INTRODUCTION: THE PROBLEM

One of the essential tasks of vision is to determine the three-dimensional (3D) shape of objects in the world. Once such information is available, a useful 3D model of an object can be constructed, which is suitable for recognition or manipulation, for example. Unfortunately, neither stereopsis nor motion parallax alone provides enough information to recover the correct 3D disposition or shape. Each method suffers serious defects unless other information is brought into play.

The critical defect with stereopsis is that the same rigid configuration of points seen at different distances will elicit different angular disparities on the two retinas. To recover the correct distance relations between the points using only horizontal disparity requires knowledge of the fixation distance. Let an observer view an equilateral triangle lying in the horizontal plane at distance \( D_A \) as illustrated in Fig. 1(a). If the altitude of the triangle is \( z_A \), then the angular disparity \( \delta x_A \) of the nearer point with respect to the farther base points will be

\[
\delta x_A = z_A / (D_A^2),
\]

where \( I \) is the interpupillary separation between the two eyes (cameras) and small angle approximations are taken. Now, if the triangle is moved farther away to position \( D_B \), then clearly the angular disparity \( \delta x_B \) of the near vertex will be reduced by the factor \( D_A^2 / D_B^2 \). However, the angular width of the base will have decreased by only \( D_A / D_B \). The triangle that previously appeared equilateral should thus appear squashed by the factor \( D_A / D_B \) as it is moved farther away. The triangle that appears equilateral based on (horizontal) disparity information alone must thus have a greater altitude, as shown in Fig. 1(b). In sum, the configuration or shape of a rigid set of points is not uniquely determined from stereopsis alone.

Recovering the 3D configuration from motion also presents problems unless information other than the (orthographic) motion of the points is provided. To illustrate the difficulty, let us assume that the motion-parallax solution [or equivalently the structure-from-motion (SFM) solution] requires at least three points and two views [for example, Hoffman and Flinchbaugh and Bobick show conditions and constraints under which the 3D configuration can be recovered from the two-dimensional (2D) projection of three points]; Ullman used four points, and Prazdnik used five points. With one exception, all these solutions, including those velocity fields, are to a set of second-degree equations, which means that there is a duplicate solution that is a reflection about a plane. (More recently, Tsai and Huang have obtained a linear solution for eight points.) For the given minimum number of points, therefore, each group containing this minimum has at least two solutions, one being a reflection of the other. Consider then the configuration of six points shown in Fig. 2(a). The triplets of points joined by solid lines are in a rigid relation, but the link between the two groups of triplets is not rigid (dashed line), as if the two parts are joined by a flexible rod. Because each of the two groups of triplets has a reflection ambiguity, other SFM interpretations of the entire configuration are possible, such as the one shown in Fig. 2(b). [Two other possible interpretations are the reflections of Figs. 2(a) and 2(b) about the horizontal line of four points.] A unique SFM solution thus requires removal of the reflection ambiguity.

By combining stereopsis with SFM we shall see that both ambiguities in the 3D interpretations can be eliminated. Stereopsis provides the sign needed to tell whether the ambiguous points seen with SFM are behind or in front of the others; SFM, on the other hand, correctly interprets the angular relations between the points, thus aiding stereopsis by eliminating the fixation-distance dependency.

STRUCTURE FROM STEREO PROPOSITION FOR TWO POINTS

Discrete Case

We will begin by considering the simple discrete case in which a stereo observer views a rigid configuration of points from one position (frame 1) and then moves to another position to obtain a second view (frame 2). Thus, although these discrete views do not make explicit the instantaneous velocities of the points, a measure of the relative velocities of the points can be obtained by keeping the temporal intervals between views...
STEREOSIS DEFECT: SCALE

Fig. 1. Two kinds of failings in the recovery of 3D structure. For stereopsis, a given disparity will indicate a different distance, depending upon the observation distance, D. Thus (a) the near vertex of the equilateral triangle at distance $D_A$ has the same disparity as (b) the near vertex of the isosceles triangle $D_B$.

SFM DEFECT: REFLECTION

Fig. 2. When structure is recovered motion, there is a reflection ambiguity. This ambiguity becomes a problem as the structure becomes increasingly nonrigid, such as when there is a flexible link (dashed line) between two rigid components.

constant. (Below we treat the case for which the instantaneous velocities are available.) This is the approach used by Ullman$^5$ in his classical monocular SFM solution. The problem here, then, is to determine how many points P and how many stereo views V are needed to recover the correct configuration of points.

Figure 3 shows the viewing conditions and the coordinate system used. The bisector of the lines of sight is taken as the Z axis (note direction); the XZ (horizontal) plane is defined as including the two lines of sight. (The solution will assume that the horizontal axes of the two retinas or cameras lie in the XZ plane.) The Y axis is normal to the XZ plane at the fixation point O. The point $P(x, y, z)$ and the origin O of the coordinate systems are assumed to be far away so that perspective information is nil; hence the projections are orthographic onto the separate frontal planes of the two eyes.

The basic problem is to recover the distance $OP(x, y, z)$ and the orientation $\sigma$, $\tau$ that the ray makes with the Z and Y axes. Because the views are orthographic and epipolar, $\tau$ appears in the image plane as does the elevation of P, namely, $y_P$. Because the azimuth of P, namely, $x_P$, also appears in the image, the problem reduces to recovering $\sigma$ and the distance $OP_x = (x_P^2 + z_P^2)^{1/2}$. Our two unknowns, $\sigma_P$ and $z_P$, are thus entirely confined to the horizontal plane. Let us then consider only the top view of the situation, as shown in Fig. 4.

Here the projection of $P(x, y, z)$ onto the XZ plane is denoted as $P_1$ for our first point, with the subscript 1 indicating our first view. The complementary angle $\theta_1 = (\pi/2) - \sigma_1$ has replaced $\sigma$. For any single view and point P, our unknown is either $\theta_1$ or $z_1$. Of course we know $x_1$, which appears in the image, and because the viewing is stereoscopic, we also know the angular disparity of point $P_1$ with respect to O. Let this disparity be designated as $\delta x_1$.

Unfortunately, knowledge of the angular disparity of $P_1$ is not sufficient to solve for its $z$ coordinate, because by Eq. (1) we do not have knowledge of the interpupillary separation or the fixation distance to O. This was the fatal defect of stereopsis alone. However, if we move our head (or cameras) slightly to one side, keeping the distance to O constant, then we have a second stereo view of $P$, namely, $P_2$ seen at azimuth $x_2$ with the observed disparity $\delta x_2$. Although this lateral motion has introduced a new unknown, namely, $z_2$, the ratio $z_1/z_2$ will equal that of the observed disparities $\delta x_1/\delta x_2$, as can be seen readily from Eq. (1). Appendix A shows that this information is then in principle sufficient to recover the distance OP and its orientation to the viewer. Specifically, we can solve for the angle $\theta_2$ in Fig. 2 as follows:

$$\theta_2 = \tan^{-1}\left[ \frac{|x_1z_2^2 - x_2z_1^2|}{1 - r_P} \right]^{1/2},$$

where $r_P = \delta x_1/\delta x_2$. Because OP$_2$ is simply $x_2$ sec $\theta_2$, we can calculate OP from $y_P$, which appears in the image plane. Hence we have the following SFM and stereo claim for two points:

Claim 1: Given two coplanar orthographic stereo views of two rigid points, their correct 3D disposition can be recovered uniquely, independent of fixation distance.

Note that the above claim speaks only of the disposition of the two points (i.e., the angle $\theta_1$). Although we have taken the azimuth $x_1$ and elevation $y_P$ of P to be distances, in fact they are seen only as angles on the retina. Thus the correct configuration, or angular relations between a set of points, can be
determined uniquely from two stereoscopic views, but not the actual absolute distances.

**Continuous Case**

Our visual system is remarkably sensitive to directional motion. Rather than simply taking snapshots of a configuration of points as we move our heads, let us now assume that the instantaneous retinal velocity of any point is available, as well as its position. Under these conditions, Appendix B then shows that once again the angle $\theta$ may be recovered by using the following relation:

$$\theta = \tan^{-1} \left( \frac{\Delta s/s}{\Delta h_x/h_x} \right)^{1/2}$$  \hspace{1cm} \text{(3)}

We thus make the following second claim:

Claim 2: Given one orthographic stereo view of two rigid points and their velocities, their correct 3D disposition can be recovered uniquely independent of fixation distance.

Thus we now have two methods of recovering the correct angular relations between a set of points.

**THE INTERPRETATION RULE**

The above two claims specify the minimal input required in order to obtain a unique solution for the 3D configuration of a rigid set of points, as seen in the 2D image. Should we then apply our solution for the 3D configuration of points to all pairs of points seen on our retinas? Clearly not, for some pairs will not be rigidly linked in three dimensions, and our interpretations will be incorrect. We thus need to be able to test from the image data whether or not a given pair of points is indeed rigidly linked. Specifically, we are required to identify false targets.

Appendices A and B analyze the false-target possibility and show that either one more point or one more (stereo) view will allow the observer to eliminate point pairs that do not arise from rigid 3D configurations. Thus we may test and verify our rigidity hypothesis from the sense data. If the points pass the rigidity test, then we propose that the points be interpreted as arising from a rigid configuration. We then have the following four interpretation rules:

**Rule 1:** (Discrete Case): If three coplanar stereo views of two points have a fixed separation according to the application of Eq. (2), then these points should be interpreted as being in a rigid configuration.

**Rule 2:** (Discrete Case): If three points and two coplanar stereo views have a fixed separation according to the application of Eq. (A2), then these points should be interpreted as being in a rigid configuration.

**Rule 3:** (Continuous Case): If two independent stereo views of two points plus their velocities suggest a fixed separation between these points according to Eq. (3), then these points should be interpreted as being in a rigid configuration.

**Rule 4:** If any of the above rules fail to apply (within certain as yet unspecified signal-to-noise considerations), then the points are not in a rigid configuration.

**SENSITIVITY TO IMPERFECT DATA**

The above analysis shows that, in principle, stereo and motion can be combined to recover the correct 3D configuration of points from 2D-image data. However, what kind of precision is required of the data in order that the 3D configuration can be reconstructed with reasonable accuracy? Will only a slight amount of noise in the data cause gross changes in the interpreted 3D shape, or do the estimation errors increase monotonically as the data become increasingly unreliable?

Figure 5 shows the result of one simulation using two stereo views of two points. (For this analysis, it is not necessary to add a third view or point.) The test case was two points separated by 1.4 m at 10 m, with one point lying at a 45° orientation to the other in the horizontal plane. The interpupil distance was taken as 6.4 cm, and the two stereo views corresponded to a 2° movement about the fixation point (i.e., a lateral motion of about 0.3 m). Figure 5(A) shows the result of increasing the disparity-measurement error; Fig. 5(B) shows the result of increasing the error of the angular-size measurements. The crosshatched regions indicate the range of the misestimation error, which depends upon whether positive- or negative-measurement errors are introduced. Note that all estimation errors rise monotonically as the measurement errors increase. Hence the interpretation process will be well behaved under measurement error.

Returning to Fig. 5(A), we see that for the particular conditions chosen, a disparity error of $10^{-5}$ rad will introduce a 7° error in one’s estimate of the configuration or orientation of the second to the first point. (The error in estimating the
Proposition 1. Proportion of reaction time.

A proportion of reaction time may be required implicitly in the process of cognitive arithmetic. Given two common arithmetic operations of two points, the time required to perform the calculation is influenced.

SUMMARY

The combination of configuration and model can be derived by combining the two modules. The combination of configuration occurs when the two models are combined. The configuration of the model is determined by the interaction between the two models. The interaction between the two models is influenced by the configuration of the model.

PSYCHOLOGICAL PREDICTIONS

The predictions are based on the assumption that the system is able to combine the two models. The predictions are derived from the interaction between the two models. The interaction between the two models is influenced by the configuration of the model.

References:

William R. Higgins.
From the fact that each view is stereoscopic, we obtain the distance-disparity relation

$$\frac{\delta x_{p1}}{\delta x_{p2}} = \frac{z_{p1}}{z_{p2}} = r_p, \quad (A2)$$

where $\delta x_{p1}$ is the measured disparity, thereby making $r_p$ a known constant. This relation follows from the fact that the horizontal disparity of P relative to O is given by

$$\delta x_{p1} = z_{p1}(D/2) \quad (A3)$$

where $I$ is the interpupil distance and $D$ is the line-of-sight distance to O, and given that the distance OP is much smaller than D. Taking the ratio of Eq. (3) for $i = 1, 2$ eliminates the $(D/2)$ dependency.

We now have two equations (A1) and (A2) in two unknowns, $z_{p1}, z_{p2}$, which can be solved for $\theta_2$:

$$\theta_2 = \tan^{-1} \left( \frac{x_{p1}^2/x_{p2}^2 - 1}{1 - r_p} \right) \quad (A4)$$

The length $OP_2$ is then simply $x_{p2}$ sec $\theta_2$, from which OP can be calculated because $y_{p2}$ appears in the image plane.

**Uniqueness**

The square root in the solution (A4) for the angle $\theta_2$ allows only positive values for $\theta_2$. Yet the correct value for $\theta_2$ may be either positive or negative, depending on whether point P2 lies in front of or behind the frontal plane containing the fixation point O. The solution (A4) for $\theta_2$ is thus not unique unless the sign of $z_{p2}$ is known. However, the sign of $z_{p2}$ is known. The sign of $z_{p2}$ is the same as that for the disparity of P2, namely, $\delta x_{p2}$. Hence the position of P1 and thus also P(x, y, z) can be determined uniquely.

**Degeneracies**

Under some conditions, Eq. (4) cannot be solved for $\theta_2$. The only case is when the denominator $(1 - r_p)$ is zero. This corresponds to $\delta x_{p1} = \delta x_{p2}$, or when P1 and P2 both lie in the same frontal plane. (This can be shown to be the only singular condition by evaluating the Jacobian of Eqs. (A1) and (A2).) The value of this determinant will be zero only when $r_p = z_{p3}/z_{p1}$. But because $r_p = z_{p1}/z_{p2}$, this singularity corresponds to $z_{p1} = z_{p2}$, as before.

**False Targets**

Is it possible that another pair of points not in a rigid configuration will also satisfy Eq. (A4)? If so, then a valid interpretation of this equation is not possible, because the observer would have no way of determining whether the solution came from a rigid configuration.

Let us assume that points O and Q also satisfy Eq. (A4) and thus appear rigid, although they are not. Let the competing rigid solution be O, P. Then, as seen in the image plane, P and Q must be coincident:

$$x_{p1} = x_{q1}; \quad y_{p1} = y_{q1}. \quad (A5)$$

The only ambiguity is in the Z values of P and Q. For two views, we may relate these Z values by the parameter $a_3$ as follows:

$$z_{q1} = a_3 z_{p1}, \quad z_{q2} = a_3 z_{p2}. \quad (A6)$$

However, because the disparity ratios for the two views of P and Q are known, they must also be identical for P and Q to appear the same. Hence from Eq. (A2) we have

$$\frac{z_{q1}}{z_{q2}} = \frac{r_q}{r_p} = \frac{z_{p1}}{z_{p2}}. \quad (A7)$$

Thus, combining Eq. (A7) with Eqs. (A6), we have

$$\frac{z_{q1}}{z_{q2}} = \frac{a_3 z_{p1}}{a_3 z_{p2}} = \frac{z_{p1}}{z_{p2}} \quad (A8)$$

requiring that $a_3 = a_2$. Thus the only false-target condition is when

$$z_{q1} = a \cdot z_{p1}, \quad z_{q2} = a \cdot z_{p2}. \quad (A9)$$

To explore this single false-target possibility, we will determine the values of a that lead to false targets. Recall that $x_{p1}$ must equal $x_{q1}$. Hence we may combine Eq. (A1) renotated to point Q with expressions (A9) to obtain

$$\overline{OQ_1}^2 = x_{q1} + z_{q1}^2 = x_{p1}^2 + a^2 \cdot z_{p1}^2,$$

$$\overline{OQ_2}^2 = x_{q2}^2 + z_{q2}^2 = x_{p2}^2 + a^2 \cdot z_{p2}^2. \quad (A10)$$

The difference in length $\overline{OQ_1}^2 - \overline{OQ_2}^2$ is thus

$$\overline{OQ_1}^2 - \overline{OQ_2}^2 = (x_{p1}^2 - x_{p2}^2) - a^2 \cdot (z_{p1}^2 - z_{p2}^2). \quad (A11)$$

But because OP is rigid (of fixed length), we may eliminate the $z_{p1}$ term using Eq. (A1) to obtain the conditions on $Q_1$ and $Q_2$ required to produce a false target, namely,

$$\overline{OQ_1}^2 - \overline{OQ_2}^2 = (x_{p1}^2 - x_{p2}^2)(1 - a^2). \quad (A12)$$

From Eq. (A12) we see immediately that there is no rigid false target $OQ$ because then the left-hand side of Eqs. (A9) would be zero, forcing $a = 1$, which from Eqs. (A9) makes point Q identical to P. How then can nonrigid false targets be excluded?

If the distance between a pair of points is nonrigid, then the value of $a$ will be different from 1. Furthermore, because the distance between O and Q will change from one view to the next, so must the value of $a$ (otherwise $OQ$ is a rigid configuration). Thus the simplest strategy to eliminate false targets is to add an extra (third) view and determine whether the distance OP indeed remains constant. If it does, then a must have been constant. The probability of this occurrence by chance for arbitrarily chosen values of $a$ is zero, except if the configuration is rigid.

Alternatively, a third (rigid) point R may also be included in the configuration. In this case, the angle POR must be consistent with the lengths OP, OR, and PR, again overconstraining the solution.

This result now leads to the following two interpretation rules:

**Rule 1**

If three coplanar stereo views of two points have a fixed separation according to the application of Eq. (A4), then these points should be interpreted as being in a rigid configuration.

**Rule 2**

If three points and two coplanar stereo views have a fixed separation according to the application of Eq. (A4), then these
points should be interpreted as being in a rigid configuration.

**APPENDIX B: STRUCTURE FROM STEREO PROPOSITION FOR TWO POINTS PLUS VELOCITIES**

**Proposition 2**
Given one orthographic stereo view of two rigid points and their velocities, their 3D disposition may be recovered uniquely independent of fixation distance.

**Proof**
Once again, the relations between the viewer and point $P(x, y, z)$ are as shown before in Fig. 3. Because the projections $x_p$ and $y_p$ are known, the problem reduces to recovering $s$ or $P_{zx}$, the projection of $P(x, y, z)$ onto the $XZ$ plane.

From above, the projection of $P(x, y, z)$ onto the $XZ$ plane is shown in Fig. 2 as before, with the substitution $\theta = (\pi/2) - \sigma$. Because more details about the geometry of $OP$ are required, this portion of Fig. 4 is further expanded to become Fig. 6. The notations here have also been simplified by dropping the subscript $p$. The problem now is to show how $\theta$ can be measured from the projection of $P$ onto the $X$ axis.

As the observer rotates about the fixation point $O$ by an angle $\phi$, the $XZ$ axes will rotate by the same angle because they are defined with respect to the observer's position. Let $R$ be the projection of $P$ onto the $X$ axis, lying at distance $x_1$ from $O$. Then for any fixed angle of observer rotation $\phi$, $R$ moves to $R'$, causing $x_1$ to increase to $x_2$ and $z_1$ to decrease to $z_2$. Note that both $R$ and $R'$ will lie on the same arc because $OP$ is fixed and $z_1$ is perpendicular to $x_1$ by definition. Thus, at any instant, the motion of $R$ will be tangent to the circle $ORP$. As $\phi \rightarrow 0$, this tangent then describes the direction of change $R$ in the $XZ$ plane. As is shown in Fig. 6, the tangent vector will have a length $\Delta x$ in the $X$ axis and $\Delta z$ in the $Z$ axis. From the geometry,

$$\tan \theta = \frac{\Delta x_1}{\Delta z_1} = \frac{z_1}{x_1}. \quad (B1a, b)$$

Recalling now from Eq. (A3) of Appendix A the relation between disparity $\delta x_{1p}$ and distance $z_{1p}$,

$$z_1 = \delta x_{1p}/D^2/I, \quad (B2)$$

where $D$ is the fixation distance and $I$ the interpupill separation. Noting that the same relation (B2) holds between $\Delta z_1$ and $\Delta x_{1}$, we can eliminate the $(D^2/I)$ term by division to obtain

$$\frac{\Delta x_1}{\delta x_1} = \frac{\Delta z_1}{z_1}. \quad (B3)$$

We now have three equations, Eqs. (B1a,b) and (B3), in the three unknowns $\Delta z_1$, $z_1$, and $\theta$. Solving for $\theta$, we find that

$$\theta = \tan^{-1}\left(\frac{\Delta x/x}{\Delta \delta x/\delta x}\right)^{1/2}. \quad (B4)$$

where the expression in parentheses is simply the ratio of the increment of the projection of $OP$ onto the $X$ axis to its relative disparity increment. Or, in terms of velocities, it is the ratio of the $x$ component of the velocity of $P$ to the rate of disparity change, both normalized by their distances from $O$.

To recover the length $OP_1$, we note that $\cos \theta = x_1/OP_1$. Hence

$$OP_1 = x_1 \sec \theta = x_1 (1 + \tan^2 \theta)^{1/2}. \quad (B5)$$

Substituting Eq. (B4) into Eq. (B5), we find that

$$OP_1 = x \left(1 + \frac{\Delta x/x}{\Delta \delta x/\delta x}\right)^{1/2}. \quad (B6)$$

Thus the disposition and the length between two points $O$ and $P$ are recoverable from one dynamic stereo view that generates relative motion of disparity and angular extent.

**Uniqueness**
As before in Appendix A, although there is a square-root solution for $\theta$, Eqs. (B4) and (B6) will yield unique solutions because the sign of $z$ is the same as that for $\Delta x$ and is known. Hence the position of $P_{xx}$, and hence $P(x, y, z)$, can be determined uniquely.

**Degeneracies**
Equation (B5) cannot be solved when $x$ or $\Delta x$ is zero, corresponding to $\theta = \pi/2$. Referring to Fig. 6, we see that this condition is equivalent to point $P$ lying in the sagittal $YZ$ plane. (Note that this degeneracy would not occur if perspective, rather than orthographic projection, were assumed.) As long as the observer’s motion is such that the configuration $OP$ will undergo some rotation, this degenerate condition will not occur in practice.

**False Targets**
Here we wish to determine the conditions in which a point other than $P$ will also satisfy Eqs. (B4) and (B6). Let us assume that there is such a point $Q$, with position coordinates $(x, y, z_{Q})$ and velocities $(\Delta x/\Delta t, \Delta y/\Delta t, \Delta z_{Q}/\Delta t)$. Because the $x, y, \Delta x, \Delta y$ values appear in the image, the $z_{Q}$ and $\Delta z_{Q}$ are the only unknowns. These unknowns for point $Q$ can be related to the corresponding values $z_{p}$ and $\Delta z_{p}$ for point $P$ as follows:

$$z_{Q} = a_{1}z_{p}, \quad \Delta z_{Q} = a_{2}\Delta z_{p}. \quad (B7)$$

However, Eq. (B3) gives us the relation between the known disparity ratios for points $P$ and $Q$:

$$\frac{\Delta x_{1Q}}{\delta x_{1Q}} = \frac{\Delta z_{1Q}}{z_{1Q}} = \frac{\Delta x_{1P}}{\delta x_{1P}} = \frac{\Delta z_{1P}}{z_{1P}}. \quad (B8)$$
\[
\frac{\Delta x_p}{\Delta x_p} = \frac{\Delta z_p}{z_p}, \quad \text{(B8a)}
\]

\[
\frac{\Delta x_q}{\Delta z_q} = \frac{\Delta z_q}{a_1 z_p}. \quad \text{(B8b)}
\]

But the disparities \(\Delta x_{p,q}\) and \(\Delta z_{p,q}\) are observables and hence must be the same. Equating Eqs. (B8a) and (B8b), we see that \(a_2 = a_1\). Thus the only false-target condition is when

\[
\Delta z_q = a_2 \Delta z_p, \quad z_q = a_2 z_p. \quad \text{(B9)}
\]

To explore this single false-target possibility, we will determine the values of \(a\) that lead to false targets.

Referring to equation (B1), the angular values \(\theta_p\) and \(\theta_q\) for P and Q satisfy

\[
\tan \theta_p = \frac{\Delta x_q}{\Delta z_p} = \frac{z_p}{x}, \quad \text{and} \quad \tan \theta_q = \frac{\Delta z_q}{\Delta z_q} = \frac{z_q}{x}. \quad \text{(B10)}
\]

Thus

\[
x \Delta x = z_p \Delta z_p, \quad x \Delta z = z_q \Delta z_p = a_2^2 z_p \Delta z_p, \quad \text{(B11)}
\]

where Eqs. (B9) have been used to express the \(z\) values for Q in terms of those for P. But Eqs. (B11) force \(a^2 = 1\) for all Q’s. Hence from Eqs. (B9) we see that Q is identical to P and there are no false targets. (This result may have been anticipated, because the solution for the configuration of OP was based on instantaneous values of the position and velocity of P.) This result now leads to the following interpretation rules:

**Rule 1**

If two independent stereo views of two points plus their velocities suggest a fixed separation between these points according to Eq. (B6), then these points should be interpreted as being in a rigid configuration.

**Rule 2**

If Rule 1 fails to apply (within certain yet-to-be-specified signal-to-noise considerations), then the two points are not in a rigid configuration.

Thus, because Proposition B is based on an instantaneous analysis of the sensory data, it provides the basis for a potentially more powerful scheme for interpreting the structure of both rigid and nonrigid configurations.

**ACKNOWLEDGMENT**

This report describes research done in the Natural Computation Group at the Department of Psychology, utilizing the facilities of the Artificial Intelligence Laboratory of the Massachusetts Institute of Technology. Support for this work is provided by the National Science Foundation and the U.S. Air Force Office of Scientific Research (AFOSR) under a combined grant for studies in Natural Computation, grant 792910-MOS, and by the AFOSR under an Image Understanding contract F49620-83-C-0135.

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