Resolving Over-constrained Probabilistic Temporal Problems through Chance Constraint Relaxation

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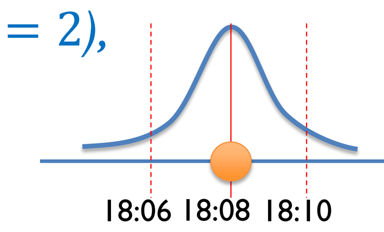
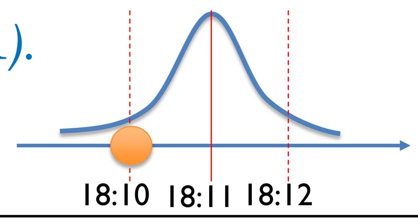
Motivation & Objective

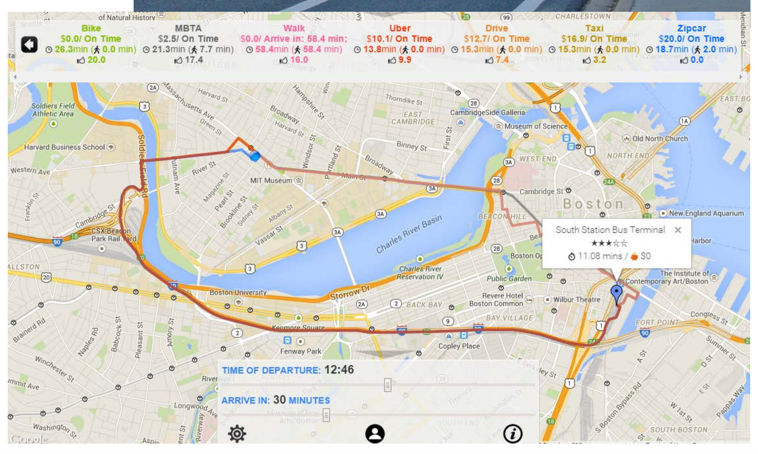
Uncertainty is hard to plan with: when planning for trips, we human beings can hardly evaluate the uncertainty accurately and plan for it.

*"It is 6pm now. I want to arrive home in 40 minutes. I can take Bus #3 at 18:08 ($\sigma = 2$), **8** mins walk to stop, and **24** mins on bus.*



*or Bus #934 at 18:11($\sigma = 1$). **10** mins walk to stop, and **27** mins on bus."*





Objective: Resolve infeasible chance-constrained temporal problems through making **trade-offs** between **risk taken** and **timing requirements**:

"You have 85% chance of catching Bus #934 and arrive home 3 minutes late."
"Or, take Bus #3 and arrive home on time, but there is a 50% risk of missing the bus, if it arrives early."

Application: This project is in support of a personal commute advisor and an AUV mission advisory system.

- We model travel and mission plans using chance-constrained probabilistic Simple Temporal Problems (cc-pSTPs), in which:

A subset of temporal durations are **probabilistic**, the chance of feasibility is restricted by a **chance constraint**, and both temporal and chance constraints can be **relaxed**.

Temporal and Chance Constraint Relaxations

Temporal Relaxations

- A *continuous relaxation*, TR_i, weakens a temporal constraint $[LB, UB]$ to $[LB', UB']$ where $LB' \leq LB$ and $UB' \geq UB$.

"Delay your arrival by 5 minutes."

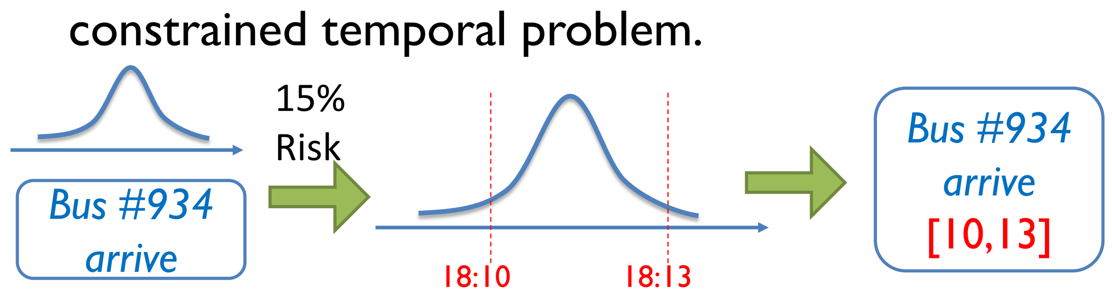
Chance Constraint Relaxations

- A *chance constraint relaxation*, CR_i, increases the chance constraint from CC to CC' where $CC' \geq CC$.

"Can you accept 15% risk of missing the deadline instead of 5%."

Resolving Over-constrained Problems

- A valid set of continuous and chance constraint relaxations restores the feasibility of a chance-constrained temporal problem.



- It presents a risk allocation over probabilistic temporal constraints that meets the chance constraint and enables a **dynamically controllable STNU** for execution.

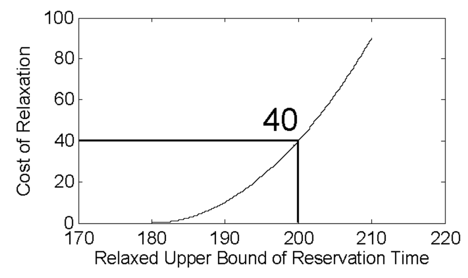
Defining Preferred Solutions

1) Each choice is mapped to a positive reward using function f_p, and computed using addition.

Bus	#3	40
	#934	100

Assignment: $Bus = \#3$
Reward: 40

2) Chance constraint relaxation is mapped to a positive cost using function f_{cr}, and each *temporal relaxation* is mapped to a positive cost using function f_{tr}.

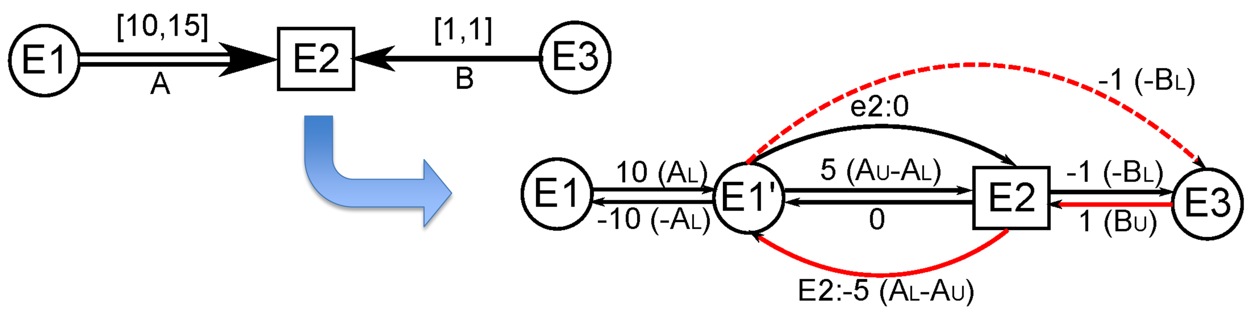


- *RiskTaken*= 5% → 15%.
  Cost: $\boldsymbol{f_{cr}}(0.15 - 0.05) = 50$
- *ArriveIn*$[0,180] \rightarrow [0,200]$
  Cost: $\boldsymbol{f_{tr}}(200 - 180) = 40$

Generating Continuous Relaxations for "Continuous" Conflicts

Step 1: Learning Conflicts

- A conflict composes of an inconsistent set of temporal constraints.
- Given a risk allocation, which converts probabilistic uncertainty to set-bounded uncertainty, we can learn conflicts from the negative cycles detected by controllability checking algorithms[1][2].



Step 2: Mapping to Continuous Constraints

- Each edge in the cycle represents a linear expression defined over the lower and upper bounds of constraints.
- To eliminate the conflict, the sum of these expressions must be **larger or equal to zero.**

$$E1' \rightarrow E3: -B_L;$$
$$E3 \rightarrow E2: B_U;$$
$$E2 \rightarrow E1': A_l - A_U;$$

$$B_U + A_L$$
$$-A_U - B_L \geq 0$$

Step 3: Computing Optimal Relaxations

- Given a set of conflicts, we formulate a constrained optimization problem and compute the optimal resolution using IPOPT, subject to the continuous constraints.

$$\min f_{cr}(cc' - cc) + \sum_{i=1}^{|tc|} f_{tc}(tc_i' - tc_i)$$

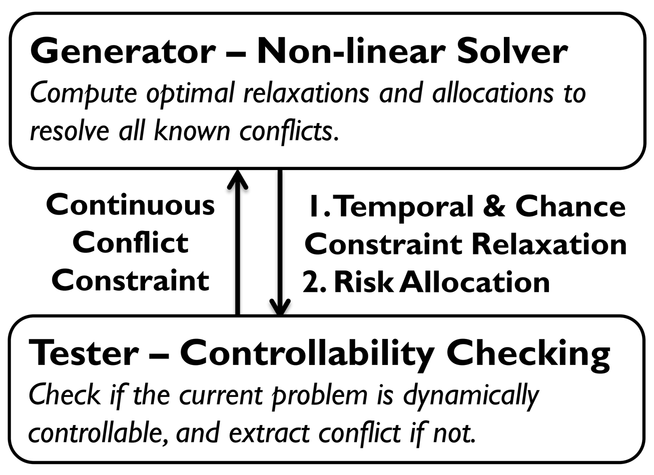
$$s.t. \quad Conflict_m \geq 0$$

$$\sum_j Risk(pt_j) \leq cc'$$

- Minimize the **cost** of temporal (tc_i') and chance constraint (cc') **relaxations**.
- Make all **conflict** expressions non-negative.
- Bound the risk allocated over all **probabilistic** temporal constraints (pt_j).

Best-first Enumeration of Relaxations

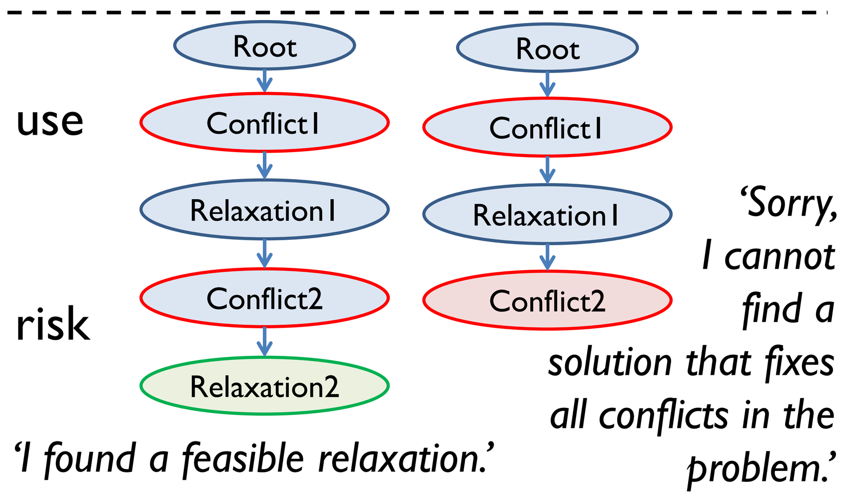
We developed the Conflict-Directed Chance-Constraint Relaxation (CDCR) algorithm for enumerating relaxations to chance-constrained temporal problems in best-first order. It takes a generate and test approach for discovering conflicts and refine candidate relaxations.



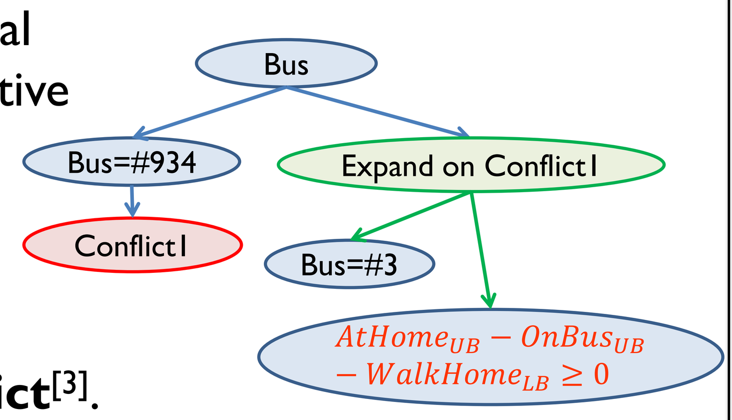
Each time a conflict is detected, we compute relaxations for it and all known conflicts, then use them to extend the search tree.

The search terminates if

- a relaxation is found that enables a feasible risk allocation and a **controllable STNU**,
- or when a conflict cannot be resolved.



If there are choices in the problem (such as temporal problems with alternatives), we can also use alternative assignments that suspend constraints to resolve conflicts.

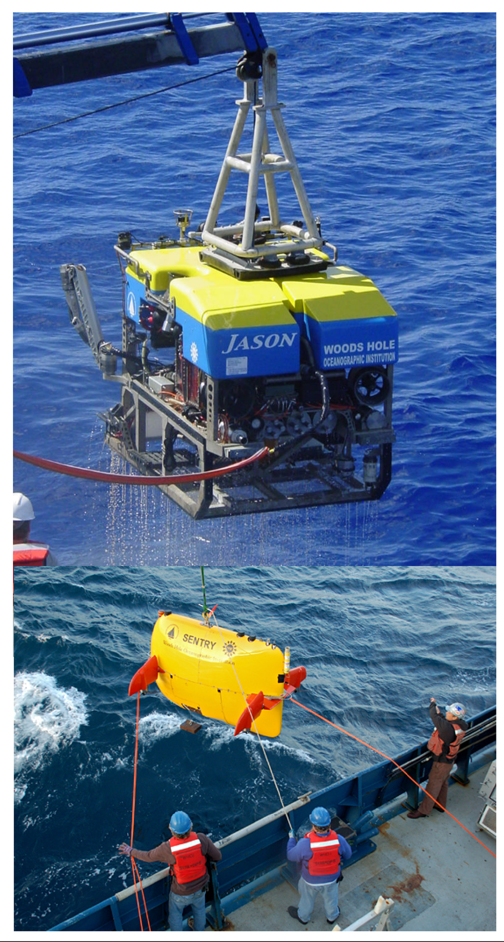


The search space will be explored using both **expansion on variable** and **expansion on conflict**[3].

Applications & Experiments

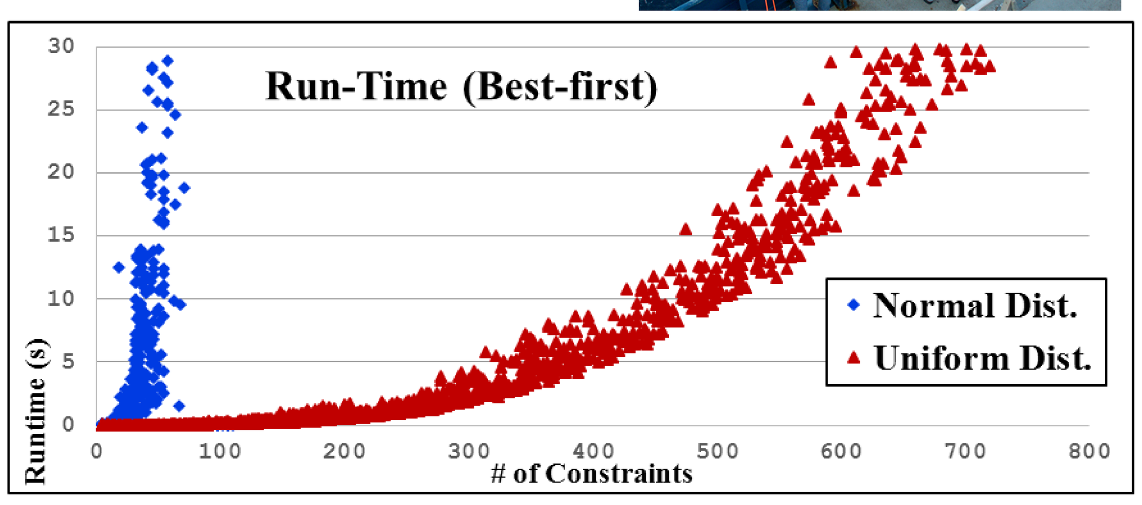
- In addition to transit advisor, CDCR has been integrated into a mission advisory system to help oceanographers schedule autonomous underwater vehicle (AUV) operations with high uncertainty. The goal is to improve the robustness of their plans and reduce their workload.
- To benchmark its performance, we simulated a set of AUV missions using randomly generated target locations and mission constraints, by varying:

a. Number and length of activities in a mission.
b. Risk bounds and uncertainty distributions of activities.
c. Costs over temporal and chance constraint relaxations.



- We tested CDCR on problems with two types of uncertainty distributions: uniform and normal.

- CDCR performs much better on problems with uniform distributions, since conflict resolution is much more costly for non-linear distributions.
- Hence tractability must be considered when selecting density functions for modeling temporal uncertainty.



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[1] Morris, P. 2006. A structural characterization of temporal dynamic controllability. In Proceedings of the 12th International Conference on Principles and Practice of Constraint Programming, 375–389.
[2] Yu, P.; Fang, C.; and Williams, B. 2014. Resolving uncontrollable conditional temporal problems using continuous relaxations. In Proceedings of the Twenty-fourth International Conference on Automated Planning and Scheduling, 341–349.
[3] B. Williams and R. Ragno. Conflict-directed A* and its role in model-based embedded systems. In *Discrete Applied Mathematics*, 155(12):1562–1595, 2007.