# Exact Decoding of Phrase-Based Translation Models through Lagrangian Relaxation: Supplementary Material 

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## A An Example Run of the Algorithm in Figure 3

Figure 1 gives an example run of the algorithm. After 31 iterations the algorithm detects that the dual is no longer decreasing rapidly enough, and runs for $K=10$ additional iterations, tracking which constraints are violated. Constraints $y(6)=1$ and $y(10)=1$ are each violated 10 times, while other constraints are not violated. A recursive call to the algorithm is made with $\mathcal{C}=\{6,10\}$, and the algorithm converges in a single iteration, to a solution that is guaranteed to be optimal.

## B Speeding up the DP: A* Search

In the algorithm depicted in Figure 3, each time we call Optimize $\left(\mathcal{C} \cup \mathcal{C}^{\prime}, u\right)$, we expand the number of states in the dynamic program by adding hard constraints. On the graph level, adding hard constraints can be viewed as expanding an original state in $\mathcal{Y}^{\prime}$ to $2^{|\mathcal{C}|}$ states in $\mathcal{Y}_{\mathcal{C}}^{\prime}$, since now we keep a bit-string $b_{\mathcal{C}}$ of length $|\mathcal{C}|$ in the states to record which words in $\mathcal{C}$ have or haven't been translated. We now show how this observation leads to an A* algorithm that can significantly improve efficiency when decoding with $\mathcal{C} \neq \emptyset$.

For any state $s=\left(w_{1}, w_{2}, n, l, m, r, b_{\mathcal{C}}\right)$ and Lagrange multiplier values $u \in \mathbb{R}^{N}$, define $\beta_{\mathcal{C}}(s, u)$ to be the maximum score for any path from the state $s$ to the end state, under Lagrange multipliers $u$, in the graph created using constraint set $\mathcal{C}$. Define $\pi(s)=\left(w_{1}, w_{2}, n, l, m, r\right)$, that is, the corresponding state in the graph with no constraints $(\mathcal{C}=\emptyset)$. Then for any values of $s$ and $u$, we have

$$
\beta_{\mathcal{C}}(s, u) \leq \beta_{\emptyset}(\pi(s), u)
$$

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That is, the maximum score for any path to the end state in the graph with no constraints, forms an upper bound on the value for $\beta_{\mathcal{C}}(s, u)$.

This observation leads directly to an A* algorithm, which is exact in finding the optimum solution, since we can use $\beta_{\emptyset}(\pi(s), u)$ as the admissible estimates for the score from state $s$ to the goal (the end state). The $\beta_{\emptyset}\left(s^{\prime}, u\right)$ values for all $s^{\prime}$ can be calculated by running the Viterbi algorithm using a backwards path. With only $1 / 2^{|\mathcal{C}|}$ states, calculating $\beta_{\emptyset}\left(s^{\prime}, u\right)$ is much cheaper than calculating $\beta_{\mathcal{C}}(s, u)$ directly. Guided by $\beta_{\emptyset}\left(s^{\prime}, u\right), \beta_{\mathcal{C}}(s, u)$ can be calculated efficiently by using A* search.

Using the A* algorithm leads to significant improvements in efficiency when constraints are added. Section 6 presents comparison of the running time with and without $\mathrm{A}^{*}$ algorithm.


Figure 1: An example run of the algorithm in Figure 3. At iteration 32, we start the $K=10$ iterations to count which constraints are violated most often. After $K$ iterations, the count for 6 and 10 is 10, and all other constraints have not been violated during the $K$ iterations. Thus, hard constraints for word 6 and 10 are added. After adding the constraints, we have $y^{t}(i)=1$ for $i=1 \ldots N$, and the translation is returned, with a guarantee that it is optimal.

