Continuously Relaxing Over-constrained Conditional Temporal Problems

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Robotic Personal Transportation System

- Video that highlights the interaction during trip planning.

Key features

- Desired capabilities in the video:

  - Minimize the perturbations to users' requirements;

  "You have to shorten the dinner from 15 to 10 minutes".

  - Explain both cause of failures and resolutions;

  "Because of the extended driving time".

  - Prioritize different alternative solutions;

  "then how about arriving home 5 minutes late".

  - Adapt to modified requirements during interactions.

  "if you want to spend at least 12 minutes on dining, then you will arrive at home 2 minutes late".

Best-first Conflict-Directed Relaxation

- Motivation:
  - Provide **helpful** and **parsimonious** alternative solutions with **insights** into cause of failure.
  - Like an experienced assistant.

- Objectives:
  - Enumerate only **continuous relaxations**;
  - Record the **conflicts** detected in the requirements;
  - Explore the candidate space in **best-first** order;
  - Be **reactive** to newly added requirements.

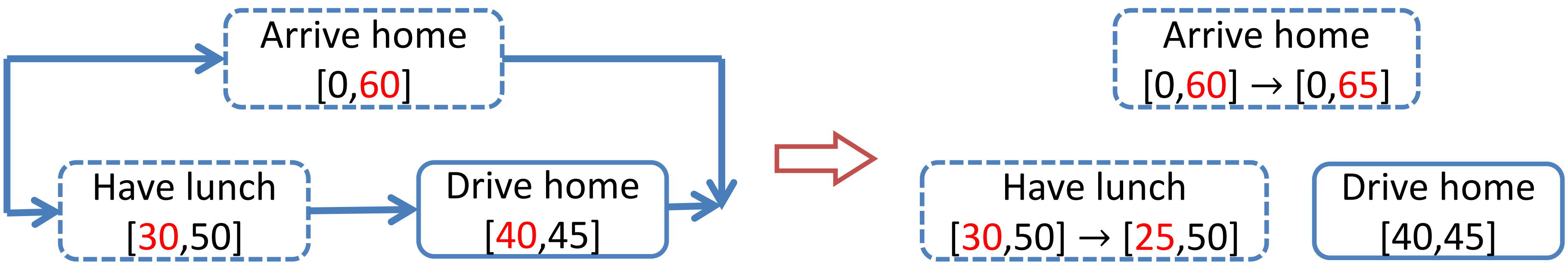
Continuous Relaxations: Definition

- Previous approaches resolve over-constrained temporal problems by **suspending** constraints.
  - "Remove your trip duration requirement" or "eliminate your stay at the restaurant".
  - Unnecessary in most scenarios.

- We only relax constraints continuously to reduce the perturbation:
  - Definition: a continuous relaxation, CR, is a set of relaxed constraints, $rc_1, rc_2, \ldots, rc_n$, such that the temporal problem is consistent.

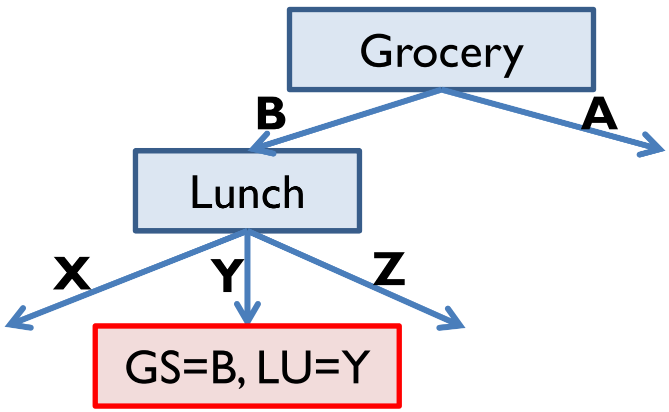
Best-first Conflict-Directed Relaxation

- Model: (Over-constrained) Controllable Conditional Temporal Problems.
  - All variable are controllable.
  - Allowing temporal constraints to be relaxed to restore consistency;
    - A subset of the temporal constraints, $RE \subseteq E$, are relaxable.
    - A relaxable temporal constraint, re_i: $[t_L, t_U]$, can be relaxed to $\mathrm{re_i}'$: $[t_L - \Delta_L, t_U + \Delta_U]$ if necessary.

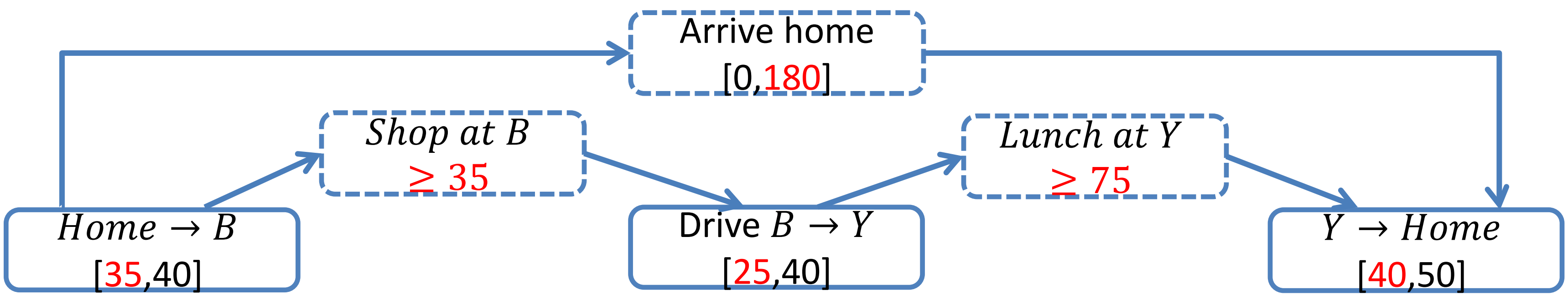


From conflicts to relaxations

- A conflict composes of an inconsistent set of temporal constraints and their required assignments.
- We learn conflicts from the negative cycles detected by temporal consistency algorithms.



Conflict:
GS = B, LU = Y;
$\text{Home} \to \text{B} \geq 35$;$\text{Shop at B} \geq 35$;
$\text{Drive B} \to \text{Y} \geq 25$; $\text{Lunch at Y} \geq 75$;
$\text{Y} \to \text{Home} \geq 40$;$\text{Arrive Home} \leq 180$.



Conflict Resolution

- Compute both *discrete* and *continuous* constituent resolutions to a conflict.

Conflict:
GS = B, LU = Y;
Home → B $\geq$ 35;Shop at B $\geq$ 35;
Drive B → Y $\geq$ 25; Lunch at Y $\geq$ 75;
Y → Home $\geq$ 40;Arrive Home $\leq$ 180.

Discrete Resolutions

GS=A;
LU=X; LU=Z;

Continuous Resolutions

Shop at B $\geq$ 5;
Lunch at Y $\geq$ 45;
Arrive Home $\leq$ 210;
Shop at B $\geq$ 25 and Lunch at Y $\geq$ 55;
… …
and many more

- Note that only relaxable temporal constraints can be relaxed to resolve conflicts.

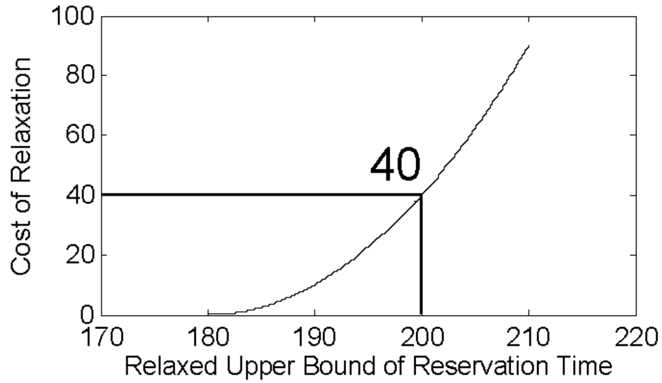
Preferences over relaxations

- Defining preference functions over variable assignments and constraint relaxations.
  - Each variable assignment is mapped to a positive reward value by function f_p.
  - Each constraint relaxation is mapped to a positive cost value by function f_e.

Grocery	A	40
	B	100
Lunch	X	70
	Y	80
	Z	30

Assignment reward:

$$\alpha = \{Grocery = B, Lunch = Y\}$$
$$f_p(\alpha) = 100 + 80 = 180$$



Relaxation cost:

$$\mathrm{r} = ReservationTime[0{,}180] \rightarrow [0{,}200]$$
$$f_p(r) = f_p(200 - 180) = 40$$

Compute Preferred Continuous Relaxations

- Using the cost function over relaxable constraints, we compute only the most preferred continuous relaxation.
- This is framed as a linear optimization problem.

Objective Fn: $min \sum_{e_i \in Conflict} (f_{ei}(UB'_{ei} - UB_{ei}) + f_{ei}(LB_{ei} - LB_{ei}'))$

Constraints: $s.t. \sum_{e_i \in Conflict} [(UB'_{ei} - UB_{ei}) + (LB_{ei} - LB_{ei}')] \geq -1 \times V_{ncycle}$

- For example:

Home $\rightarrow$ B $\geq$ 35;Shop at B $\geq$ 35;
Drive B $\rightarrow$ Y $\geq$ 25; Lunch at Y $\geq$ 75;
Y $\rightarrow$ Home $\geq$ 40;
Arrive Home $\leq$ 180.

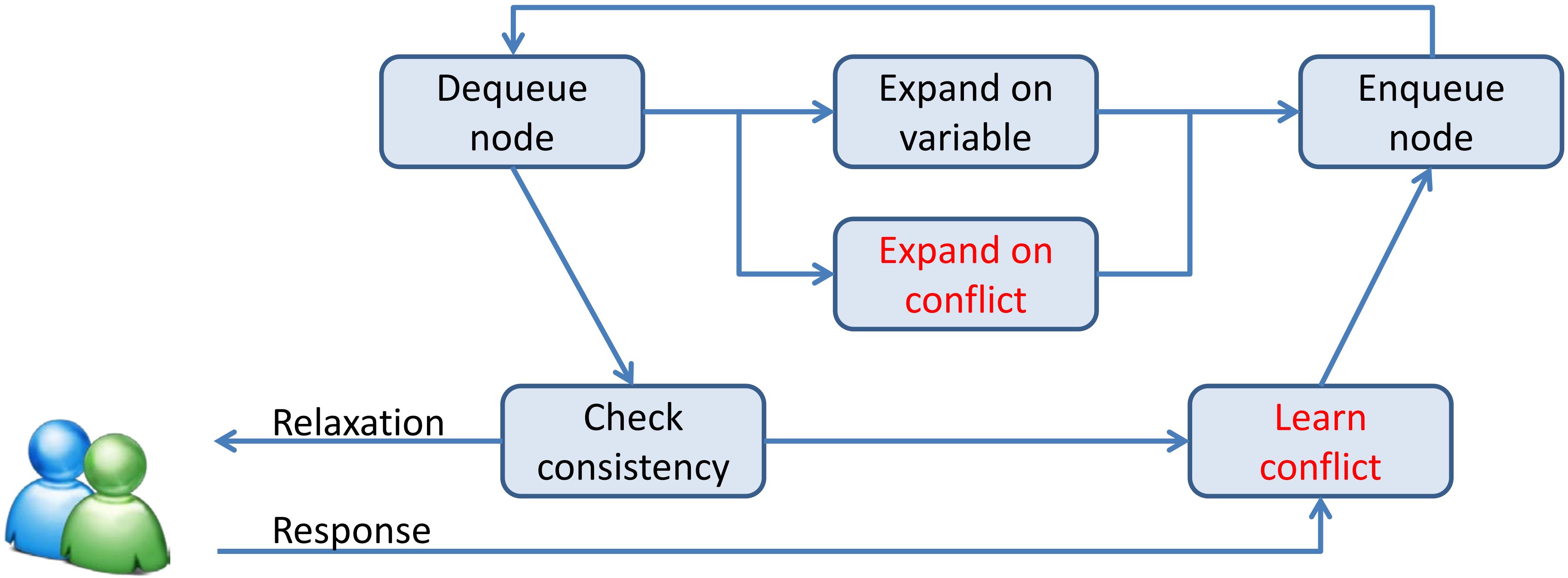
$$\min(f(\Delta_{Shop\ at\ B}) + f(\Delta_{Lunch\ at\ Y}) + f(\Delta_{Arrive\ Home}))$$

$$\text{s.t. } \Delta_{Shop\ at\ B} + \Delta_{Lunch\ at\ X} + \Delta_{Arrive\ Home} \geq 30$$

- We restrict the cost functions to be convex so that the relaxation process is always tractable.

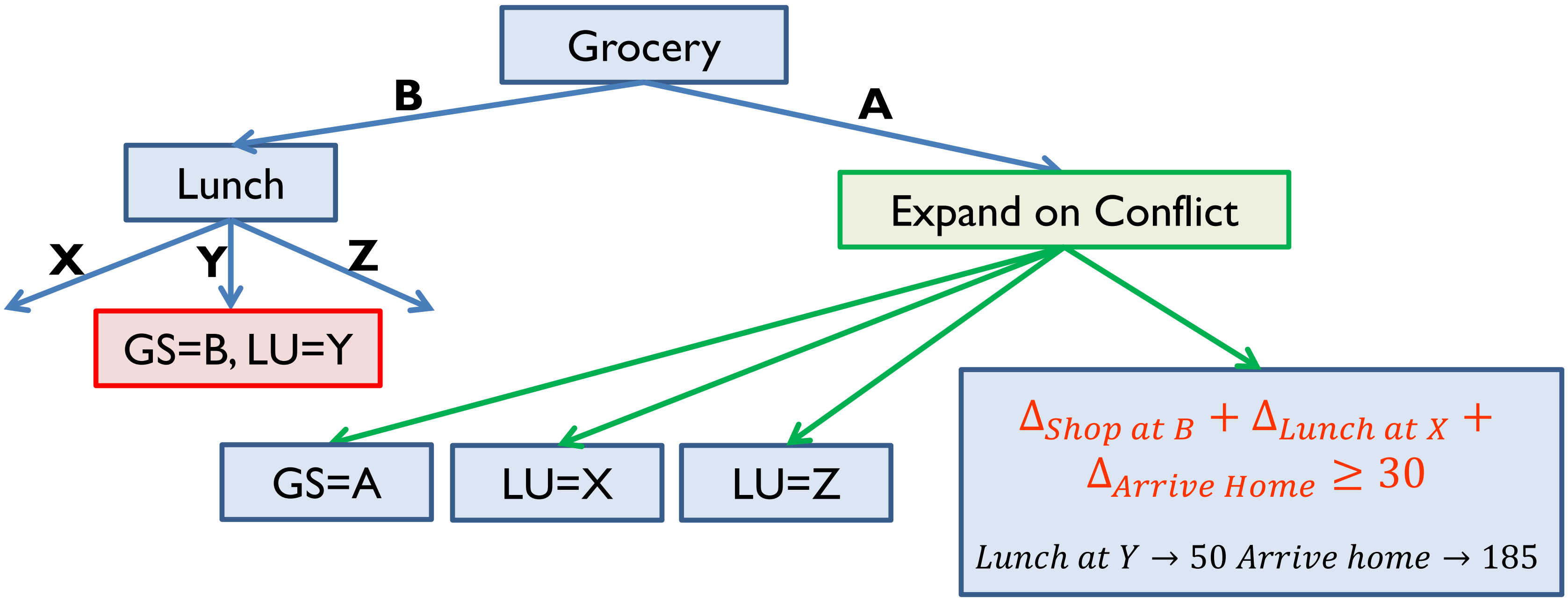
Generalize CD-A* to Continuous Relaxations

- BCDR enumerates relaxations in best-first order:
  – It searches over subsets of constraints by making different variable assignments.
  – It resolves a conflict by relaxing a constraint, partially and completely.



Expand on Unresolved Conflicts

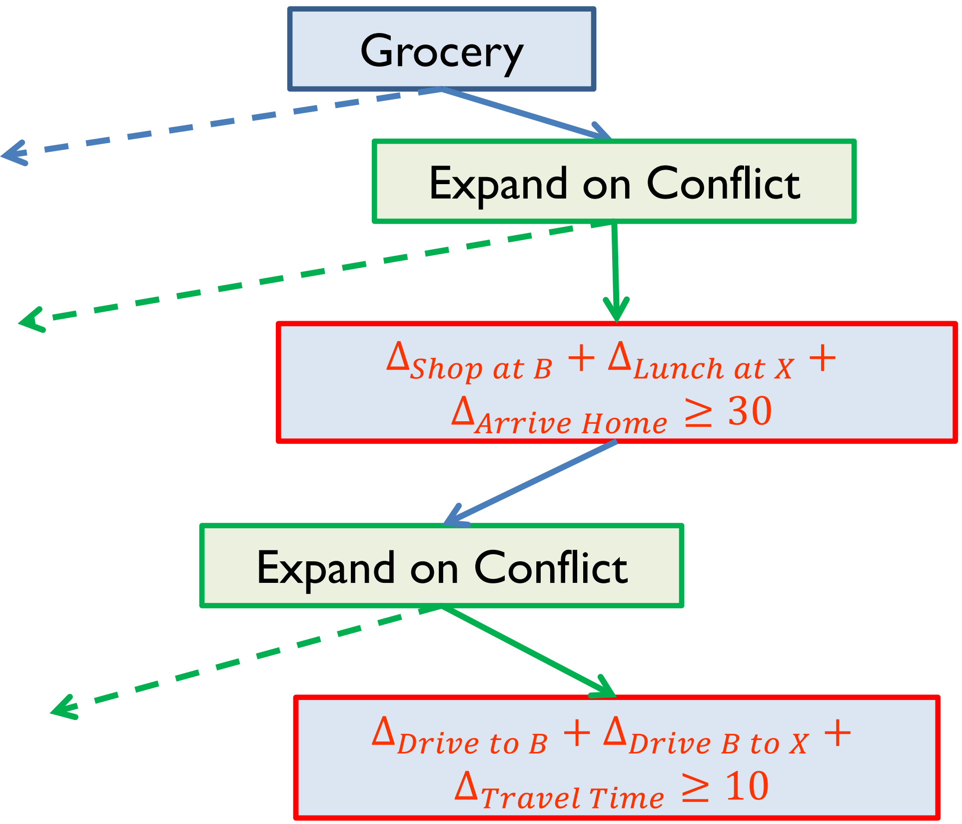
- If a node has an unresolved conflict, we expand it using both continuous and discrete constituent relaxations.



- The utility of the continuous relaxation is computed using the grounded solution of the lowest cost.

Continuous Relaxations for Multiple Conflicts

- For two or more continuous relaxations on the same branch, the utility is determined by the ground solution that respects both inequalities.



$$\min(f(\Delta_{Shop\ at\ B}) + f(\Delta_{Lunch\ at\ Y}) + f(\Delta_{Arrive\ Home}) + f(\Delta_{Drive\ to\ B}) + f(\Delta_{Drive\ B\ to\ X}) + f(\Delta_{Travel\ Time}))$$

s.t.

$$\Delta_{Shop\ at\ B} + \Delta_{Lunch\ at\ X} + \Delta_{Arrive\ Home} \geq 30$$

and

$$\Delta_{Drive\ to\ B} + \Delta_{Drive\ B\ to\ X} + \Delta_{Travel} \geq 30$$

Incorporating User Responses

- BCDR is reactive to newly added user requirements.
- We encodes these requirements as new conflicts, and follow them as constraints in the continuous relaxation.

No, I do not want to extent my reservation time.

No, I want to spend at least 25 minutes on shopping.

Arrive Home ≤ 180;
Shop at B ≥ 25;

$$\min(f(\Delta_{Shop\ at\ B}) + f(\Delta_{Lunch\ at\ Y}) + f(\Delta_{Arrive\ Home}))$$

$$\text{s.t.}\ \ \Delta_{Shop\ at\ B} + \Delta_{Lunch\ at\ X} + \Delta_{Arrive\ Home} \geq 30;$$
$$\Delta_{Arrive\ Home} \leq 0;$$
$$\Delta_{Shop\ at\ B} \leq 10.$$

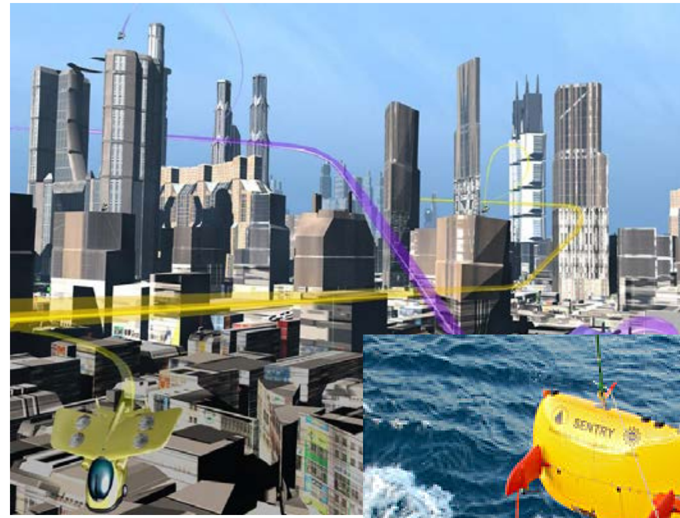
Applications

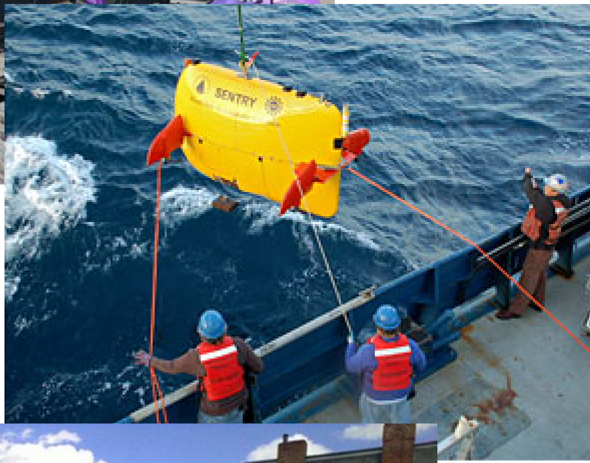
- Personal Transportation System.

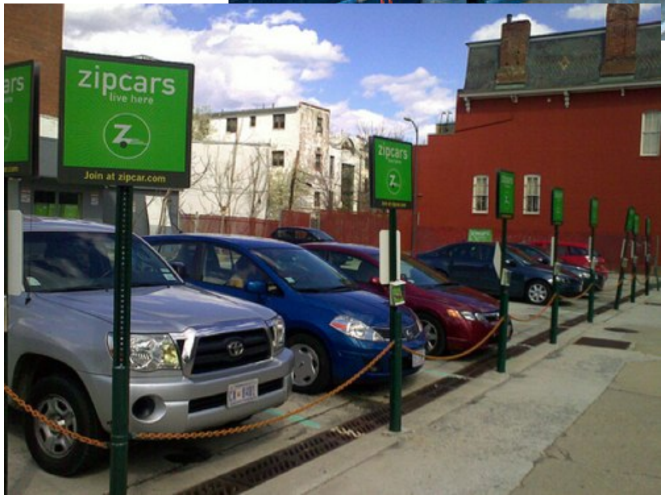
- Mission advisory system for autonomous underwater vehicles.

- Scheduling assistant for factory floor operations.

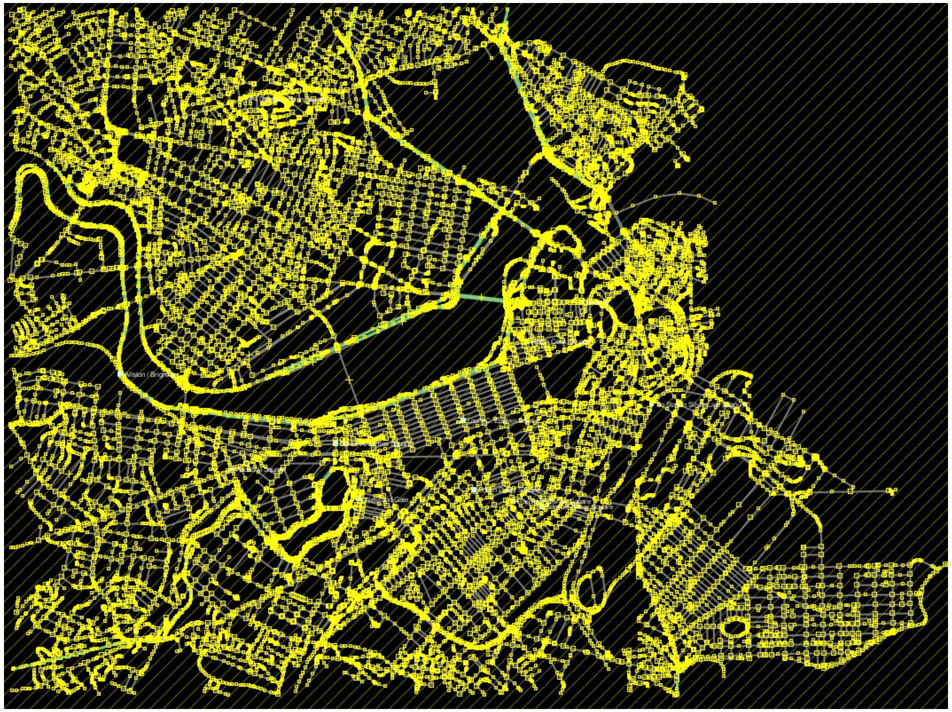
- Trip advisor for car-sharing network users.







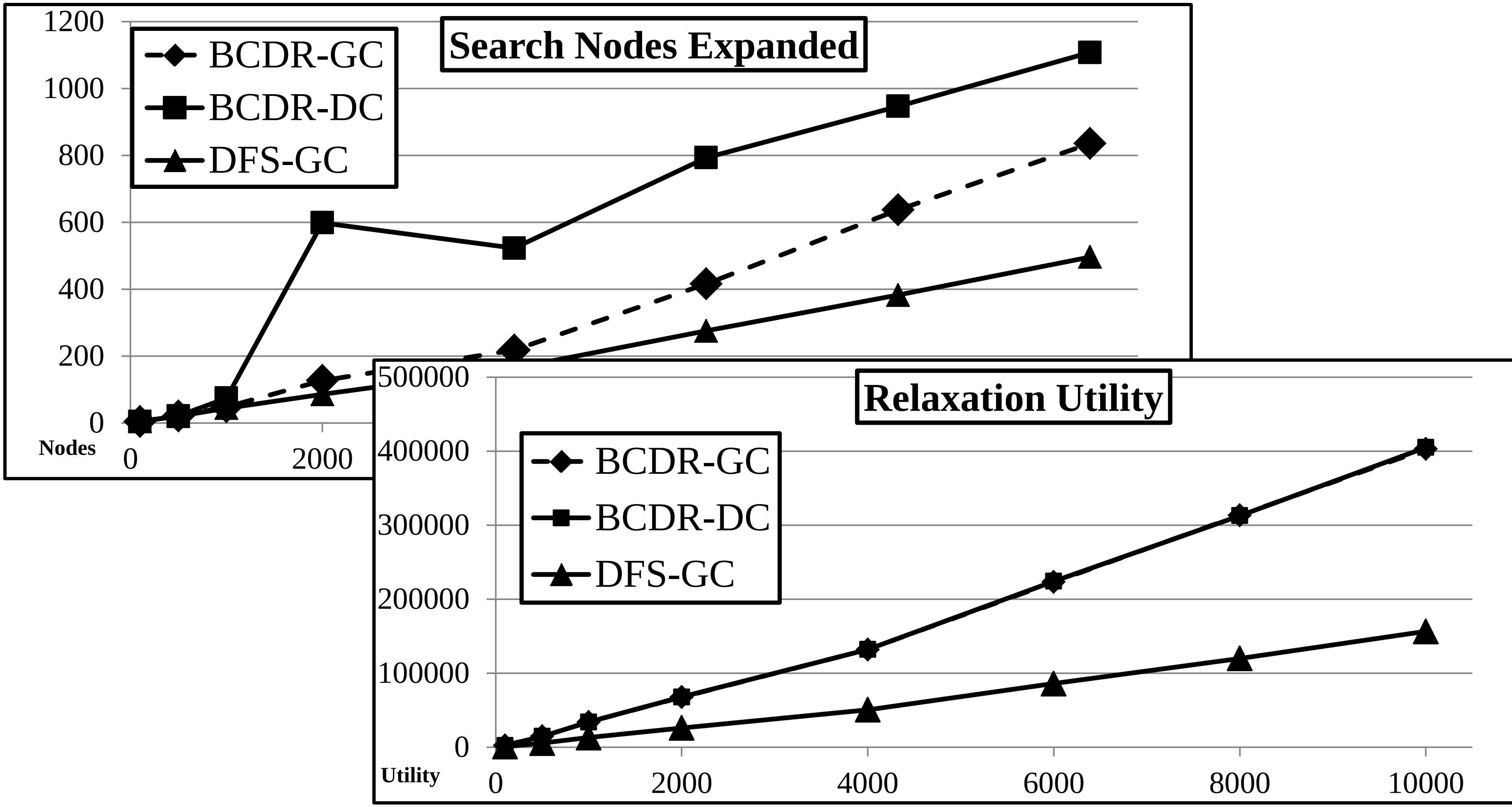
Empirical Validation - Setup



- We simulated a car-sharing network in Boston using randomly generated car locations and destinations.
- Test cases are characterized by:
  - Number of reservations per car.
  - Number of cars in the network.
  - Number of activities per reservation.
  - Number of alternative options per activity.
- Time change may affect neighboring reservations.

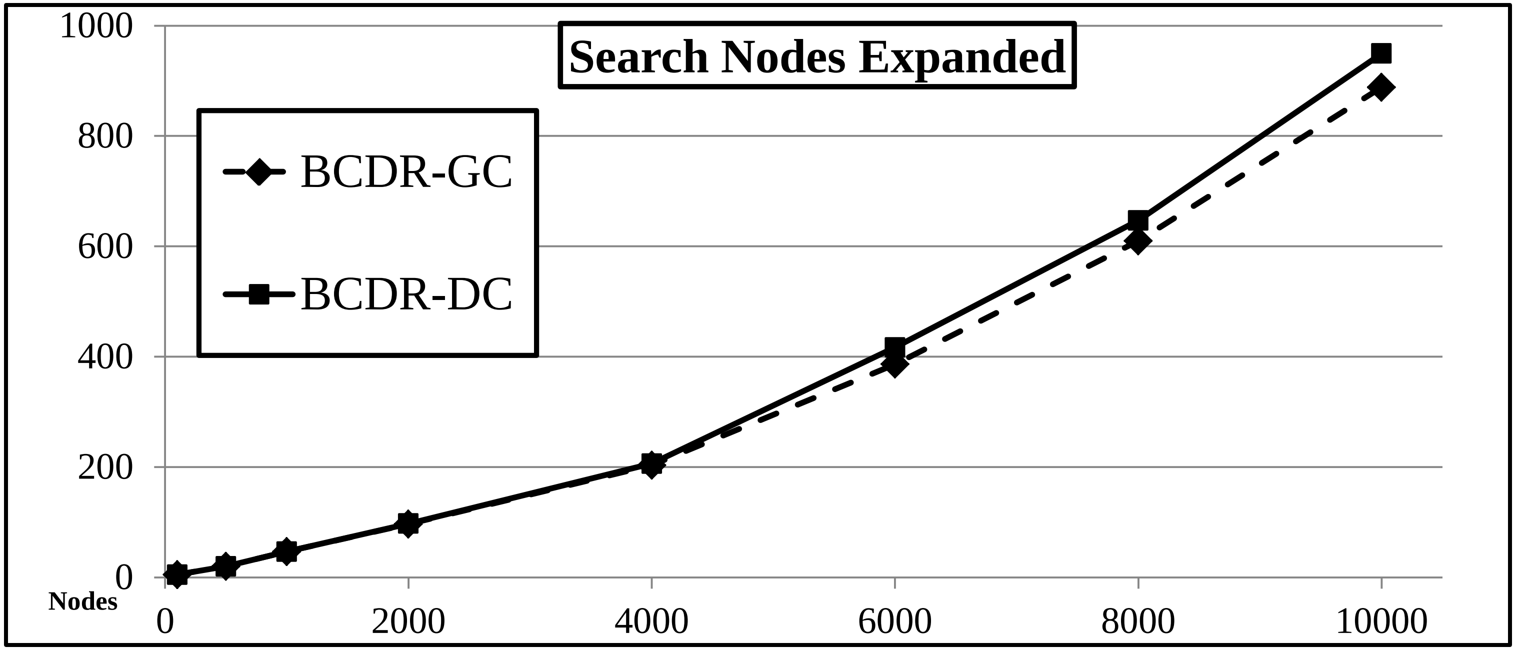
Empirical Validation - Results

- We compare BCDR-GC,BCDR-DC and DFS-GC in finding the first relaxation to a CCTP.



Empirical Validation - Limitations

- The advantage of generalized conflict resolution is limited when the relaxation cost is significant lower than rewards.
  - We reduced the range of cost values in the previous test by 100 times.



Contributions

- In this paper, we presented the Best-first Conflict-Directed Relaxation approach to:
  - Resolve over-constrained temporal problems using continuous relaxations.
  - Minimally relax constraints for minimal perturbation.
  - Efficiently enumerate relaxations using generalized conflict learning and resolutions.
  - Be reactive to newly added user requirements.

- Executable, test cases and slides are available at:

  http://people.csail.mit.edu/yupeng/software.html

Backup slides

Objectives

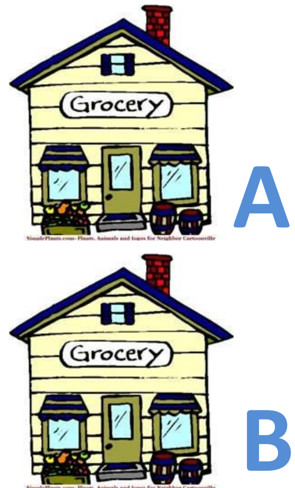
- To resolve an over-constrained CTPs:
  - Preferably: using the most preferred relaxations;
  - Minimally: minimizing the perturbations to the problem;
  - Efficiently: speeding up the search for feasible relaxations.

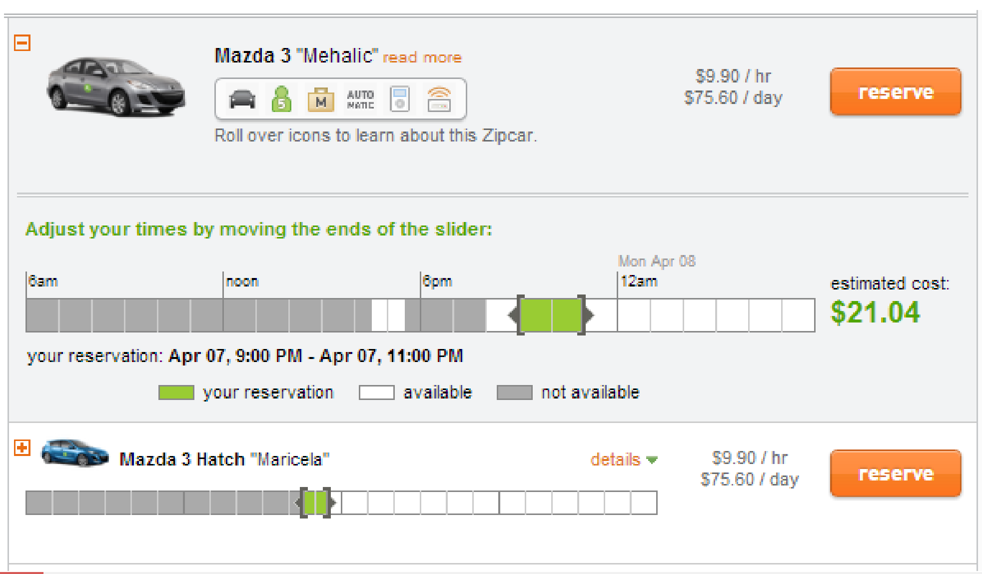
- BCDR extends the Conflict-Directed A* algorithm to continuous variables, constraints and preference functions for best-first relaxation enumeration.

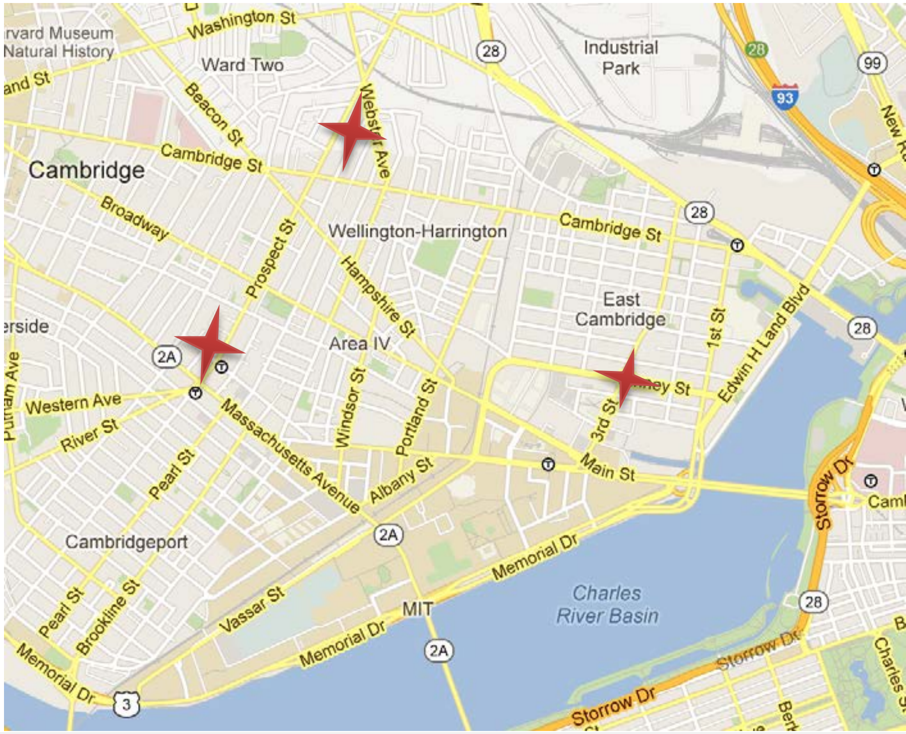
- Implemented as a reservation advisory system for car-sharing networks, BCDR demonstrates significant improvements in performance and solution quality.

Motivating Example

- Planning a weekend trip using a car-sharing network.

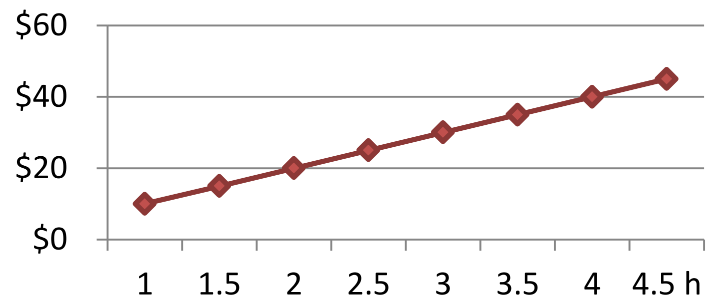






Trade-off Between Trip Parameters

- Which store/restaurant to visit?
- How much time should the shopping/dinning time be?
- How long should I reserve the car?
- How much am I willing to spend for the trip?



Driving Time in minutes			
$A \rightarrow X$	[30,40]	$Home \rightarrow A$	[35,50]
$B \rightarrow X$	[35,40]	$Home \rightarrow B$	[35,40]
$A \rightarrow Y$	[25,30]	$X \rightarrow Home$	[45,50]
$B \rightarrow Y$	[25,40]	$Y \rightarrow Home$	[40,50]
$A \rightarrow Z$	[20,25]	$Z \rightarrow Home$	[50,60]
$B \rightarrow Z$	[30,35]		

Shopping/Dining Time in minutes	
A	40
B	35
X	50
Y	75
Z	100

Intelligent Reservation System

- Automatically find and negotiate a solution with the user that satisfies the user's requirements to the maximal extent.

OK, that's fine.

OK. How about eating at Y for 55 minutes?
You can then shop at B for 25 minutes.

Approach

- Model: Controllable Conditional Temporal Problem.
  - Captures the users' requirements, preferences, alternatives and environment constraints.

- Algorithm: Best-first Conflict-Directed Relaxation.
  - Enumerate solutions/relaxations to a CCTP in best-first order.
    - Assignments: where to go and what to do.
    - Relaxations to temporal constraints: alternative goals/requirements to ensure consistency.

Minimal Continuous Relaxations

- Minimal **discrete** relaxations:
  - A valid relaxation, whose proper subsets are not valid relaxations.
  - Generated by computing the minimal covering sets of minimal conflicts.

- Minimal continuous relaxations:
  - A set of continuous relaxations whose
  - Generated by solving a linear optimization problem against the conflicts.

Solution

- A solution to a CCTP T is a pair $\langle A, R \rangle$:
  - A: a set of assignments to discrete variables.
  - R: a set of relaxations for some temporal constraints.

  such that no variable in T is left unassigned and T is temporally consistent.

- The most preferred solution has the highest utility, evaluated by:

$$\sum_{a_i \in A} f_{p_i}(a_i : p_i \leftarrow v_i) - \sum_{r_j \in R} f_{e_i}(r_j : e_j \rightarrow e_j{}')$$