of different tasks. Successful models for one situation may be quite inappropriate for another. Fortunately, given even minimal environmental pressures, we find that offspring are not carbon copies of their parents. As models grow to deal with new events and challenges in the world, the offspring’s view continues to depart more and more from that of its parents. Diversity evolves, even in the presence of a common core. We have, then, the evolution of a colony with shared goals, but with members of different talents. Examples include insect societies, football teams, or a jazz combo. Within each, the members become united into one entity through an almost unconscious bond. How different may any two members be, and still experience such a bond? The answer sets the stage for studying how a population of anigrafs with different models may still be able to form productive alliances.

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same goals, then their choices will be the same only if their graphical forms are the same. An opposite extreme would be two anigrafs with the same goals, but with different preference orderings derived from a completely different model. To illustrate, Fig 8.1 shows two graphical models that are complementary: each are subgraphs of K4 and together form the complete graph. None of the preference orderings agree. Hence, even given the same set of weights on alternatives, the final choices would almost always differ except in special circumstances. Our aim is to have groups that engage in coordinated actions, yet have a diversity in anigraf forms.

To capture our intuition about similarity and dissimilarity (or equivalently, diversity), we can measure the fraction of agents in both anigrafs who have identical preference orders. This measure automatically takes into account dissimilarities introduced by differences in goals.

Definition: The similarity (overlap) between two anigrafs An and Bm of degrees n and m is the fraction of all (n+m) vertices that have the same goals (alternatives) and the same edges to identical goals.

The patterns in Fig 8.2 exhibit pairs with similarities of 1/3rd. To generate these anigrafs, the edge set of the first was chosen with probability 1/2. A core for the second anigraf was then created by choosing at random 1/3rd of the vertices, with their edges preserved. The remaining 2/3rds of the vertices were then assigned edges with probability 1/2 using the reduced edge set. For these small graphs, we thus expect on average, a 50% overlap in the edges associated with the four remaining vertices \{C,D,E,F\} or \{R,S,T,U\}. Structural
similarity for anigrafs is thus dictated by vertex similarities and only indirectly by the edge sets.

8.2 Overlap vs Outcomes

As the overlap between two anigraf decreases, so will the percent of outcomes that are the same. To explore this relationship in detail, generate a set of random graphs \( \{A_i\} \) with \( n \) vertices and with edges chosen with probability one-half. For each of these graphs we now revise a fraction, \( f \), of the edges to create a new set of graphs \( \{B_i\} \). For large \( n \), we find that the chance of \( A_i \) and \( B_i \) having the same Condorcet winner is inversely proportional to the structural dissimilarity between \( A_i \) and \( B_i \), as shown by the diagonal line in Fig 8.3. (See appendix for a formal proof for large \( n \).)

One might ask whether all tally procedures will generate a result similar to Fig 8.3. The answer is no. If the agent (node) with maximum weight is always chosen as the winner, then all curves will be flat at 100% regardless of overlap. Hence any similarity and dissimilarity measure must incorporate information about second or higher order preferences. Also critical is the
knowledge depth used to determine an agent’s preference orders. If the knowledge depth is increased, the results of Fig 8.3 will change for small graphs. The reason is because deeper preference rankings will now include relations for vertices previously chosen to have fixed edges to their neighbors. Consider the right pair of anigrafs in Fig 8.2. If knowledge depth is increased from one to full, then the preference order for P changes from \([P>R>Q,S,T,U]\) to \([P>R>Q\sim S>T\sim U]\). Hence the Condorcet (and Borda) tallies will be change, subject to the particular way the second anigraf is augmented. The exception will be for graphs with diameter two. (See also appendix on how outcomes are affected when local inconsistencies are introduced in preference rankings.)

8.3 Coordination and Diversity

Our objective is to create a diverse group of anigrafs that still maintain bonding and coordination through shared goals and preferences. But figure 8.3 presents a dilemma: as graphical forms become more diverse, the odds for a pair of anigrafs having the same winner go down at the same rate. How can we optimize both at the same time?

Although the linear relation between common winners and diversity implies only one underlying variable, this is not the case. There is another variable obscured by the deceivingly simple figure. Absent is just how tightly the common winners are distributed near the reported mean. The cross-hatched envelope presents such data, showing one standard deviation of variations from the mean for anigrafs of size 24. We now use this variation to advantage.

Let us repeat the initial experiment where \(A_i\) and \(B_i\) have different graical forms, but now also compare winners with a third, \(C_i\) or fourth \(D_i\) variation of \(A_i\), where all three variations have the same dissimilarity measure with respect to \(A_i\). Using Fig 8.3, we can now estimate the probability that at least one of the pairings with \(A_i\), namely \(A_iB_i, A_iC_i,\) etc. will have the same winner. This pairing probability gives an indication of the strength of bonding for the group. (Pairs \(B_iC_i, B_iD_i\), are correlated, and hence excluded from the
estimate.) Multiplying this bonding probability times the dissimilarity of form, gives a measure of “coordinated-diversity.” Maximizing this product yields the optimal balance between coordination, or group bonding, and diversity.

Fig. 8.4 shows this “coordinated-diversity” product for small groups of size 2, 3, and 5. The maximum moves towards a two-thirds dissimilarity of form, based on simulations using random graphs with edge probability of one-half. An appendix presents a more rigorous information-based treatment. Thus we estimate that an optimal diversity among members of an anigraf population is when similarity between pairs is about one-third.

8.4 Clique Formation

Within any population of anigrafs, benefits to some can be gained by forming subgroups, or cliques, where members agree to vote as a block. Obviously, members of a clique will resemble each other and will have strong
similarities in preferences and choice of winners. The principal constraint on clique formation is thus the anigraf form itself, not the super-set of these forms that represent all relations in the world they inhabit. For example, if the superset were Kn, then all anigraf forms of size n or less would be possible. Thus, constraints and regularities created by Mother Nature gain their greatest effect not from any global model for Nature herself, but rather on the designs she imposes on her inhabitants.

To show the important role anigraf form plays in clique formation, again consider random anigrafs with edge probability one-half. From Fig 8.3 we know that even for anigrafs of size 12, there is only a 10% chance that any two anigrafs will have at least one identical goal with the same preference orders (to knowledge depth one.) In such a population, the clique size will be small (relative to the population size), and conversely, the number of cliques will be large. However, if we were to increase the edge probability toward one, then the anigrafs will become increasingly more similar, eventually all being identical, forming one large clique. This effect of edge probability is shown in Fig 8.5.

![Graphs showing the effect of edge probability on clique size and number of cliques.](image)

In the right panel, for edge probability one-half, almost all individual anigrafs are isolated, and the maximum clique size is very small. As edge probability goes to one, clique size increases to the size of the population (20).
Along with this trend, as shown in the left panel, the average number of cliques with at least two members will increase, except at the limiting value when the network is completely self-connected.

The surprise is when anigraf edges become sparse, with edge probability going toward zero. Now maximum clique size increases again and more and more cliques appear with at least two members. (In the limit for large graphs, a conjecture is that this distribution is very modal, with asymptotes of Dirac spikes at edge probability 0 and 1.) This multi-modal distribution is a key to the formation of Social Networks. If a population is to consist of dissimilar “random” anigrafs, then the division of this population into cliques of any significant size will require sparse anigraf designs.

Definition: Members of a **minimally compatible clique** will have in common with at least one other member at least two goals with identical level-two preference orders.

Thus, any anigraf who is a member of a minimal clique, will be seen as a node of at least degree two in the clique network, or, equivalently having at least two edge-similarity links to at least one other anigraf within the clique.

When random graphs are constructed to have small edge probabilities, they begin to resemble trees with additional edges between branches. Even more sparse are chains or rings, with vertices of degree no more than two. However, if the anigraf population consisted entirely of rings (or chains) \( n > 3 \) with vertices labeled consecutively, then all anigrafs would have at least two vertices in common, namely 2 and 3. Hence a collection of ring anigrafs would form one big, minimally compatible clique, and would be of little interest. Trees, then, provide a better form to explore the interplay between clique formation and connectivity within a population. To ensure at least some connectivity throughout the entire social network, however, we choose a special form of anigrafs with “heads” consisting of a ring with chords, plus one or more limbs that have the form of a random tree graph. (See Fig 8.6 for choices.) What then, will be the distribution of cliques in this population?
Which of the many forms will tend to affiliate, and which will be isolated without significant group support?

8.5 Social Networks

Definition: A social network is a set of loosely connected, minimally compatible cliques, with each member of the population belonging to only one clique.

By “loosely connected”, we mean that the density of edges between cliques is much less than the density of the links between members within a clique. To insure this tighter connectivity within a clique, a second constraint is imposed on anigraf forms, namely that their cliques are compact.

Definition: A compact clique consists of anigrafs who share the same complete (labeled) subgraphs.

We are now in a position to explore some aspects of anigraf social networks. The principal claim is that populations of compact cliques of sparse anigrafs of different sizes will tend to form scale-free networks. Such networks are very prevalent in Nature and Social systems (Barabasi, 1999.)

To test this claim, colonies of 50 anigrafs of sizes 6, 8a, 8b, 10 and 12 were generated, with heads fixed as shown in Fig. 8.6, but limbs and fingers added as tree graphs. Note that the head designs assure the formation of some compact cliques, whereas the trees add sparse vertices that support similarity relations between different types of anigrafs. New vertices for the limbs are added to previously connected vertices, chosen with equal probability.

For convenience, the different species are identified as types I, II, III, IV, and V. The nodes for each anigraf were labeled, proceeding clockwise around the head, then continuing to the limbs. Hence all heads of the same species had identical labels. Limbs were always added to the same nodes. The head design insured that the social network would be connected with at least one link. However, because a two (or more) link criteria was imposed for cliques, many anigrafs were left isolated outside the network.
Some of the more common clique members
The chart in Fig 8.6 shows how the different types of anigrafs came together into cliques, and the resulting social network. At the top, where the cliques are minimal, we seen that members of type I can find strong affiliations with anigrafs of type II and III, but not IV and V. Similarly, anigrafs of type II and III can find strong connections to type IV anigrafs, but not type V. Of greater interest is the structure of the network as stronger affinities are required (i.e. more links.) Obviously, this will reduce the number of members in a clique, as well as weakening the connectivity between cliques (dashed lines.) When four links are required for our small population, on average, only eight type I anigrafs survive as pair-wise members of four cliques (as shown in brackets.) The remaining types of anigrafs continue to coalesce into their maximal clique sizes of 10. As more links are added, slowly the smaller anigrafs drop off, because the number of possible similarities becomes exceeded. With eight links required for a member of any clique, only the large anigrafs of with limbs having six nodes have a decent chance of forming a group.

Fig 8.7 shows that the distribution of clique size for this anigraf population follows a power law, with an exponent roughly $1/e$. Here we have,
then, a scale-free social network with small world properties (Watts and Strogatz, 1998) This means that there are few cliques of large size, increasingly more cliques of smaller size, with all cliques only loosely connected. The cluster coefficients are about 0.8. In other words, the cliques have an internal connectivity that is 80% of that comparable to the local complete graph. It is the form of the limbs and fingers that undoubtedly leads to this network property, where “features” to be linked are associated with vertex degrees. Vertices of degree one are easier to link, those of degree two next, etc. Hence it is no surprise that a set of 50 sparse anigrafs of size 12 with edge probability of only 1/6 also will be linked as a scale-free network – albeit there are many fewer links between members. (A maximum of 5 compared with a maximum of 10 for pairs of head+limb anigrafs.)

The difference in connectivity is also reflected in the diameters of each of the different networks: with even one link, the set of 12 vertex sparse random graphs with edge probability 1/6 will have a diameter of 4; whereas the diameter of the set of head+limb anigrafs is 2. For the anigraf colony, other diameters are 4 and 6.5 for 2 and 3 similarity links respectively.

8.6 Cliques as Anigrafs

Obviously, any clique can be seen as a complex anigraf in itself, with the clique members being the constituent agents. But for a society of cliques to be integrated, edges between all cliques are necessary. In addition, we have the problem of assembling the votes within one clique, and then passing the consensus on to obtain the social outcome. Presumably, weights on this second stage of voting will be proportional to the size of the clique. Note that such an aggregation procedure need not reflect the wishes of the society that directly counts one vote for each agent. Given this new social entity, an intriguing problem is to insure that meaningful communications take place between cliques – or the representatives of any clique. Here, severe constraints become imposed on clique representatives. On the one hand they must understand the diversity within their group, but on the other should have strong similarity relationships to representatives of other cliques. Clique leaders will not be your average anigraf.