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Reduction from Variance Matting to Triangulation Matting

Definitions:

$$\textbf{var}[x_1, \ldots, x_N] = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

$$\bar{x} = \textbf{mean}[x_1, \ldots, x_N] = \frac{1}{N}\sum_{i=1}^{N} x_i$$

Triangulation Matting has two images, two background, foreground is unchanged:

$$I = [I_1, I_2],\ B = [B_1, B_2]$$

$$\textbf{var}(I) = \frac{1}{2-1}\sum_{i=1}^{2}(I_i - \bar{I})^2 = (I_1 - \bar{I})^2 + (I_2 - \bar{I})^2,\ \bar{I} = \frac{I_1 + I_2}{2}$$

Thus,

$$\textbf{var}(I) = \left(I_1 - \frac{I_1 + I_2}{2}\right)^2 + \left(I_2 - \frac{I_1 + I_2}{2}\right)^2 = \left(\frac{I_1 - I_2}{2}\right)^2 + \left(\frac{I_2 - I_1}{2}\right)^2$$

$$\textbf{var}(I) = \left(\frac{I_1 - I_2}{2}\right)^2 + \left(\frac{I_2 - I_1}{2}\right)^2 = \left(\frac{I_1 - I_2}{2}\right)^2 + \left((-1)\frac{I_1 - I_2}{2}\right)^2 = 2\left(\frac{I_1 - I_2}{2}\right)^2$$

$$\textbf{var}(I) = \frac{(I_1 - I_2)^2}{2}.$$

Similarly,

$$\textbf{variance}(B) = \frac{(B_1 - B_2)^2}{2}.$$

Solving for alpha,

$$\alpha = 1 - \sqrt{\frac{\textbf{var}(I)}{\textbf{var}(B)}} = 1 - \sqrt{\frac{2}{(B_1 - B_2)^2} \cdot \frac{(I_1 - I_2)^2}{2}} = 1 - \sqrt{\frac{(I_1 - I_2)^2}{(B_1 - B_2)^2}} = 1 - \frac{(I_1 - I_2)}{(B_1 - B_2)}$$

$$\alpha = 1 - \frac{(I_1 - I_2)}{(B_1 - B_2)}.$$

This final formula is the triangulation matting formula [Smith and Blinn 1996].