Chance-constrained Probabilistic Simple Temporal Problems

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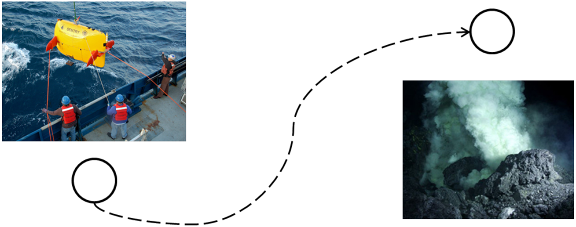
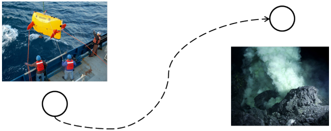
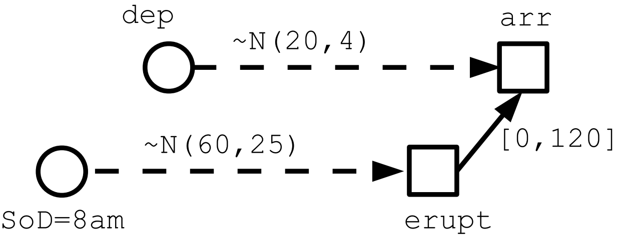
Problem characteristics

- Uncertain durations in logistics
  - Traversal time of AUV $\sim N(20, 4)$;
  - Eruption relative to 8am $\sim N(60, 25)$;
- Timing requirements
- Require: Guarantee on probability of success



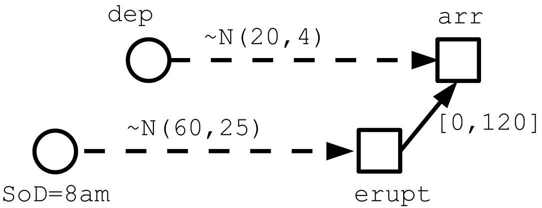
Example problem





Require 99% probability of success

Example: Risk averse approach



- Minimize probability of failure
- Resulting: close to 7.70×10^{-5}% probability of failure
- Departure: 84 minutes after SoD

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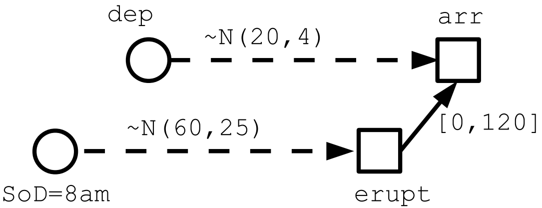

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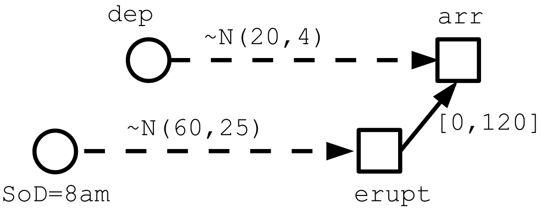

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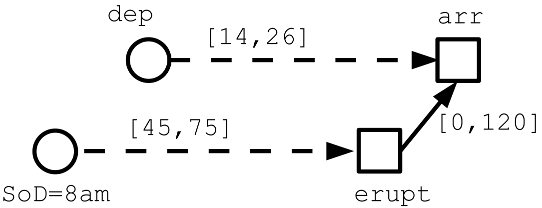
- Probably: much safer than it needs to be

Example: STNU approach



- Map distributions to set-bounded uncertainty
  - Unbounded distribution - try mapping 3-σ bound
- Solve with standard STPU algorithms
- Departure: 61 minutes after SoD

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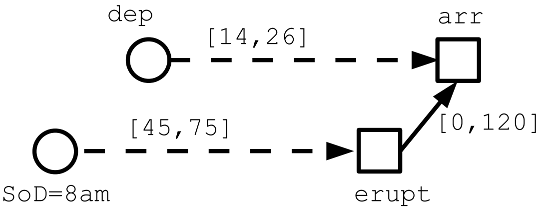

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- Still conservative? Are these the best uncertainty bounds?

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  - Set of uncontrollable durations
  - Set of temporal constraints
  - Δ upper bound on the probability of failure

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- Such that:
  - Time points optimal for utility function V
  - Temporal constraints consistent with probability at least Δ

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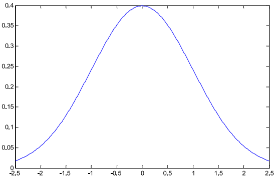
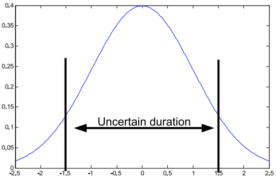
- Risk allocation: makes best use of available risk

Intuition behind risk allocation



- Intuition:

Intuition behind risk allocation



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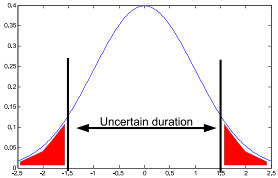

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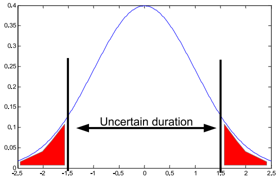

- Intuition:
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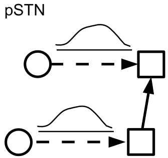
  3. Test set bounded durations for strong controllability

Solution of CC-pSTP



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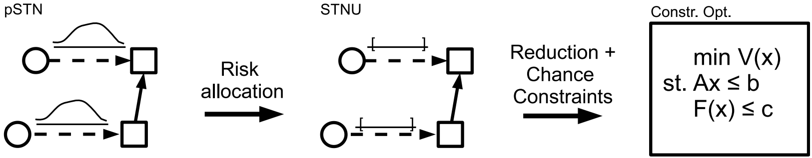
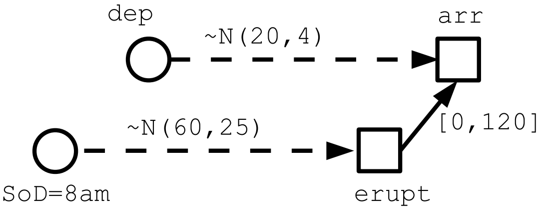


Example: CC-pSTP approach



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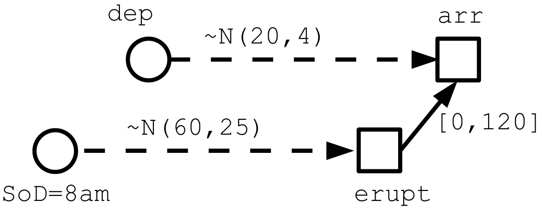

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- Comparison:
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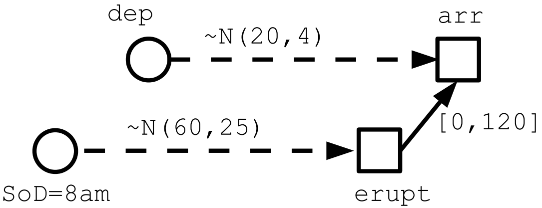

- Optimised time, for 99% probability of success: **57.775** minutes after SoD
- Comparison:
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- Uncertainty bounds: [14,30] and [36,72]

Simulation sets

- Based on Zipcars
- Schedule for 6 hour period
- Up to 20 cars
- Up to 5 users each car
- Up to 3 goal locations for each user
- Traversal durations based on a map of Boston
- 1800 cases

Number of solutions v constraints

		Number of activities to schedule			
		≤ 10	$11-20$	$21-30$	>31
Risk 10%	Even distribution solutions	0	0	0	0
	cc-pSTP solutions	**143**	**16**	**1**	**1**
Risk 20%	Even distribution solutions	0	0	0	0
	cc-pSTP solutions	**146**	**17**	**1**	**1**
Risk 40%	Even distribution solutions	28	0	0	0
	cc-pSTP solutions	**151**	**19**	**1**	**1**
	Risk minimisation solutions	161	22	2	1
	Total number of Scenarios	428	230	165	977

Probability of success

Method	$P(Success)$ $(\pm 1 - \sigma)$
10% cc-pSTP	0.9012 ± 0.0018
20% cc-pSTP	0.8059 ± 0.0051
40% cc-pSTP	0.6250 ± 0.0198
Min. Risk	0.9372 ± 0.1801

Conservatism

Method	Improvement over risk min
10% cc-pSTP	5.37%
20% cc-pSTP	6.58%
40% cc-pSTP	6.82%

Contributions

- Currently: set-bounded uncertainty in scheduling
- Previous probabilistic approach: risk minimisation
- New approach: chance-constrained probabilistic scheduling
- Key: set uncertainty bounds via chance-constrained optimisation
- Outcomes:
  - Guarantee on success
  - Less conservatism

References I

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I. Tsamardinos. A probabilistic approach to robust execution of temporal plans with uncertainty. *Methods and Applications of Artificial Intelligence*, pages 751–751, 2002.

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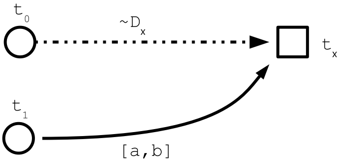
Reduction constraints



Reduction:

- l_x, u_x bounds for probabilistic $t_x - t_0$
- $t_0 - t_1 + l_x \geq a$
  $-t_0 + t_1 - u_x \geq -b$

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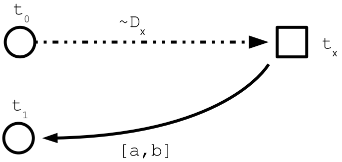

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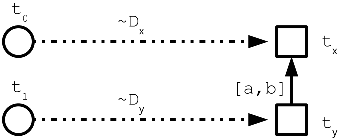

Reduction:

- l_x, u_x bounds for probabilistic $t_x - t_0$
- l_y, u_y bounds for probabilistic $t_y - t_1$
- $t_0 - t_1 + l_x - u_y \geq a$
  $-t_0 + t_1 - u_x + l_y \geq -b$

Chance constraints

- Let Δ upper bound on failure probability
- Each uncertain duration d has cdf F_d, bounds l_d, u_d
- Require

$$\sum_{d \in R_d} F_x(l_d) + (1 - F_d(u_d)) \leq \Delta$$

R_d set of all uncertain durations