Continuously Relaxing Over-constrained Conditional Temporal Problems

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Take Away Messages

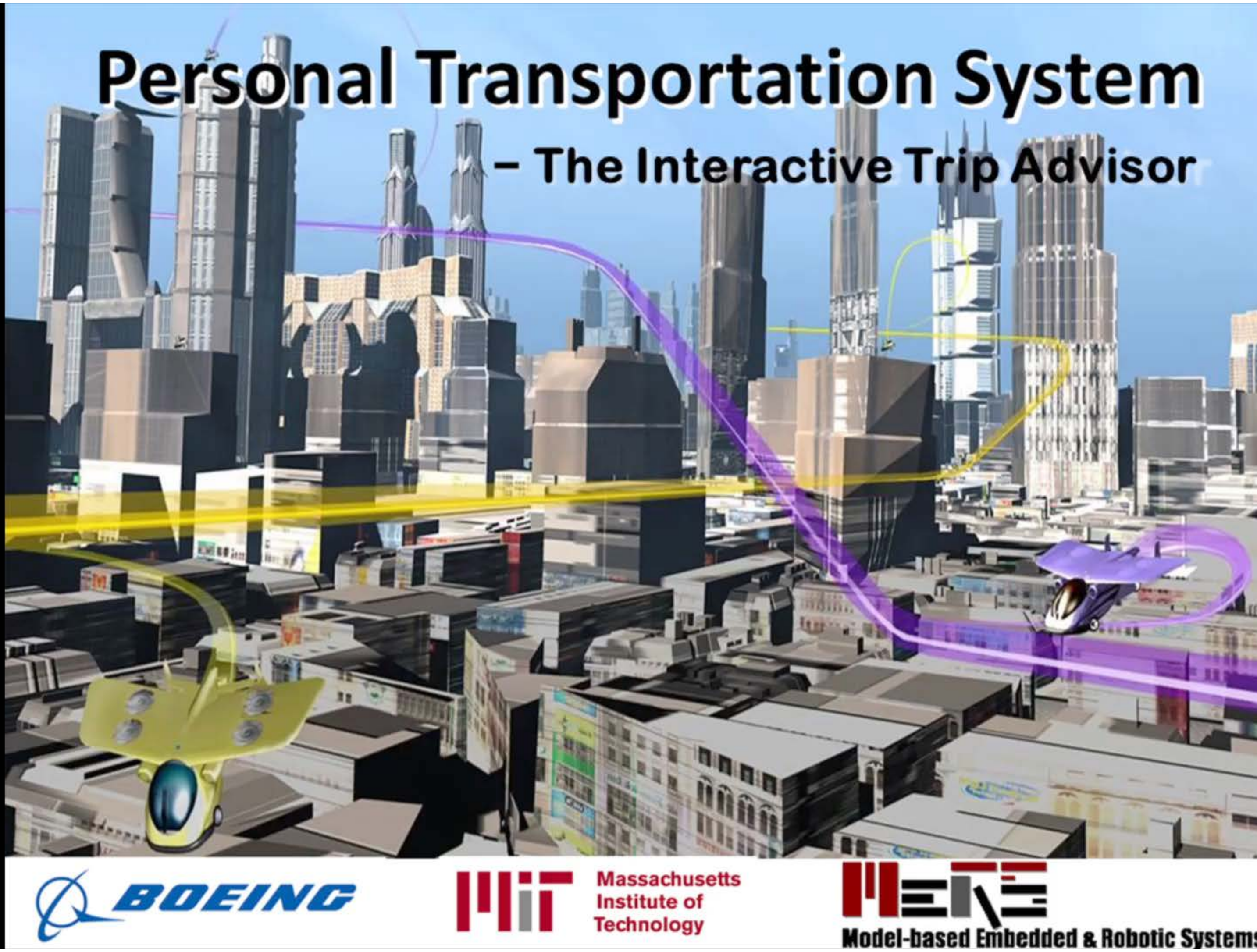
- Over-constrained temporal problems can be better resolved by relaxing the temporal constraints continuously, instead of removing them discretely.

- The fundamental concepts of conflicts and minimal relaxations naturally generalize to the continuous case.

- We can efficiently enumerate discrete and continuous relaxations in best-first order, by generalizing the Conflict-Directed A* algorithm, first developed for diagnosis.

Robotic Personal Transportation System

- A personal air taxi with an intelligent trip advisory system.



Key features

- Find alternative solutions that are **simple** and **preferred**.
- Provide **insights** into cause of failure and its resolution.

  – Minimize the perturbations;

  "Delay your arrival by 5 minutes".

  – Prioritize alternatives;

  "OK, then how about having lunch at restaurant Y".

  – Explain the cause of failure;

  "Because of the extended travel time".

  – Adapt incrementally to new constraints.

  "if you want to shop for at least 25 minutes, you can have lunch at restaurant Y for 55 minutes".

Contents

- Relaxations of Conditional Temporal Problems;
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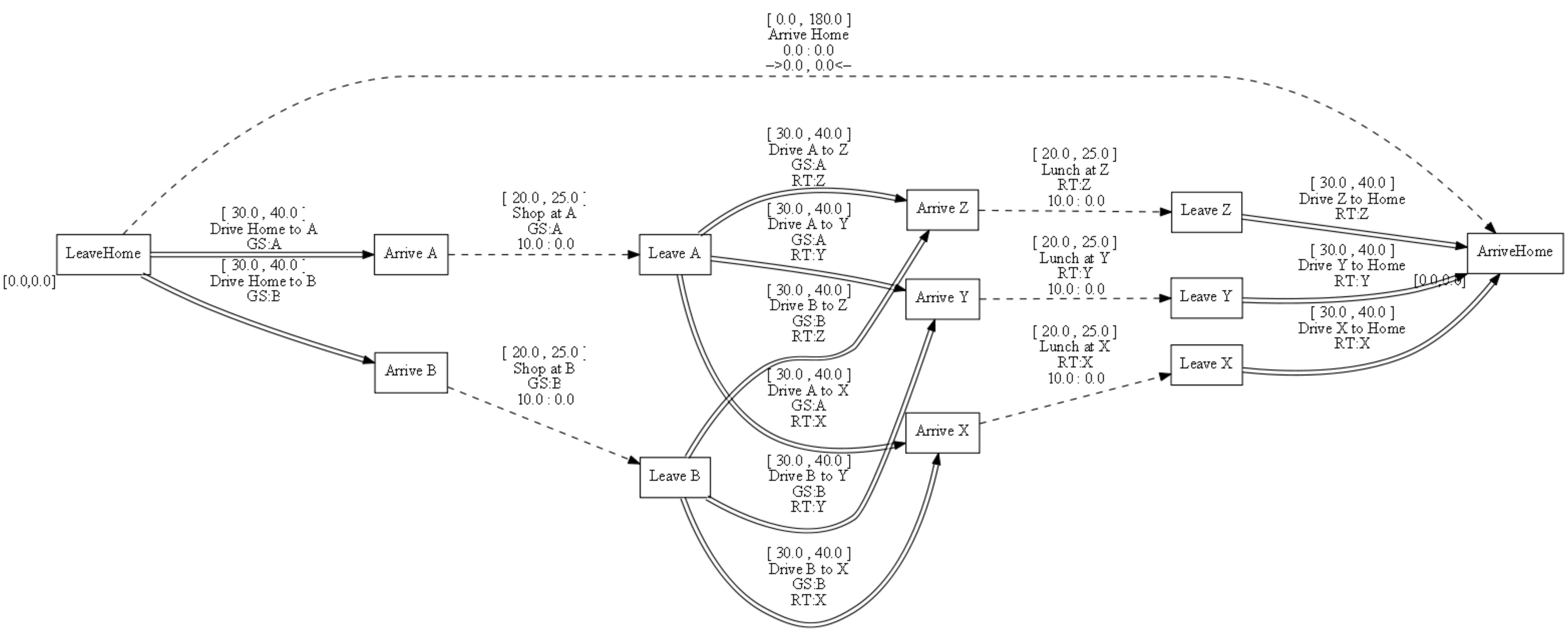
- Best-first Enumeration through Conflict-directed Relaxation;
- Applications.

Problem Formulation

- Model: (Over-constrained) Controllable Conditional Temporal Problems.
  - All choices are controllable.
  - Allowing temporal constraints to be relaxed:

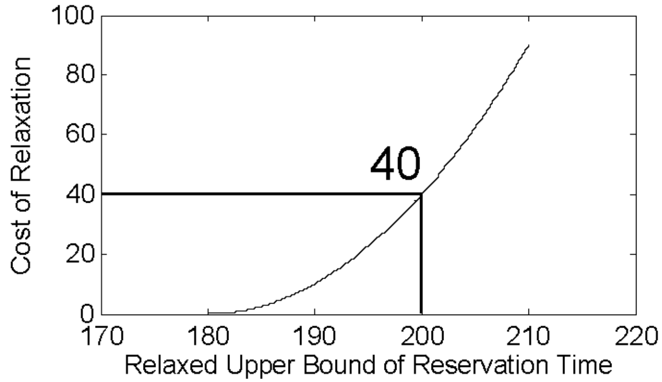
  a subset of the temporal constraints, $RE \subseteq E$, are relaxable.



Define User Preferences

- Preference functions are defined over decisions and constraint relaxations.
  - Each decision is mapped to a positive reward by function f_p.
  - Each constraint relaxation is mapped to a positive cost by function f_e.

Store	A	40
	B	100
Lunch	X	70
	Y	80
	Z	30



Assignment :$\{Store = B, Lunch = Y\}$
Reward: $100 + 80 = 180$

Relaxation: $Reservation[0,180] \rightarrow [0,200]$
Cost: $f_e(200 - 180) = 40$

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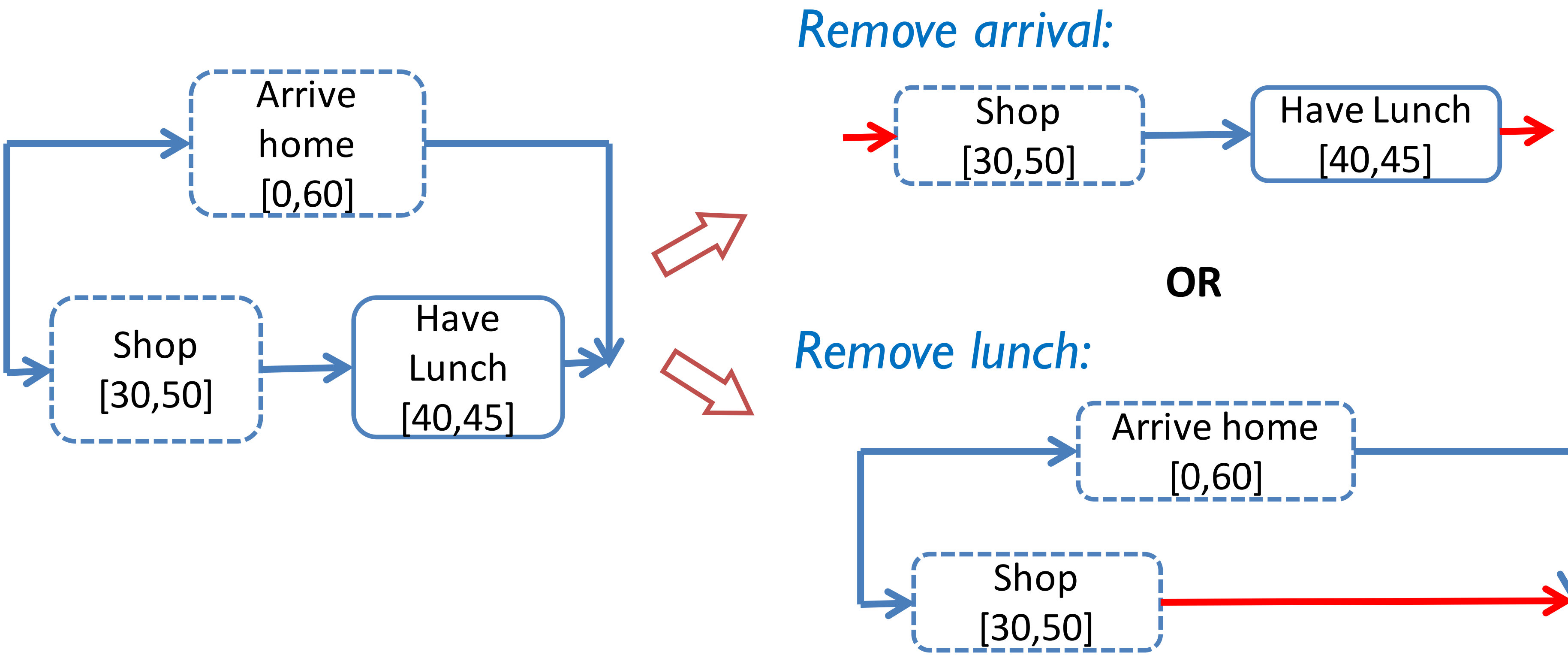
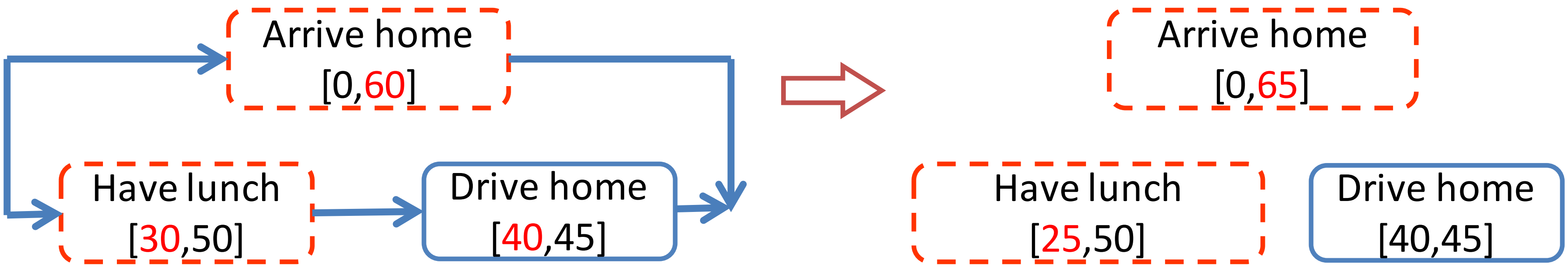
(Minimal) Discrete Relaxation

- Resolve over-constrained temporal problem C by **removing** constraints.
  - Resolved: $M \subseteq C$ *such that* $C \setminus M$ *is consistent*.
  - Minimal: $\forall c \in M\ (C \setminus M) \cup \{c\}$ *is inconsistent*.



Continuous Relaxation

- Relax a constraint partially by **continuously** modifying its temporal bounds:
  - A continuous relaxations, CR_i, weakens a temporal constraint: $[LB, UB] \rightarrow [LB', UB']$ where $LB' \leq LB \; and \; UB' \geq UB.$
  - Continuous relaxations only apply to **relaxable** constraints.



"Shorten lunch to 25 minutes and delay arrival by 5 minutes"

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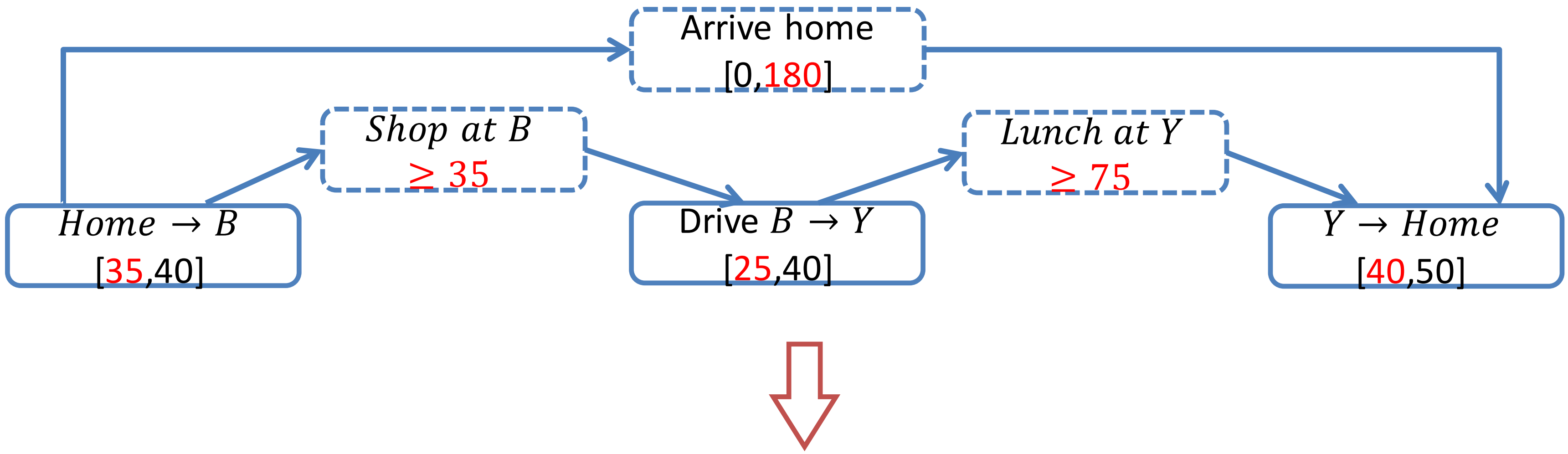
1. Learn Discrete Conflicts

- A discrete conflict is an inconsistent set of temporal constraints.

Choosing Store=B and Lunch=Y produces:



Discrete Conflict:

Store = B;
Home → B ≥ 35;
Drive B → Y ≥ 25;
Y → Home ≥ 40;

Lunch = Y;
Shop at B ≥ 35;
Lunch at Y ≥ 75;
Arrive Home ≤ 180.

2. Weaken to Continuous Conflicts

- A continuous conflict is an equation formed from the discrete conflict.
- It specifies the deviation needed to resolve the conflict.

Discrete Conflict:

$\mathrm{HometoB} \geq 35;$
$\mathrm{ShopatB} \geq 35;$
$\mathrm{BtoY} \geq 25;$
$\mathrm{LunchatY} \geq 75;$
$\mathrm{YtoHome} \geq 40;$
$\mathrm{ArriveHome} \leq 180.$

Continuous Conflict:

$$ArriveHome - HometoB - ShopatB \\ - BtoY - LunchatY - YtoHome = \mathbf{-30}$$

3. Map to Constituent Continuous Relaxations

- Relaxations specified by linear inequalities:

$$\begin{aligned} & ArriveHome - HometoB - ShopatB \\ & \quad -BtoY - LunchatY - YtoHome = \mathbf{-30} \end{aligned}$$

$$\Downarrow$$

$$\Delta_{ShopatB} + \Delta_{LunchatY} + \Delta_{ArriveHome} \geq 30$$

Discrete vs. Continuous Relaxations

- Resolve a conflict by relaxing constraints **completely** or **partially**.

Conflict:

Store = B, Lunch = Y;
Home → B ≥ 35; Shop at B ≥ 35;
Drive B → Y ≥ 25; Lunch at Y ≥ 75;
Y → Home ≥ 40; Arrive Home ≤ 180.

Discrete Resolutions

Remove Shop at B ≥ 35;
Remove Lunch at Y ≥ 75;
Remove Arrive Home ≤ 180

Continuous Resolutions

Lunch at Y ≥ 45;
Arrive Home ≤ 210;
Shop at B ≥ 25 and Lunch at Y ≥ 55;
… …
and many more

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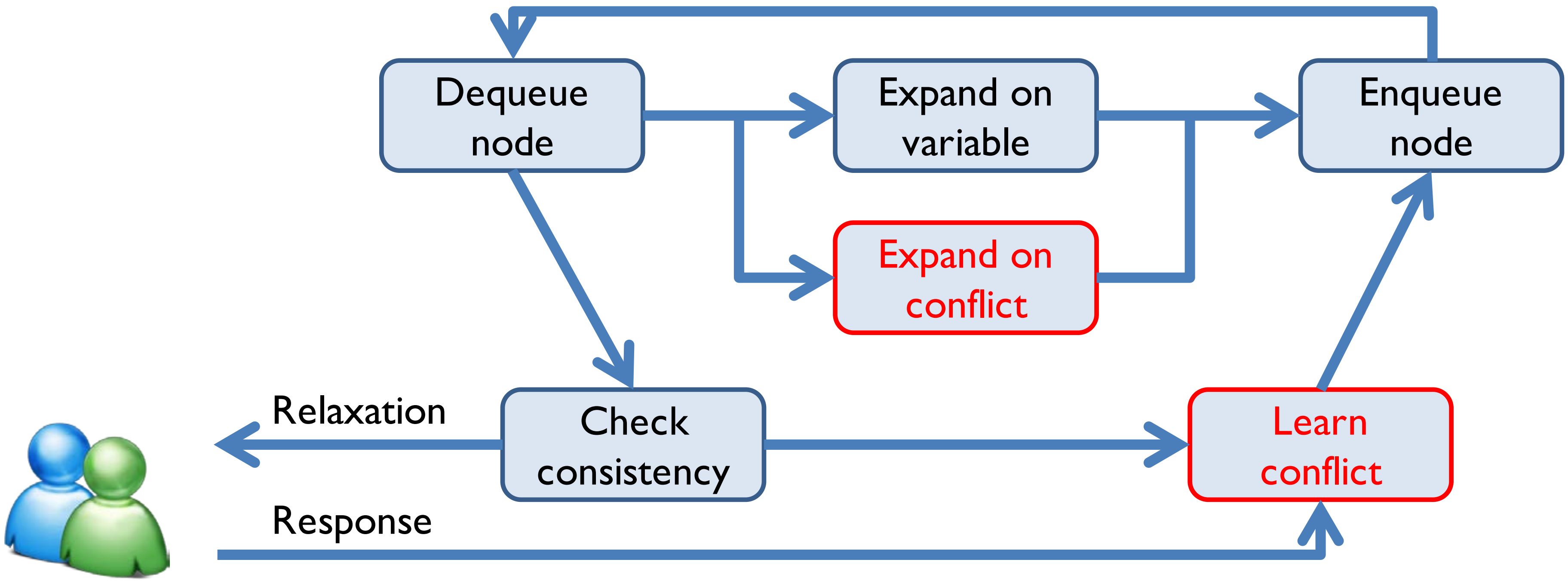
Generalize CDA* to Continuous Relaxations

- Conflict-Directed A* (Williams and Ragno, 2004) can be applied to discrete relaxation problems:
  - Efficiently prunes search space using learned conflicts (Bailey and Stucky, 2005).
  - Enumerates minimal discrete relaxations (Previti and Marques-Silva, 2013) in best-first order.

- To solve a relaxation problem:
  - Frame an equivalent constraint optimization problem.
    - A. Discrete relaxation: add binary variables.
    - B. Continuous relaxation: add non negative continuous variables.
  - The objective function represents the preference.

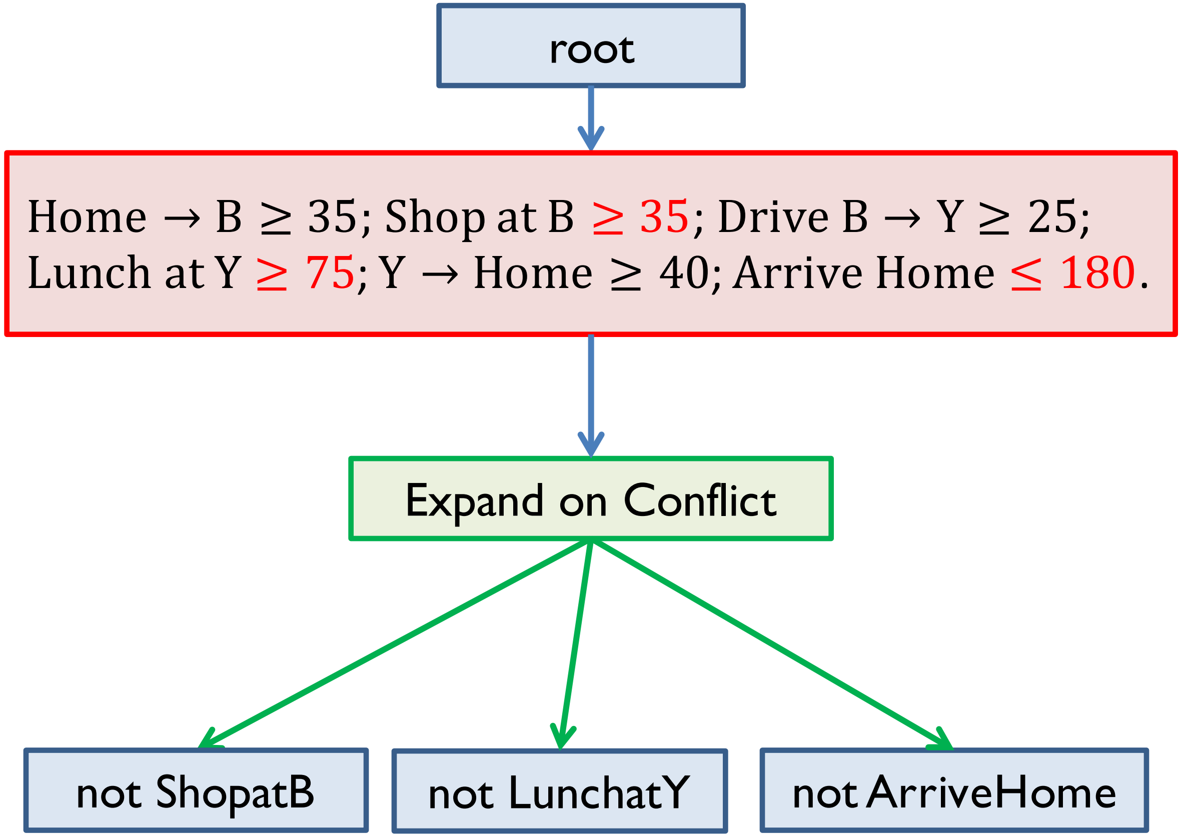
Best-first Conflict Directed Relaxation

- BCDR generalizes the conflict resolution procedure in CDA* to include constituent continuous relaxations.



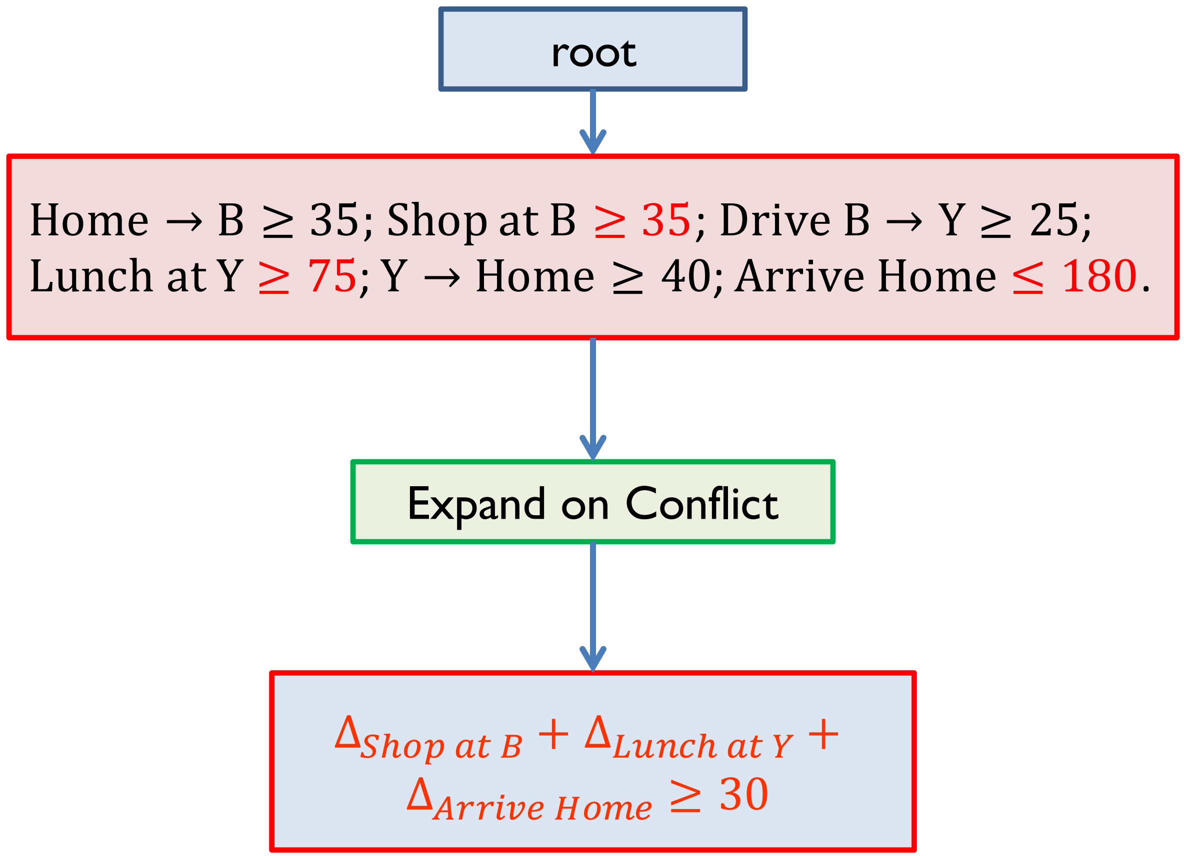
Conflict-Directed A*

- Key Ideas:
  - Split on conflict;
  - Best-first enumeration.



CDA* with Constituent Continuous Relaxation

- Split a conflict using its constituent continuous relaxations.

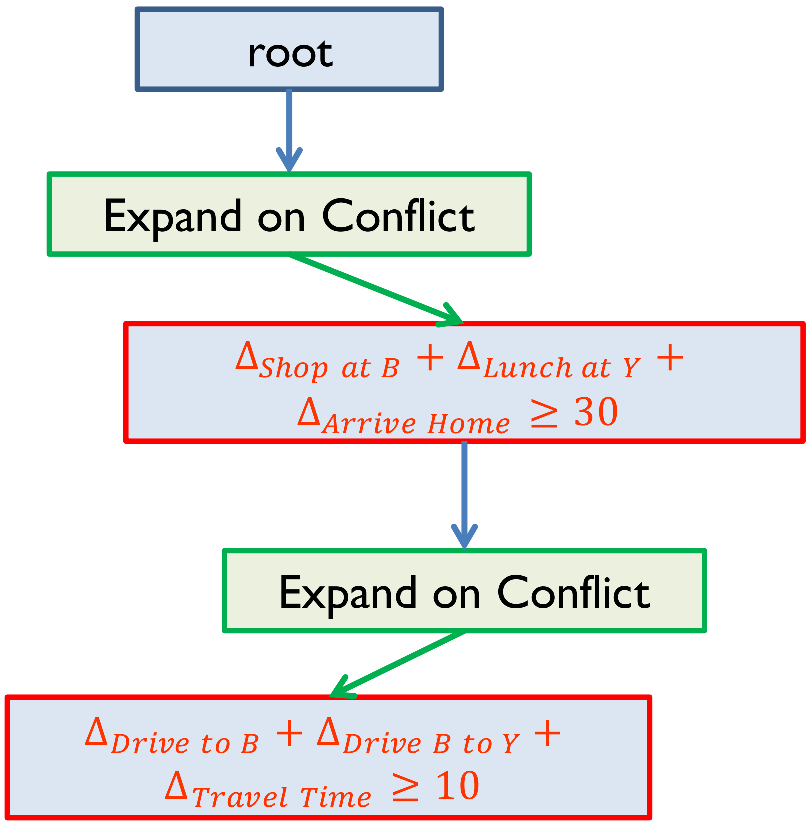


$$\min\left(f\left(\Delta_{Shop\ at\ B}\right) + f\left(\Delta_{Lunch\ at\ Y}\right) + f\left(\Delta_{Arrive\ Home}\right)\right)$$

$$\text{s.t. } \Delta_{Shop\ at\ B} + \Delta_{Lunch\ at\ Y} + \Delta_{Arrive\ Home} \geq 30$$

Continuous Relaxations for Multiple Conflicts

- For two or more continuous relaxations on the same branch, the utility is determined by the grounded solution that respects both inequalities.



$$\min\Big(f\big(\Delta_{Shop\ at\ B}\big) + f\big(\Delta_{Lunch\ at\ Y}\big) + f\big(\Delta_{Arrive\ Home}\big) + f\big(\Delta_{Drive\ to\ B}\big) + f\big(\Delta_{Drive\ B\ to\ Y}\big) + f\big(\Delta_{Travel\ Time}\big)\Big)$$

s.t.

$$\Delta_{Shop\ at\ B} + \Delta_{Lunch\ at\ Y} + \Delta_{Arrive\ Home} \geq 30$$

and

$$\Delta_{Drive\ to\ B} + \Delta_{Drive\ B\ to\ Y} + \Delta_{Travel} \geq 10$$

Incorporating User Responses

- BCDR incrementally adapts to new requirements.
- These requirements are recorded as new conflicts.

Required Continuous Relaxations

No, I **do not** want to extend my reservation time.

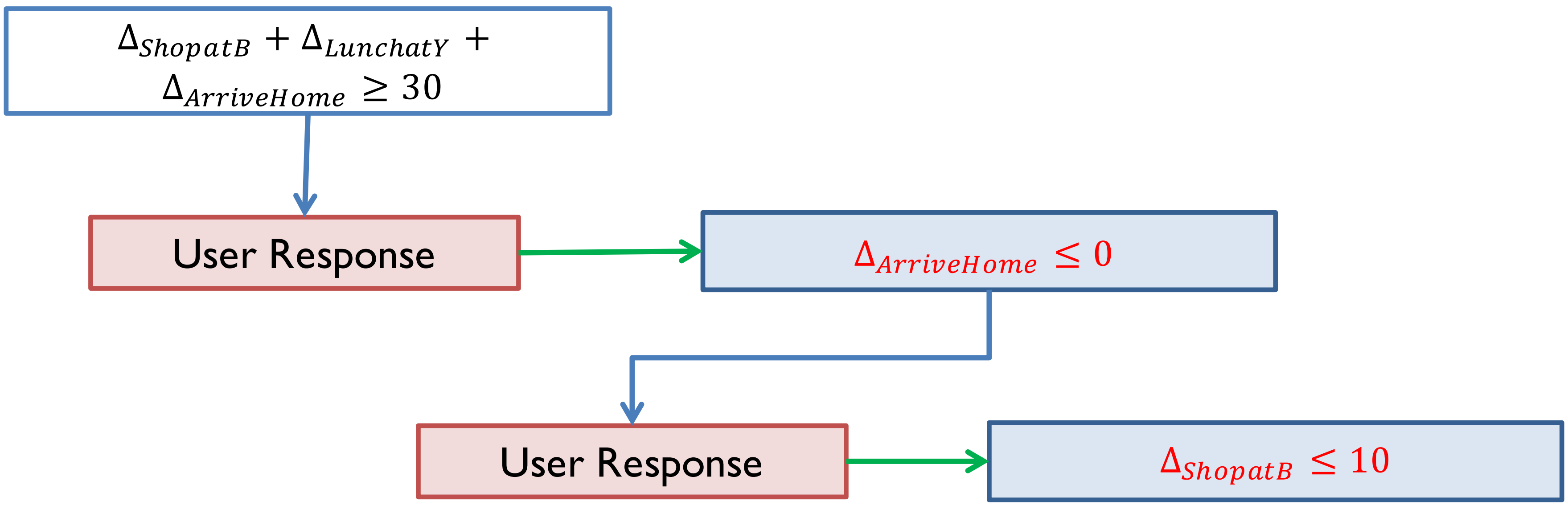
$\Delta_{Arrive\ Home} \leq 0;$

No, I want to spend **at least 25** minutes on shopping.

$\Delta_{Shop\ at\ B} \leq 10;$

New Requirements as Conflicts

- Expand search tree using user response conflicts.

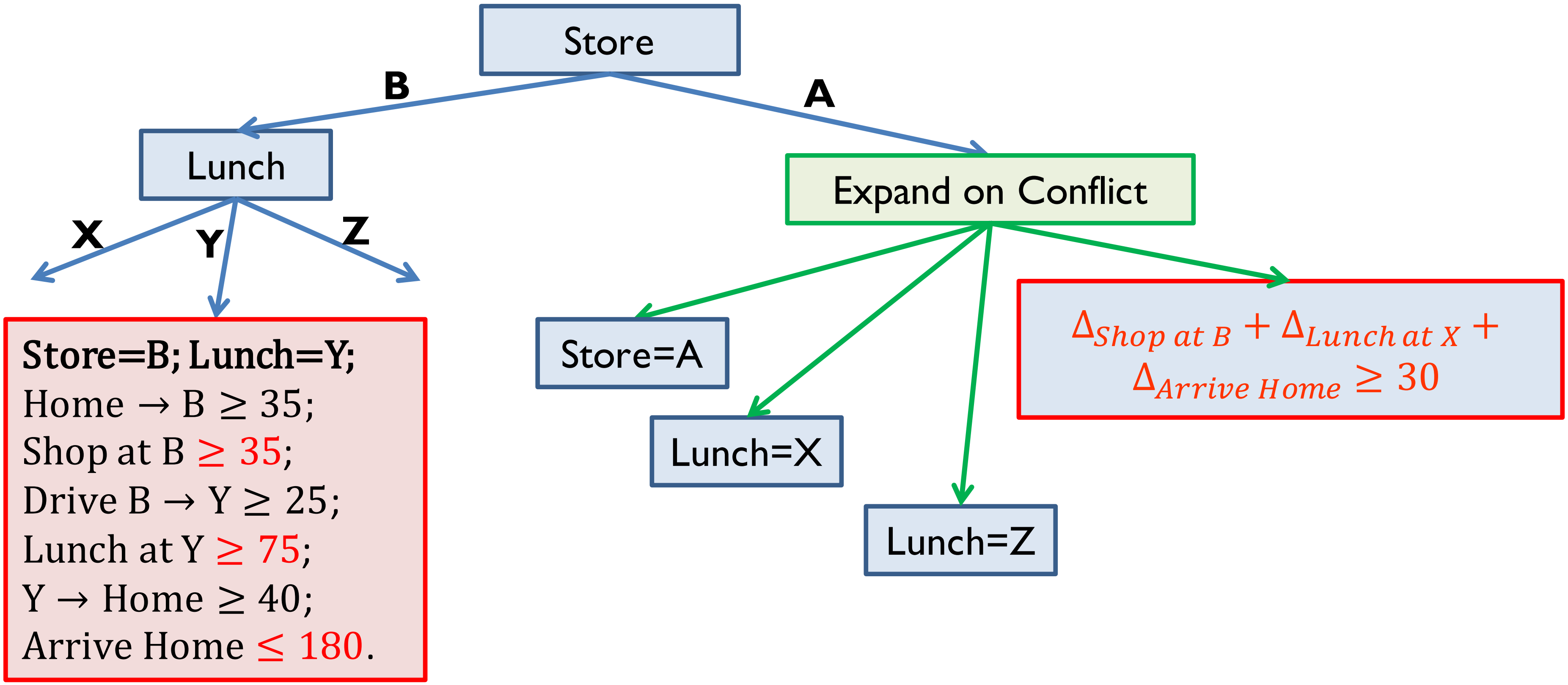


$$\min(f(\Delta_{ShopatB}) + f(\Delta_{LunchatY}) + f(\Delta_{ArriveHome}))$$

$$\text{s.t.}\quad \begin{aligned} &\Delta_{ShopatB} + \Delta_{LunchatY} + \Delta_{ArriveHome} \geq 30; \\ &\Delta_{ArriveHome} \leq 0; \\ &\Delta_{ShopatB} \leq 10. \end{aligned}$$

Split on Conflicts for Conditional Problems

- If a node has an unresolved conflict, we expand it using both constituent **continuous** relaxation and **decisions** that deactivates its constraints.



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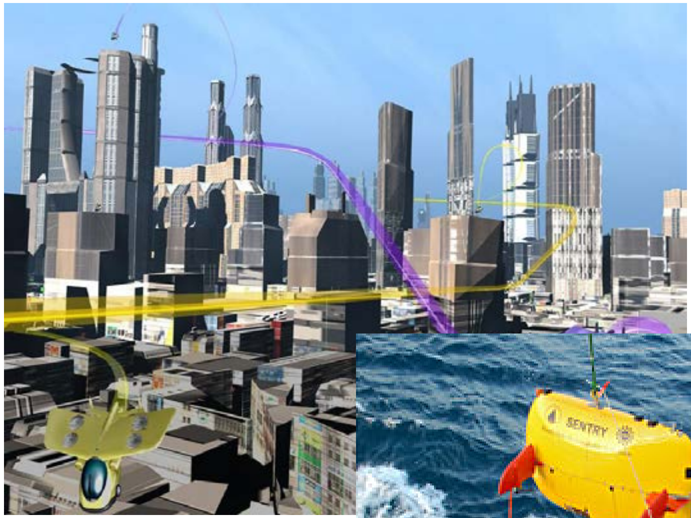
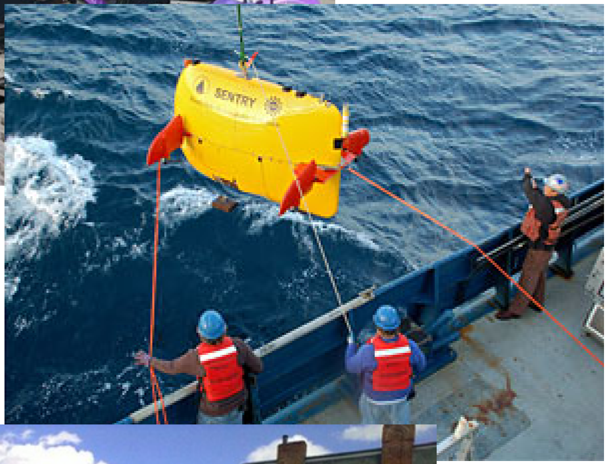
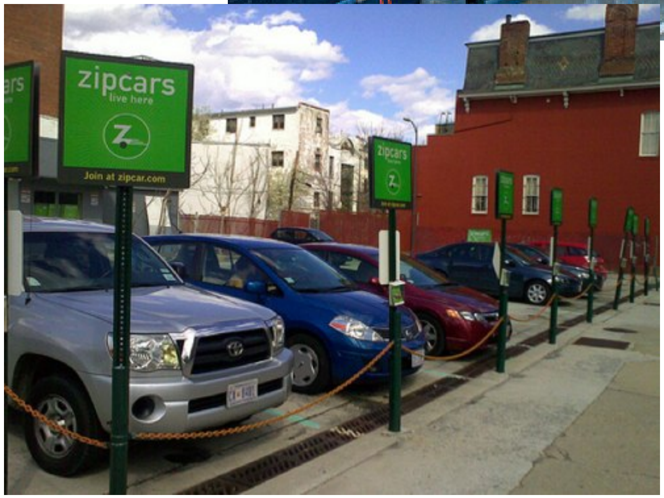
Ongoing Projects

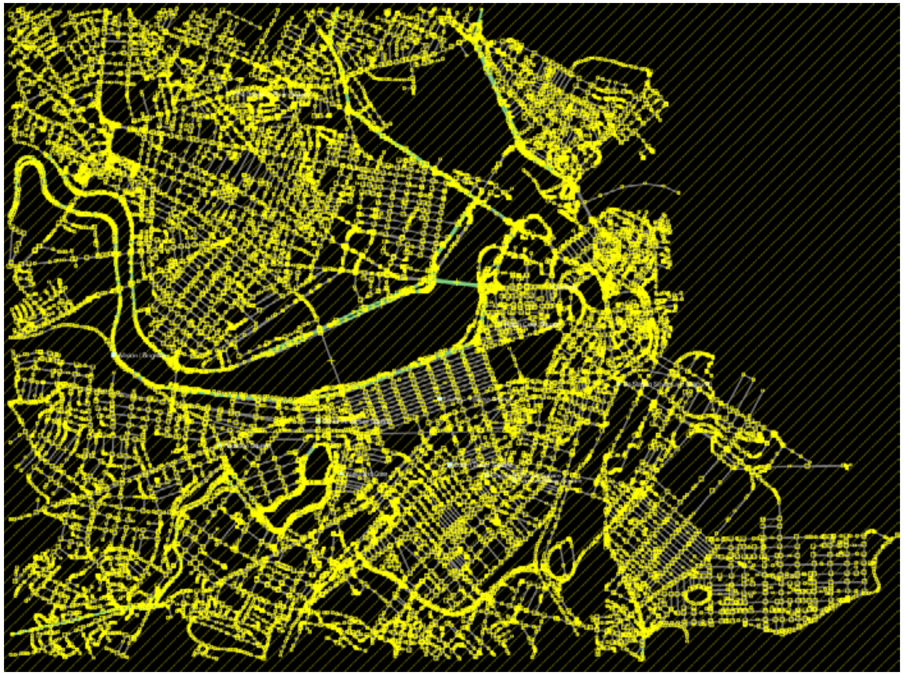
- Personal Transportation System.
- Mission advisory system for autonomous underwater vehicles.
- Trip advisor for car-sharing network users.







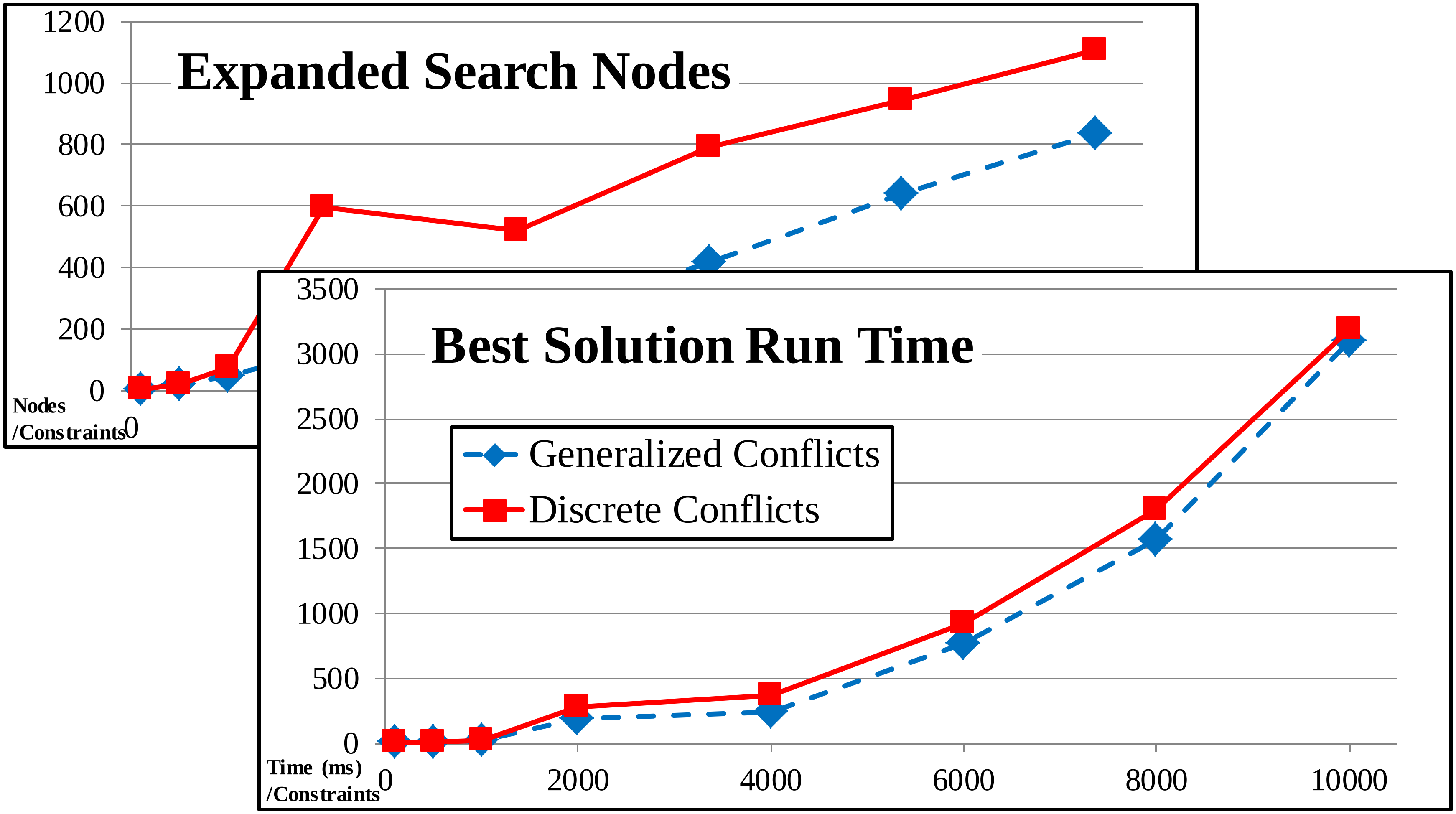
Empirical Validation - Setup



- We simulated a car-sharing network in Boston using randomly generated car locations and destinations.

- Test cases are characterized by:
  - Number of reservations per car.
  - Number of cars in the network.
  - Number of activities per reservation.
  - Number of alternative options per activity.
- Time change may affect neighboring reservations.

- We compare the efficiency of generalized vs. discrete conflict resolution in finding the best continuous relaxation.



Contributions

- Over-constrained temporal problems can be resolved by relaxing the temporal constraints continuously.
- The fundamental concepts of conflicts and minimal relaxations naturally generalize to the continuous case.
- We can efficiently enumerate discrete and continuous relaxations in best-first order, by generalizing the Conflict-Directed A* algorithm.
- Executable, test cases and slides are available at:

  http://people.csail.mit.edu/yupeng/software.html