3. Anigraf2: Swimmers
beginning to move
3.0 Movement

Cell Anigrafs had very limited behaviors, constrained for didactic purposes to simple ring forms. This simplification hid the potential complexity of designs and behaviors that could be attributed to such primitive life forms. Here we begin to expand this repertoire. The component agents will have access to different kinds of interfaces with the environment, with interface hardware that affects the behavior of the system as a whole. Agents will have control over those additions, and will “vote” depending upon the strength of its desire to achieve a preferred goal. In effect, then, we are formalizing a two tier system: one where the interface agent constitutes a member of the social system of all other agents, and another, lower level where each agent controls an effector, but only when approved by all other agents.

One of the most visible and prevalent effectors are flagella or hairy muscular fibers. For example, the single-cell Euglena has a flagellum that waves to pull the creature forward; another simple creature, the paramecium, has a host of hairy superficial fibers that can generate wave-like motions. These appendages can be entirely passive, with the movements initiated only at the point of attachment to the cellular body (See Appendix 1.) More complicated limbs may have additional agents confined to “local” social enterprises that control special sinusoidal motions, such as the movement of wings, fins, or legs. These are precursors to hands that become capable of still more complex actions, especially with the addition of claws or fingers. We begin first, however, with a description of an anigraf form that can generate chaotic-like flagellations typical of coelenterates. This little society has only one primary desire: to move.

3.1 Jellyfish
Coelenterates have very primitive neural nets that lie on the rim of a funnel-like body. When activated, this network causes a wave-like motion of tenacles attached to nodes on the rim. The jelly fish is an example. It has an
inverted cup-like form with many tenacles emerging from the rim of the cup. Typically the neural elements embedded in the ring appear in multiples of four. Each neuron drives its own tenacle, causing it to flagellate. Our objective is to retain the ring-like anigraf form, at the same time maximizing the likelihood of cyclic outcomes for a wide range of voting strengths among the agents. The longer the cycle among agents, the better, for then we achieve the greatest flagellation and potential for movement.

For bilateral communication with neighboring agents (e.g. rim nodes), the probability of cycles was shown previously to reach a peak at 8 - 12 agents. Let us then consider design possibilities for an octagonal jellyfish anigraf, considering that to satisfy other desires, we might also prefer a single, non-cyclic outcome. An example might be the coordinated constriction of the rim of the coelenterate as it captures a food particle.

Figure 3.1 shows a few modifications of the ring, and the probability of top cycles among outcomes using a Condorcet tally. If the ring is reconfigured to “wheel”, then the anigraf has one vertex that is adjacent to all others (i.e. the graph is “covered”) and there will never be cycles (p=0.) The next configuration is a regular graph, which has a top-cycle probability roughly half that of a simple ring. For bidirectional communications, the pure ring has the maximum likelihood of top-cycles of about 12% if voting weights are chosen from a uniform distribution. Unfortunately, this percent may not be high enough to insure that our jellyfish can easily engage in complex, long-cycle flagellated movements.

A simple modification of our communication channels solves this problem: make the communication channels directional. In other words, let
preferences be “passed on” from one agent to the neighboring agent, but not vice versa. Depending upon the directionality of the communication channel, we also will have the great advantage of being able to enforce either clockwise or counter-clockwise cycles among the rim agents. For the jellyfish with a ring of eight agents, clockwise directed communication will produce clockwise cycles 60% of the time; for a ring of twelve agents, the cycle probability rises to 80%. These numbers are approximated very closely for n>4 by cycle percentages shown earlier in Fig. 2.5 for random preference orderings. (See also Fig. 3.3.) The difference, however, is that now the (directional) internal model, $M_n$, is respected by all agents. Clearly we have the beginnings of a social control system that mimics a physical feedback controller, but here the condition of sharing social preferences is met.

Cyclic flagellation is but one of several actions that the jellyfish might engage in. Even if the agents all have identical effectors, all effectors need not perform identical movements, and consequently, all agents need not have identical goals for the system. Some agents might prefer that the jellyfish move to another, more favorable region in its environment (more “food” or perhaps “less hostile”). Another might want to ingest nutrients, another might wish to release a toxin. As in our primitive cell anigraf, some of these actions might entail a change in shape -- perhaps to be carried out by activating other internal fibers or membrane properties. In a very simple scenario, we could define a unique “zero” agent position on the coelenterate’s ring network, where even numbered agents would create an inward current whereas odd numbered agents would create outward currents, or tighten the membrane supporting the ring. All these choices entail societal consensus that is reached after conducting the Condorcet tally among the agents. Hence the coelenterate ring of agents can easily be regarded as a very primitive brain.

3.2 The Flagellum

Consider next yet another primitive creature, such as the sperm cell or Euglena, where a single flagellum controls movement of a simple, cell-like “body.” If this appendage itself is passive, driven by forces applied to its
base, the effect will be as if someone were whipping or twirling a cord. Such a cord will have some kind of natural frequency of motion when forces are applied at one end. Let the particular agent who energizes the flagellum also be sensitive to any external forces applied to this appendage. In other words, if the motion becomes more difficult to execute, this will be noted.

![Symmetric and Asymmetric Flags](image)

Our anigraf creature now appears as a ring of agents, which comprise the “body”, plus a flagellum attached to the body agent A. (See Fig. 3.2.) Let the communication channels be bi-directional in either pentagonal or hexagonal bodies. We know from Table 2.3 that if voting strengths of the agents are suitably chosen, cyclic outcomes will occur that will include agent A’s preferences for system behavior -- to initiate movement by flagellation of the “tail.” We might also expect the rate of flagellation to be dependent, in part, on the body size, because larger bodies will require larger flagella. Longer periods for flagellation in turn may require more agents to participate in the cycle. In other words, for an octagonal ring of agents, A - H, one possible cycle might be A -> C -> D -> F -> H -> A. If the tally machine counts votes at fixed time intervals for anigrafs of similar species, then clearly we can change the time between the recurrence of outcome A simply by increasing the length of the cycle, which in turn will depend on the size of the ring. The downside, however, is that
the selection of proper voting weights becomes more difficult for the longer cycles, and the cyclic pattern is more susceptible to small changes in these weights. Hence we consider only small ring networks.

As previously mentioned, rings with odd or with even numbers of agents have different cycle characteristics. For bodies with odd numbers of agents (and \( R = n \)), any triple of agent combinations can exhibit cyclic behavior. For bodies with an even number of agents, however, the cycles must skip at least one agent. Specifically, with an even number of agents, the flagellum can be activated only if one of the neighbors to the tail agent is not in the cycle. So in Fig 3.2, a natural triple for a hexagonal anigraf would be ACE, which activates symmetric locations on the body. Presumably the symmetric activities would leave the body itself undistorted, and the flagellum activity would propel the body forward (or backward.) On the other hand, if asymmetric triples such as ABE constitute the cycle, and the active agents B and E also cause "body" movements such as a contracting of a portion of the membrane, then one might envision this simple Anigraf as performing left or rightward movements for stereotyped flagellum motion.

For anigraf rings with an odd number of agents, a wide variety of triples becomes possible. Consider the pentagonal anigraf with tail in Fig 3.2. For bi-directional communications between agents, there are two cycles of interest: EAB or DAC: one places the flagellum at the “head” of the ring, whereas the other is at the “base”. Ignoring the other possible cycles (e.g. BEC, BED, CDE,....) which do not include the “origin” A, a simulation shows that a flagellum placed at the “head” has a 10-fold greater possibility for activity than when it is at the “tail.” It is easy to envision that this difference in cyclic choice could govern whether the body is pulled forward (from the “head”) vs propelled forward (by the “tail” activity.) Euglena move by the flagellum “pulling” the cell, whereas sperm do the opposite. The main point is that social networks can exhibit not only species differences, here characterized by the even vs odd number of ring agents, but also variations within the species, here characterized by the location agents who control the single flagellum.
3.3 “Smart” Tails

To create creatures with more complex locomotor abilities, we can augment the flagellum by adding agents to the flagellum itself. For example, we might desire to increase the amplitude of a sinusoidal wave of a tail, or its frequency, or bias its angle to change heading. These effects can not be accomplished easily by a single, simple agent at the point of attachment of a passive appendage. "Smart" tails can alter their movement under higher-level command. These possibilities can be explored by representing the tail as a chain of agents. (See Fig 3.0.)

A chain is a simple graph consisting of a string of nodes -- i.e., a broken ring. The top cycle behavior of chains are thus very similar to that of pure rings. As before, let each node represent an agent, with the edges depicting which agents communicate preferences to one another. Consider first a simple 5-chain with agents a-e, as illustrated in the top panel of Fig. 3.4. Our objective is to create a wave through at least one part of the chain, such as when a fish swims or a snake crawls. We assume that the agents are embedded within a flexible shell that contains contractile tissue such that when an agent is active, the muscular springs will contract to create part of a
wave motion. A cycle among agents, say agents \(a, b, c\) or \(a, b, d\) can drive this kind of behavior.

It is easy to show that if there is no sharing of information between agents, then there can be no cyclic behavior. Intuitively, if all agents are independent, then in a pair-wise competition among choices, the agent with maximum clout will win. Similarly, although less obvious, if all agents share their current preferences with each other \((R = n)\), then again there can be no cyclic behavior (Black 1958.) Cycles among agent choices are most likely when information is shared only among immediate neighbors. Furthermore, like the jellyfish, cycles will become even more likely if the communication channels are directional. We begin first, however, with the more socially conscious case where the channels in the chain share information bilaterally between the neighboring agents. We will also use bold lower-case letters to distinguish tail agents from body agents.

To generate an \(abc\) cycle, we require that the Condorcet comparisons between agent \(a\) and \(b\) favor \(a\), those between \(b\) and \(c\) favor \(b\), and lastly, that \(c\) will be preferred over \(a\). Inspecting the preference orderings for the agents, and assigning weights \(v_i\) to each, we see that

\[
\begin{align*}
    a \ vs \ b &= v_a - v_b - v_c > 0 \\
    b \ vs \ c &= v_a + v_b - v_c - v_d > 0 \quad (3.1a) \\
    c \ vs \ a &= -v_a + v_c + v_d > 0
\end{align*}
\]

and to insure that agents \(d\) and \(e\) will not dominate the outcome, we require:

\[
\begin{align*}
    a \ vs \ d &= v_a + v_b - v_c - v_d - v_e > 0. \\
    a \ vs \ e &= v_a + v_b - v_c - v_d - v_e > 0. \quad (3.1b)
\end{align*}
\]

To further simplify our example, let \(v_e = 0\) and hence \(b \ vs \ c = a \ vs \ d\), leaving us with only four constraint equations in four unknown weights. (Note that setting an agent's voting power to zero does not guarantee that agent's choice will NOT be chosen -- an example will appear later.)

We now have a 4-dimensional space which will be cut by half-planes, one corresponding to each of the linear constraint equations. Any particular cycle will occupy a region bounded by such planes. If all the \(v_i\)'s can occur
each with uniform probability over the interval 0 - 1000, then a simulation shows that the region occupied by an abc cycle is 0.02 of the total (i.e. this is the probability of an abc cycle.) Hence, for all practical purposes, our chain anigraf tail can't move. However, if we impose the constraints (3.1), then we can find a small portion of the 4-dimensional space that generates abc cycles. For example, the set \{v_a, v_b, v_c\} = \{ \} always produces cycles. Following a similar analysis, an abd cycle or an adec cycle can be generated by the set \{ \} or \{ \}. [Note: probably should delete equations and re-write previous two para with text only to give idea.]

3.3.1 Three-state weight space.

Allowing agent activities to assume arbitrary values over a range, say 0 - 1000, seems far too generous. Most neurons, for example, typically have a modicum of spontaneous activity near zero, and then fire near their maximum capability. Here, a rough equivalent would be to allow the spontaneous activity to fluctuate between 0 and 100, and the maximum activity at some much larger value, say 500 to 600 -- whatever the proper upper bound might be for a device with 0 - 100 noise level. We have, then in effect a "two-state" agent who is either "turned on" or is "off." With such a constraint on weight activities, there can be no cycles for our 5-agent chain. The proof is simply to run a constrained maximization program and find that no weights satisfy equations 3.1. Alternately, we can test the equations with w = x or y or z and show no solution. Hence to generate cycles in a chain, we need at least three weight states.

Again, we run a constrained maximization program to search over the 4-dimensional space to see which values of v_a, v_b, v_c, v_d can generate abc cycles. The weight states are now fixed as follows: a "noise level" of 0 - 100; a mid-level mean weight chosen from the 200 - 800 range with a spread of +/- 100; and highest level weight from 200 to 1000, with a spread of 50. The smaller spread for the large weights would correspond to (neural) activities near saturation, and hence would have a more limited dynamic range.
To achieve \textit{abc} cycles with three weight levels, we found that $v_c$ needed to be "near zero" and hence was assigned a value chosen randomly from the noise level range. We also found that $v_a$ could be set equal to $v_d$ without a significant reduction in cycles, thus allowing us to use a 2-dimensional plot to show the effect of weight choice (Fig. 3.4.) In this plot, the contours with numbers indicate the probability of an \textit{abc} cycle. Hence weights in the noise level yield \textit{abc} cycles only 2\% of the time (lower left), whereas if $v_a = v_d = 700$ and $v_b = 300$, then over 90\% of the time there would be \textit{abc} cycles. In principle, it is possible to achieve cycles with 100 \% probability by further reducing the upper range of the top weight. In other words, if agents \textit{a} and \textit{d} both shouted at their maximal capability, say 800, with essentially no variance, then \textit{abc} cycles would almost be guaranteed. However, as one increases the range of the top weight, then the cyclic behavior becomes more sporadic -- such as when a coin is flipped and one obtains sequences of tails of varying length.
3.3.2 Directed Networks

Although weights can be found that will generate cycles in a bidirectional anigraf chain, the rather limited portion of the weight-space available is crippling. Even if we allow v/e to vary, values near zero yield the greatest number of cycles. We can resolve this problem using directed networks. But this solution comes at a cost: directional networks will lack the full social awareness of bidirectional communication channels which allow neighbors to fully share preferences for the state of the system. In our “social” system, they thus have lower-tier or second-class citizenship.
The front panel shows a few directed and undirected anigraf forms, where the arrow on an edge indicates a directed preference relation. Hence, for a simple five-chain, agent \( b \)'s preference orderings will be \( b > a, c \) in the undirected case, and \( b > c \) for the directed network. Because a chain is simply an open ring, the ring results in Fig 2.4 still apply to first-order; these can be compared with Fig. 3.3 which shows actual top-cycle probabilities for chains. Note that the probability of a cycle for the five element flagellum (chain) goes from about 8% to 50% with the directed channels. However, we can be more precise about the range of weights that can generate cycles. Referring to Fig 3.5, consider the entire three-dimensional slice of the 5-dimensional weight space. Recall that the region where undirected channels can yield cycles lies near the bottom plane where \( v/e = 0 \), and occupies only one-half of this region. In contrast, the directed anigraf network fills out most of the remaining half of this volume, thereby accounting for the six-fold rise in the odds of (abc) cycles. Cycles are thus easy to generate in directed chain networks.

3.3.3 Extended chains.

The shortest active bi-directional chain has only four agents. A directional chain can be as short as three nodes. For the \( abc \) cycle in a 4-chain, activating agent \( a \) will trigger the swimming motion. As we add more elements to the chain, we can effect two different kinds of outcomes: (1) we can make it easier to generate cyclic behavior at some point in the chain or (2) we can increase the complexity of the cycle. For example, as previously mentioned for rings, an 8-chain Anigraf “worm” can have a 5-cycle wave \( abdeca \) (or whatever.) -- but at the expense of having to choose from a more restricted set of weights. The more sophisticated the tail action, the more restricted will be the allowable range of weights. For simple swimming, random choices of weights should be sufficient. For more sophisticated, powerful swimming, we are faced with an optimization problem that trades off restrictions on weights to generate cycles of interest versus power produced by having many coordinated waves in the tail. Later, we will see that chains with “limbs”, i.e. tree-graphs, can also be used to produce variations in cyclic behaviors.
3.3.4 Controlling Cycles

We would obviously like to endow our swimming (or crawling) anigraf with the ability to turn on or off the undulations at will. Two schemes have already been noted: simply shut the voting weights down (or equivalently assign equal weights to all agents), or eliminate any directionality in the network. More desirable, however, would be to have one agent invoke the undulating motion of the tail, and leave the tail agents alone, perhaps allowing them to control rate, or cycle length, etc. For a pentagonal anigraf, the natural choice for the "on/off" agent would be agent A, located at the end of the chain where it is attached to the body. If this agent adds its maximum clout to body agent E, then the ABE cycle is shut down. Alternately, we can also set the voting power of body Agent A to zero. It is thus easy to turn the tail activity on or off. If desired, we could also reverse the effect of activating agent A, so that the cycle occurs only when body agent E has significant input. This requires another regime, which occupies another portion of the weight space whose three dimensional slice will resemble Fig 3.5, but with different axes.

3.4 Brain Types

An obvious extension to swimmers (or crawlers) is to modify the neural network at the “head” end of the flagellating chain. In Fig. 3.6 we illustrate an anigraf with a fork added to the head end of the 5 chain. Other possible modifications are also shown. First consider the fork. The notion is that activation of the left (G) or the right (F) member of the fork will tighten a muscle and cause the head to turn. Let this change in “head orientation” happen in the presence of the abc cycle which is driving the anigraf forward. Hence we need either an \textbf{abcG(F)} cycle or a local \textbf{abc} cycle in the presence of a dominant G activation. Both can be easily achieved, but as the latter seems more appropriate, we explore this with an example.
To set up a dominant $G$ agent in the presence of an $abc$ cycle, we require the Condorcet comparisons as follows:

\[ a \text{ vs } b > 0; \quad b \text{ vs } c > 0; \quad c \text{ vs } a > 0; \quad G \text{ vs } a > 0; \quad G \text{ vs } d > 0; \quad G \text{ vs } F > 0. \]

hence we have six equations in seven unknowns. We also require that the agent weights take on one of three levels: noise, middle, saturated. One regime that satisfies these conditions is \{add weight example here\}.

It is easy to imagine other kinds of behaviors that might be associated with different “brain types”. A three-fork at the head end could activate not only turning to the left or right, but also perhaps local lunging by the middle fork in the presence of the abd swim. Such activation weights correspond to lunging forward to grasp a prey. More complex ring networks might provide potential for agents with sensory apparatus that triggers either body or tail movements once the adequate stimulus is sensed.

For example, one obvious manipulation of the “head” is to add edges that join symmetric agents/nodes. In a Hexagon Anigraf (see frontpiece), adding the edge BF will affect the preference orderings not only for B and F, but also C and E as well. Consequently, the cyclic probabilities change because agent’s preference orders are altered. If the edge BE is added instead, bisecting the Hexagon, then we have a symmetry between ABC and FED, and we obtain still another behavior. Surprisingly, this latter addition will prohibit cycles, whereas the first increases the cyclic possibilities.
Yet another modification, one perhaps useful for very large rings, is to add edges that create local bridges, as in regular graphs, or distant bridges to create "small world rings." Each has quite different cyclic behavior. We will return later to the relation between graph structures and cycles. Still another modification would be to have two directional rings as illustrated in the front panel, with bridges between pairs of nodes on each ring. Turning rings on or off could cause opposite cycles, or constriction, etc. But even with such more complex “brains”, these swimmers or crawlers can still move only with wave-like movements of their chain-like tails. The specific “brain type” will have little influence on this coordination. Hence, when all is said and done, these anigrafs have severe limitations on their movement abilities. Any “intelligence” with respect to the coordinated movements lies largely in the manner in which agents in the chain interact. The more restricted the weight ranges, together with the use of directed edges, the greater the movement abilities. Agents comprising tail appendages are thus definitely lower class, with limited social consciousness.

3.5 Cooperative vs Competitive Networks

Anigrafs with directed, rather than with bi-directional communication channels marginalize one of our key assumptions about social awareness. Because information is passed only one way and not back and forth, the sharing of knowledge and preferences within the system is much more limited than systems with bi-directional channels. As a species, anigraf creatures with directed networks lie somewhere between a fully aware (bi-directional) anigraf and Braitenberg’s more reflexive vehicles.

To see this connection, first consider an analogy to excitation and inhibition at the level of anigraf preferences. Refer again to Fig 3.3. The upper-most curve shows that directed anigraf networks can be very unstable, in the sense that top-cycles are prevalent. By analogy, this behavior is similar to what one might find in a network with positive, rather than negative feedback.
One reason inhibitory feedback typically produces stable, non-oscillatory behaviors, and hence is used for controllers, is that activity can be driven toward zero. So to “control” the presence of top-cycles in directed anigrafs, we could likewise consider driving the exchange activity down by introducing a competition between some agents (whereas before we only used cooperative exchanges.) Indeed, as the exchanges between agents become more limited, the probability of top cycles decreases and can be driven below the bi-directional result shown in Fig 3.3. (The inhibitory case plotted shows the attenuation effects, but not the limiting result.) The more agents whose clouts have been reduced to zero, the greater the chance of one agent getting its way. Our directed anigraf has then been morphed into a kind of hybrid between a Braitenberg vehicle and the truly social, bidirectional anigraf. The tally machine remains the same, but the presence of inhibition restricts or eliminates cooperation, and the network becomes more typical of vehicles. The most important difference between anigrafs and vehicles still remains, however: the distinction is the nature of the message that is passed between agents. In a vehicle, two agents (one may be a sensor or effector organ), pass information about one agent’s state through a directed channel. Bi-directionality occurs when the second agent sends information about its specific state back to the first agent (perhaps via another external artifact.) For anigrafs, the messages are at a different level: they deal with the state of the system as a whole. In addition, with directed communication channels anigrafs can not complete their updating of the system’s state without some kind of feedback to those agents from which they receive directed information. The extra step required to update an agent of the state of the system is simply one more feature that moves directional anigrafs much closer to vehicles.