Part IV

Metagrafs

Metagrafs are relationships between anigraf models. The simplest such relationship is the identity, when one graph is the same as another, but the nodes have different meanings in different contexts. More complex relationships between graphs would include how one model may be transformed into another, or similarity relations between non-identical graphs. These mappings can serve as a basis for the classification of types of anigraf models. Explorations of such relationships is relevant to understanding the creative process, when models become reconfigured or augmented. Relations between model classes leads also to notions of complete or “ultimate” models and the ability to comprehend complexity, beauty and self.

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9.0 Representational Forms

Mental models can not be arbitrary. Because the world has structure, so must the models that capture this structure. But structure is also in the eye of the beholder, who sees some relationships but not all, or chooses certain events as being more significant than others. Counting all relationships between objects and events from all perspectives is a hopeless task. For only 8 events, there is the potential for over 10,000 models for bidirectional, symmetric similarity relationships; for 12 events this number exceeds 10 billion. The number of different directed graphical models are yet orders of magnitude larger. (See Appendix.) Constraints imposed by cognitive abilities on the interpretation of world structures can whittle down these numbers, by favoring some classes of models over others. Perhaps most prevalent are tree graphs, which appear as categorical constructs, fractal representations, grammars, models for drainage systems, inheritance heirarchies, etc. Lists or a sequence of causal events would be other examples of very simple forms of tree graphs. Yet even with this short list, contextual variations and representational choices can still bias the interpretation of relationships. Metagrafs describe mappings between such classes of graphs or their representational forms.
Two kinds of metagrafs are immediately obvious. One arises from relations between contextual frames; the other is how graphical models are to be displayed. Fig. 9.0 is an example of the first. The graphs in each row initially appear different due to alignments of edges consistent with their frames. The metagraf is the relation between frames: rectangular, elliptical, “triangular”, and the special symmetrical square and circular frames. The second kind of metagraf is illustrated in Fig. 9.1. Here the pairs in each column have identical connectivities, yet the graphs appear different. The top row emphasizes symmetries, whereas the low row separates vertices into groups (i.e. as k-partite graphs.) This mapping from one group to another is a simple “two node” metagrafical relationship, with the bidirectional edge corresponding to the transformation that takes one group into the other. Loosely speaking, then, a metagraf is a “model of models.”

### 9.1 Graphical Orderings

Graphical models are naturally placed into categories (or classes) that share certain features. Examples mentioned earlier include planar, regular,
bi-partitite graphs, etc. and, of course, trees and rings. Within each category, an ordering can be placed on the graphs. Consider a simple chain, \( cn \). The natural ordering of the chain is \( c_1, c_2, c_3, \ldots, c_n \). This series then specifies the simple “chain metagraph” \( C*n \). Alternately, we could have a hub with \( j \) spokes, \( s_j \). Again, the obvious ordering is \( s_1 = c_2, s_2 = c_3, s_3, s_4, \ldots, s_n \). The same graph may thus be an element of more than one category, as is true of objects in general. We could also enclose the spokes \( s_j \) with a ring \( r_j \) to create a set of “wheels” \( w_j \). (See Fig. 9.2) A limiting form of this set might be the complete graph, \( k_j \), where edges connect all vertices. Each of these separate orderings, \( c_n, s_n, w_n, k_n \) are linear metagrafts. Mapping an element of one of these metagrafts into an element of another is trivial in this case because there is an integer ordering of nodes. The set of these metagrafts thus constitutes a ladder graph, as illustrated. This ladder forms yet another, higher-level metagraf. The two different mappings that create the ladder establish a frame for this graf.

The principal parameterization of the ladder metagraf is the integer sequence; the other parameter is related to the distribution of vertex degrees across columns. It is natural to use these two parameters to characterize the metagraf. More generally, when a single parameter or transformation is applied repeatedly to an object, structure is imposed on topology. The nature of the relationships becomes a defining attribute of the structure, which we use to define a “frame.”

Definition: The (coordinate) frame for a metagraf is aligned with the parameterizations that are chosen to place an ordering on the elements of the metagraf. The origin of the frame is typically specified by identifying a root node.

The parameterizations of a frame can be quite general. In Fig 9.3 we illustrate two further (linear) examples. Both end with a tree structure, but one starts with an unrooted random graph and the other a tiling. Once a root node is specified (as an “origin” for the frame), the metagraf quickly becomes a directed graph.
9.2 Frames and Parameters

All anigraf models rely on two basic relationships: “similarity” and “causality”. These notions were presented earlier (chapter 6) and can be extended to define three useful properties of grafical models: paths, forks, and landmarks. Paths are parameterizations that define the coordinate frame for a metagraf as well as a local frame for its constituents. Landmarks identify possible foci for centering frames.

9.2.1 Terminology.

A path is a causal transformation, rather than simply a single consequence of an action. It is expressed as a transitive construction of two pairs, where the effect of the first state change is carried over to a second, thus creating a sense of continuity along a direction in some event space. (Recall the role of broker agents.) If a path has two or more transformational parameters, then this path may also have branches, which will occur at a fork. The notion of path also clarifies behaviors at intersections. When two or more independent paths cross and share a common element (eg “Y” below), then two or more different
transformations through the crossing point are indicated. The element at the intersection will be a “landmark” because it is a vertex clearly distinguished from others of degree one, two or three. Note that a single causal pair would not convey a path through the landmark, and would leave ambiguous just what the succeeding state would be. Landmarks are natural candidates for origins of coordinate frames. Forks are dislocations that have the potential for refining or revising parameters. These notions will be useful in interpreting graphical models.

Path: \[ X \Rightarrow Y \Rightarrow Z \text{ or also } Z \Rightarrow Y \Rightarrow X \text{ if path is “reversible”} \]

Fork: \[ X \Rightarrow Y \Rightarrow Z(u1), W(u2) \text{ where } u_i \text{ is a parameter} \]

Landmark: \[ X(u1) \]
\[ \\backslash \]
\[ W(v1) \Rightarrow Y \Rightarrow U(v2) \text{ where “Y” has parameters } (u,v) \]
\[ \\backslash \]
\[ Z(u2) \]

In some cases, graphs will not have obvious landmarks. Examples would include rings, or the complete graph K5 (shown in Fig 9.2) or bipartite graphs. Most random graphs will have no single vertex that is the obvious root. However, in the absence of a unique landmark, a centroid can be defined:

Definition: A centroid is the vertex of highest degree, or the barycentric mean of the graphical representation, with each vertex assigned weight equal to its degree.

Note that the above definition has the consequence that when there is no unique vertex of highest degree (as in a ring), then the manner of displaying the graph affects the centroid. Hence, two graphs can be isomorphic, but have different centroids because of framing effects or choices of layout.
9.2.2 Frames

As mentioned, any path implies a transformation of one parameter in some space. In the ladder metagraf, these parameters provided a basis for a coordinate frame. Of course, by doing so, some minimal metrics were imposed which might destroy strict equivalences between topologically isomorphic graphs. The benefit, however, is that some relationships can be emphasized at the expense of others. Hence the depiction of the graphical model becomes context sensitive, reflecting the structure of the coordinate frame being used. Although perhaps not noticed in the earlier chapters, all anigraf constructions were implicitly drawn or tied to a coordinate frame. The choice of frame imposes a constraint on graphical evolution and model building.

When a graph evolves or is augmented or altered, often the old frame seems inappropriate. In Fig 9.5 for example, the “+” graph can be augmented by adding a diagonal edge “xy”. But when this happens, the original
Storylines

Once graphs are embedded in a frame (or context), paths through the graph that are aligned with the frame imply a parametric transformation, with a beginning and an end. Hence the graph becomes directed because the relation between any two vertices is not bidirectionally symmetric. The event sequences controlled by broker agents is an earlier example of such graphs, where the frame was time’s arrow. When seen as a metagraf, these sequences are just simple stories, all having much the same structure. The most common is: “boy meets girl (positive force), boy loses girl (negative force), boy gets girl” (powerful positive force.) As this basic theme is filled out and elaborated, more events and other actors enter the script. The result is a directed tree graph, with each branch adding other players, some good some evil. In some cases, the tree becomes closed to form a ring, where the end state is the same as the start; in others, the framing creates a spiral structure to the tree, when events repeat themselves in slightly different versions (eg three pigs.) The structure of stories, as described by Campbell (A Hero’s Journey), is a very common, powerful metagraf.

Parameterizations within the frame are broken. Similarly, in the “T” graph, edge “ad” appears an unreasonable leap in the context, obviating the linear a,b,c,d parameterization. Two “solutions” are considered: the first strives to retain the original frame; the second keeps the graphical form, but changes the frame, leading to a revision in context. In the case of a “+” with an added diagonal edge, the graph cannot be fully aligned with a rectangular frame. If the frame is to be preserved, a node with two edges properly aligned is “forced” to be added. However, if the rectangular frame is changed to a circle, then we have a satisfactory coordinate frame embedding of the graph, as indicated at the far right. Alternatively, we could consider a rooted dendrogram; but then the added edge (dashed) has no relevance in this particular tree.

In a similar manner, when an extreme edge “ad” is added to a “T” graph, the most compelling “solution” is to realign the edges to create a square with a forked tail. This can be embedded nicely in a new ellipse or rectangular frame, depending upon the meaning of the tail. Again, one could
also grasp either one of these new configuration by the root to accommodate the dendrogram frame. These operations support creative discovery because a new metagraf structure is needed to specify the relationships between models.

9.3 Complementarity

Models of models are highly cognitive artifacts. Like trees, certain parameters and transformations tend to re-occur, thereby specifying identical or similar metagrafs. The spectrum of love to hate, friend to foe, good to evil are examples. Each element lies at the opposite extreme of a signed parameterization. With most terminal paths through relational spaces, comes the notion of opposites, or, more generally, complementarity. These are pairs about an origin or axis that are not necessarily at the extremes, but are in some sense objects located at symmetric positions in the space. The mapping of complements is thus a metagraf.

For connected graphs, complements can be specified precisely, using a slight revision of the graph-theoretic version:

Definition: The (connected) complement $G_n$ of the graph $G_n$ is the connected edge set of $K_n$ obtained by removing the edges of $G_n$ from $K_n$, subject ot each vertex in $G_n$ or $G_n$ being at least degree one.
Figure 9.6 illustrates. The upper and lower rows are complements of each other. In the third case, $G_n$ and $G_n$ are self-complements. For these ring graphs note that their complements (without unfolding) appear to belong to a class of “star-like” graphs. The complementing operation on rings in this context has produced a new set of similar graphs which belong to the metagraf complement of the ring metagraf. The metagrafical transformation is easily understood and hence the new (context-sensitive) patterns are readily assimilated and categorized. Through use of the metagraf, the anigraf has a cognitive tool that supports creative innovation.

**9.4 From Intellect to Intuition**

Metagrafs make explicit relationships between classes of models. Like the models themselves, these higher level structures must still be grounded in the world, otherwise they have no real meaning or predictive power. Intelligent systems are rational, not random. Likewise, creative acts that explore new relationships should have some rational basis. Underlying much of intuition and creative expression is the ability to see evidence from a novel perspective, such as recognizing that a different kind of model also explains “the facts” and indeed might have greater predictive power. Metagrafs provide the anigraf with a potential for self-induced intuitive insights. Here, we are concerned not with new concepts learned “by mistake”, but rather those recognized as a result of the anigraf’s manipulation of its own concept space.

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Appendix (partial note)

| Potential Bi-directional Graphical Forms in a Default Context |
|-------------------|---|---|---|---|---|---|---|---|---|
| # vtx | 3 | 4 | 5 | 6 | 7 | 8 | 10 | 12 |
| all | 2 | 6 | 21 | 112 | 853 | ~10^4 | ~10^7 | 10^11 |
| planar | 2 | 6 | 20 | 99 | 646 | ~6000 | 2:10^5 |
| trees | 1 | 2 | 3 | 6 | 11 | 23 | 106 | 551 |
| line | 2 | 5 | 12 | 30 | 79 | 227 | 2322 | 3:10^5 |
| Ramsey | 2* | 7* | 19* |

* diagonal only.