Flow-Based Algorithms for Local Graph Clustering

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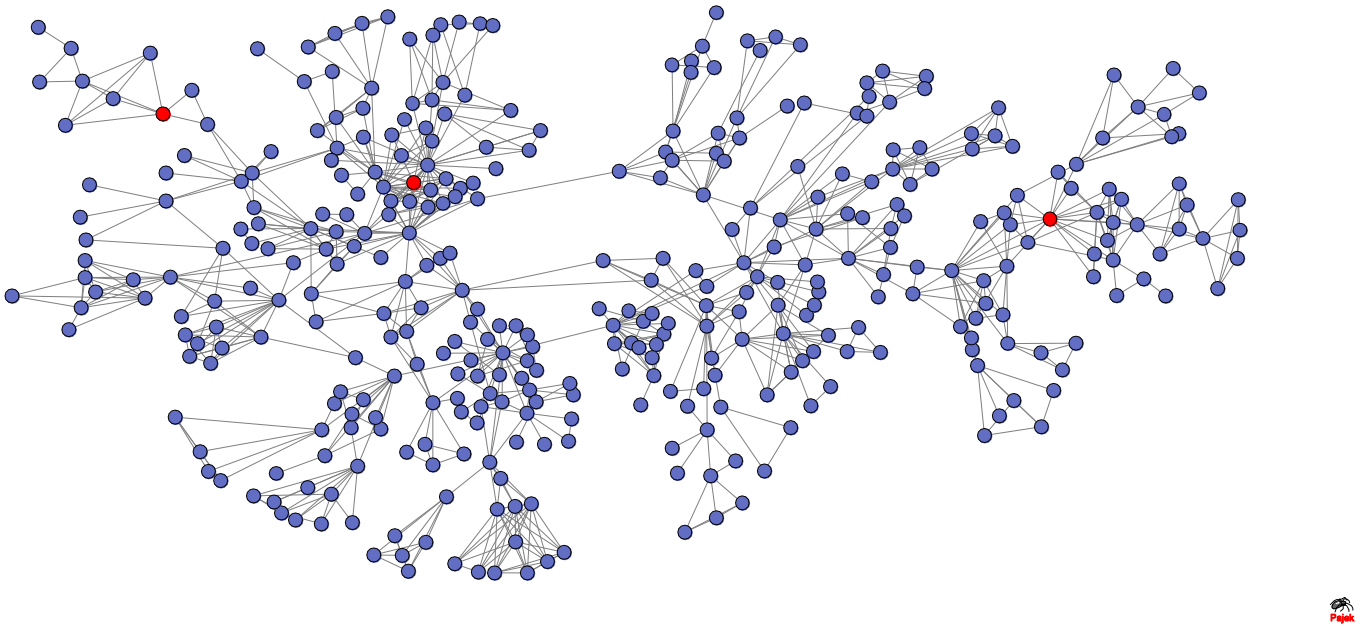
Graph Clustering for Large Networks



INPUT: Large Data or Social Network

GOAL: find clusters, i.e.,
subsets well-connected inside and poorly connected to the rest of the graph.

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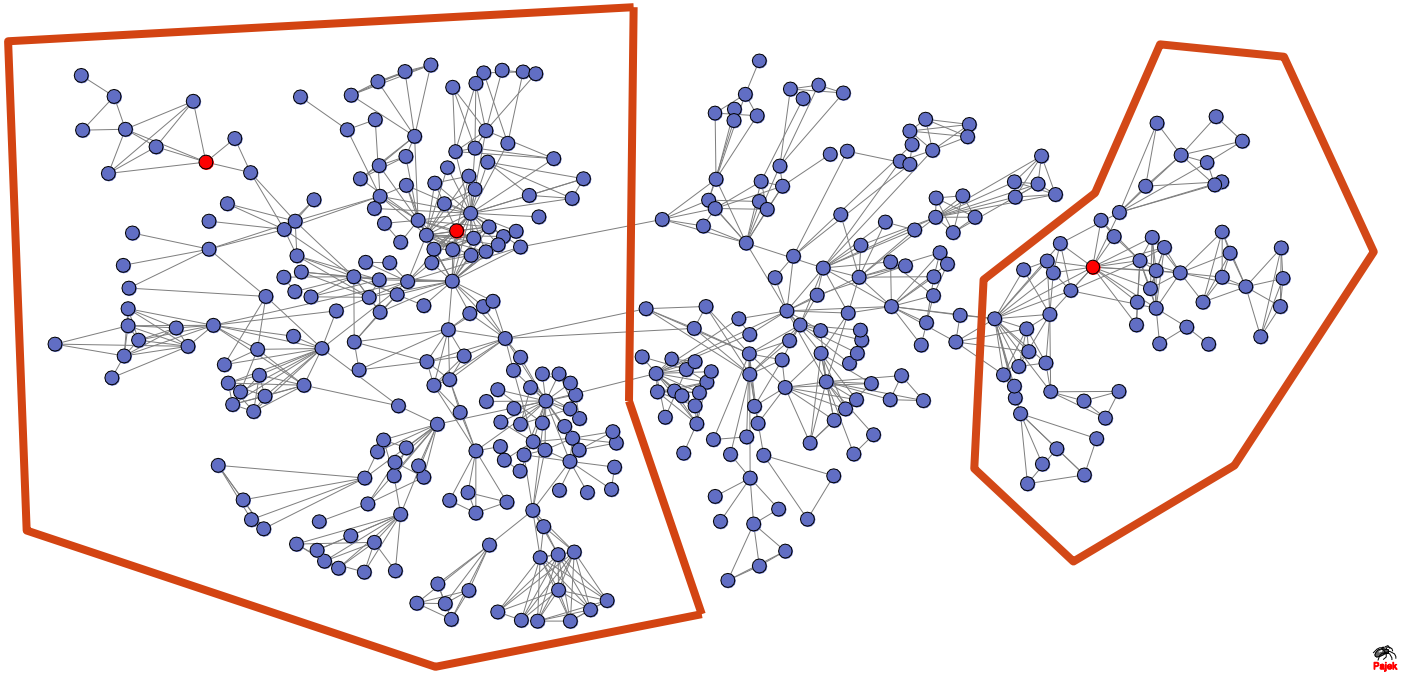

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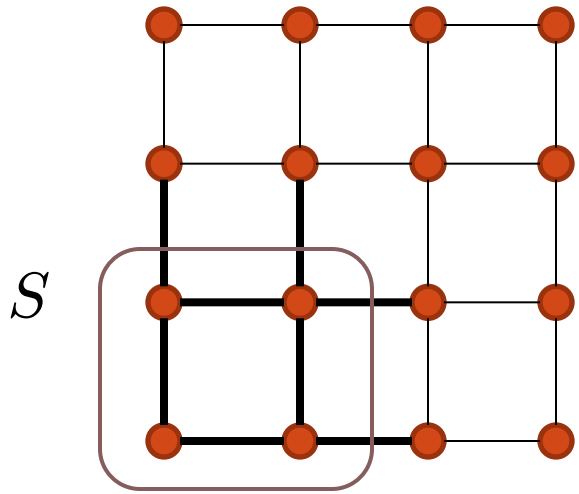
Graph Clustering and Conductance

Undirected unweighted $G = (V, E), |V| = n, |E| = m$



Measure of Cluster Quality is Conductance:

$$\phi(S) = \frac{|E(S, \bar{S})|}{\min\{\mathrm{vol}(S), \mathrm{vol}(\bar{S})\}}$$

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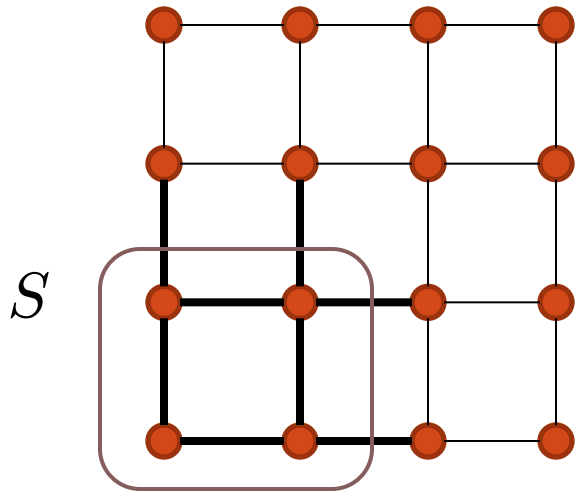

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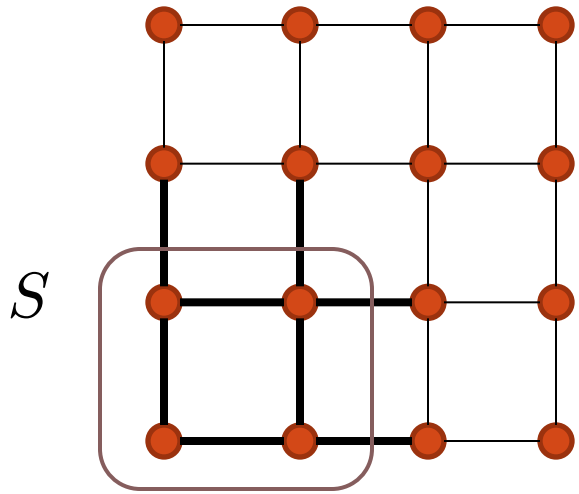

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Random-Walk Interpretation

Given uniform distribution over S,
what is probability of **exiting S in one step** of random walk over the edges of G?

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$$\min_{S \subseteq V} \phi(S)$$

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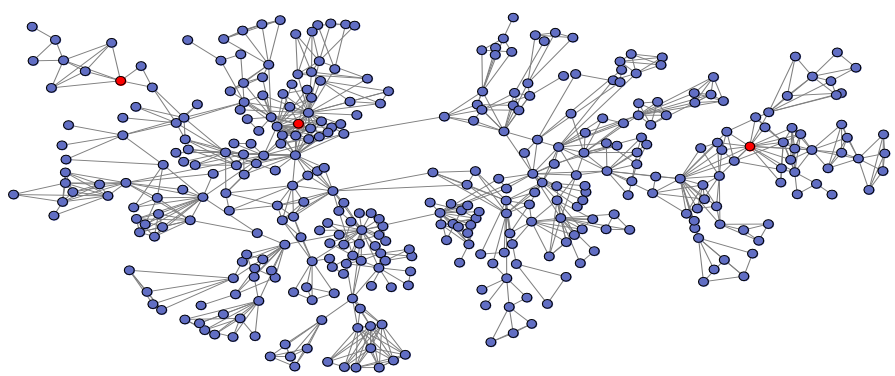

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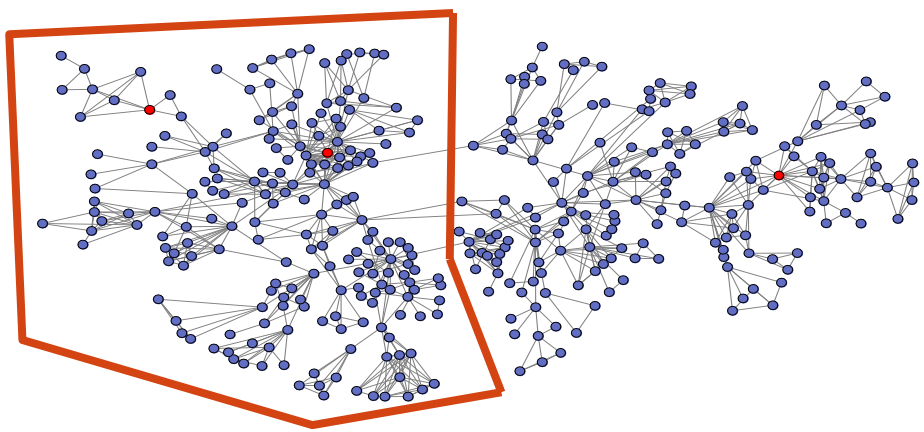

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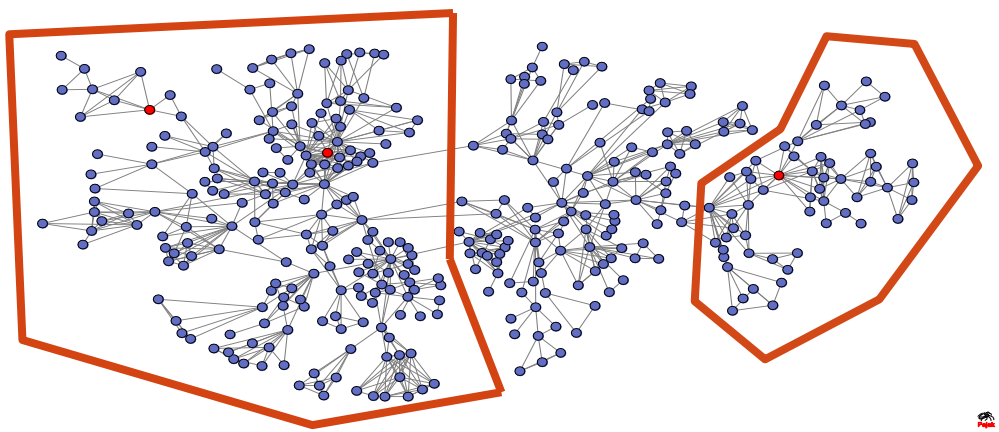

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- Fundamental NP-Complete problem
- Best known approximation algorithm achieves $O(\sqrt{\log n})$-approximation via SDP relaxation
- SDP relaxation can be interpreted as <u>combining spectral and flow techniques</u>

Local Clustering for Large Networks

$$G = (V, E),$$
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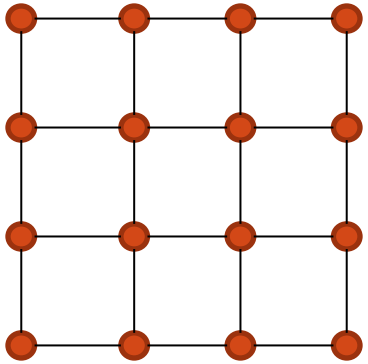
Massive Networks $\longrightarrow$ $n \to \infty,\ m \to \infty$



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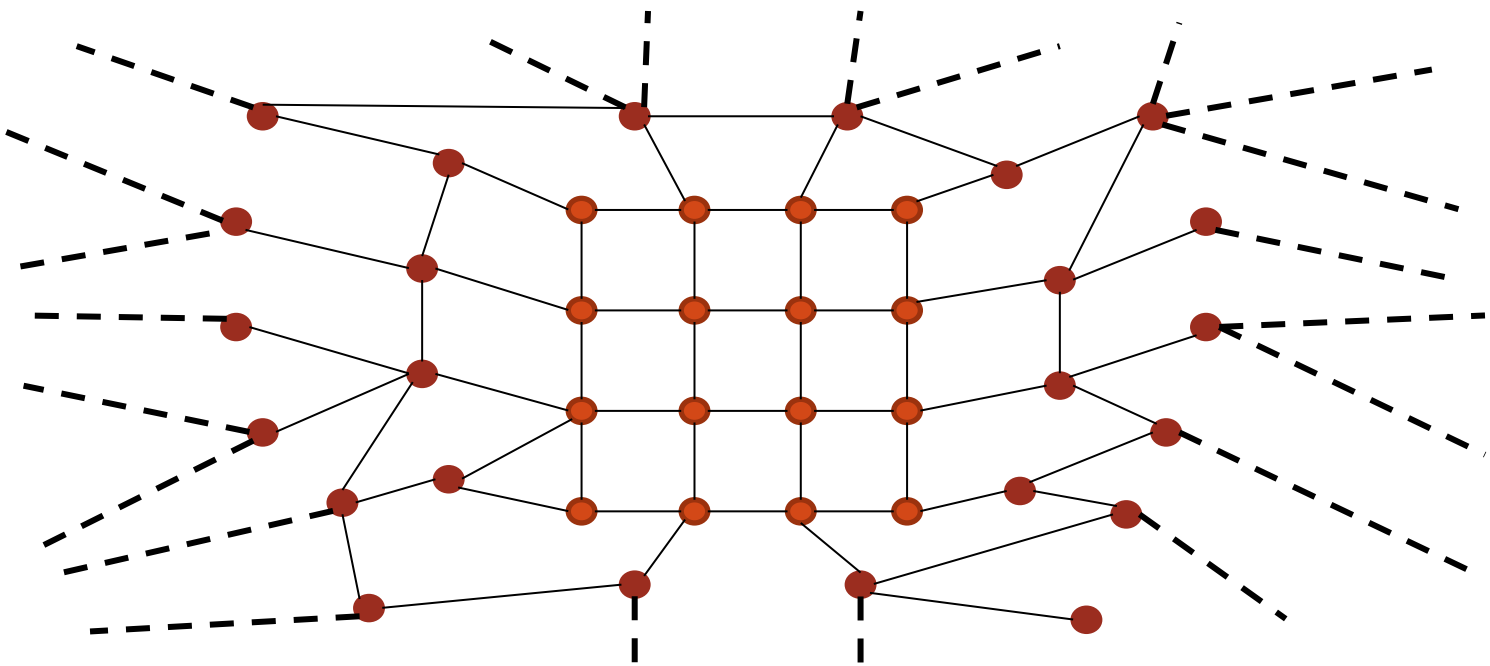

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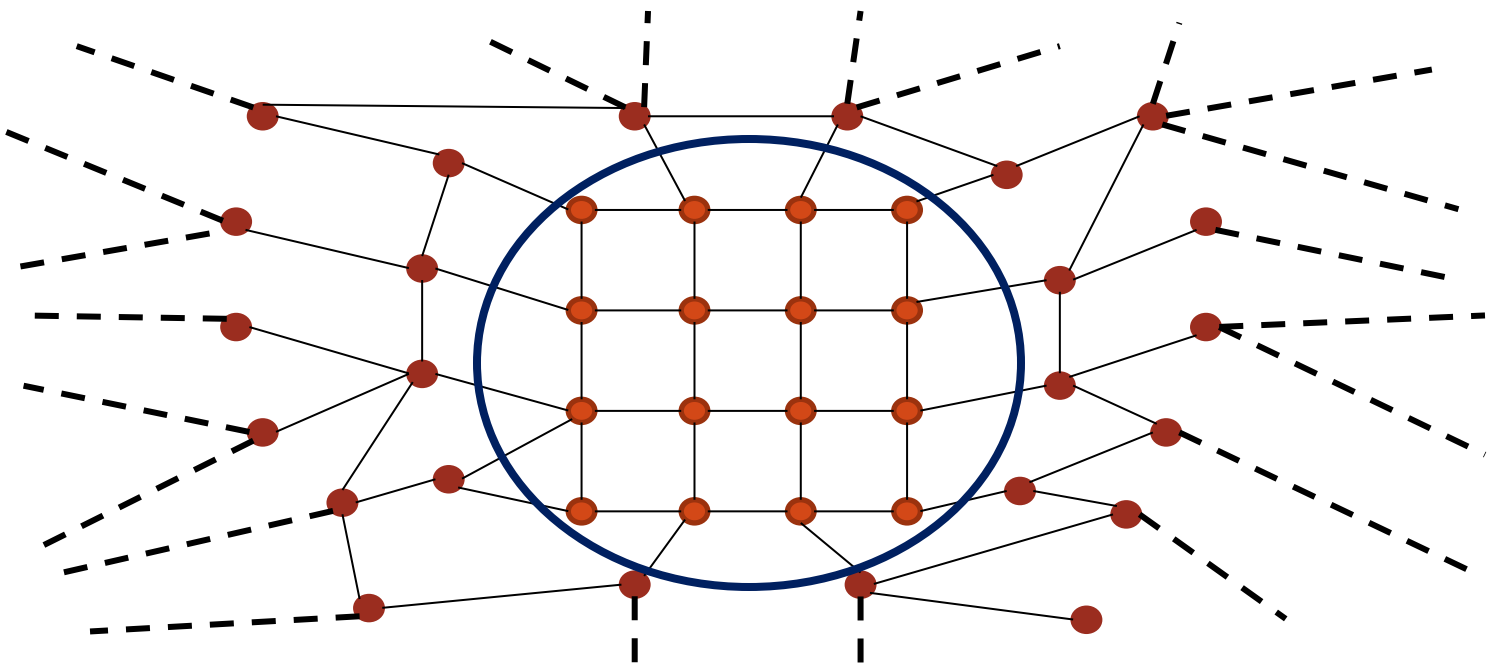
Semi-supervised Model →

Detect low-conductance cut "**near" input region** A



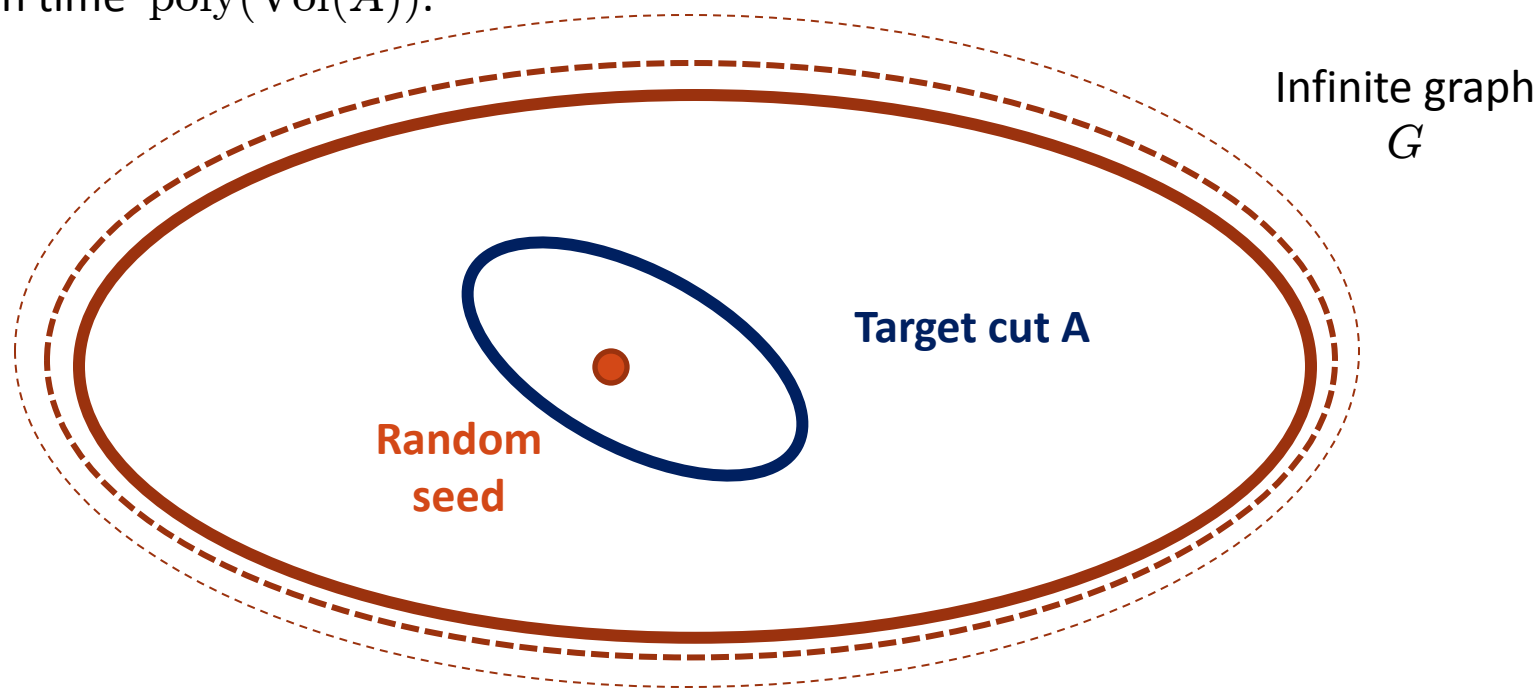
Local Graph Clustering Problem

$\min_{S \subseteq V} \phi(S)$ **Semi-supervised Model** → Detect low-conductance cut "**near" input region A**

Problem formulation:
Given random vertex from target cut A, find cut S such that

$$\phi(S) \approx \phi(A)$$

in time $\mathrm{poly}(\mathrm{Vol}(A))$.



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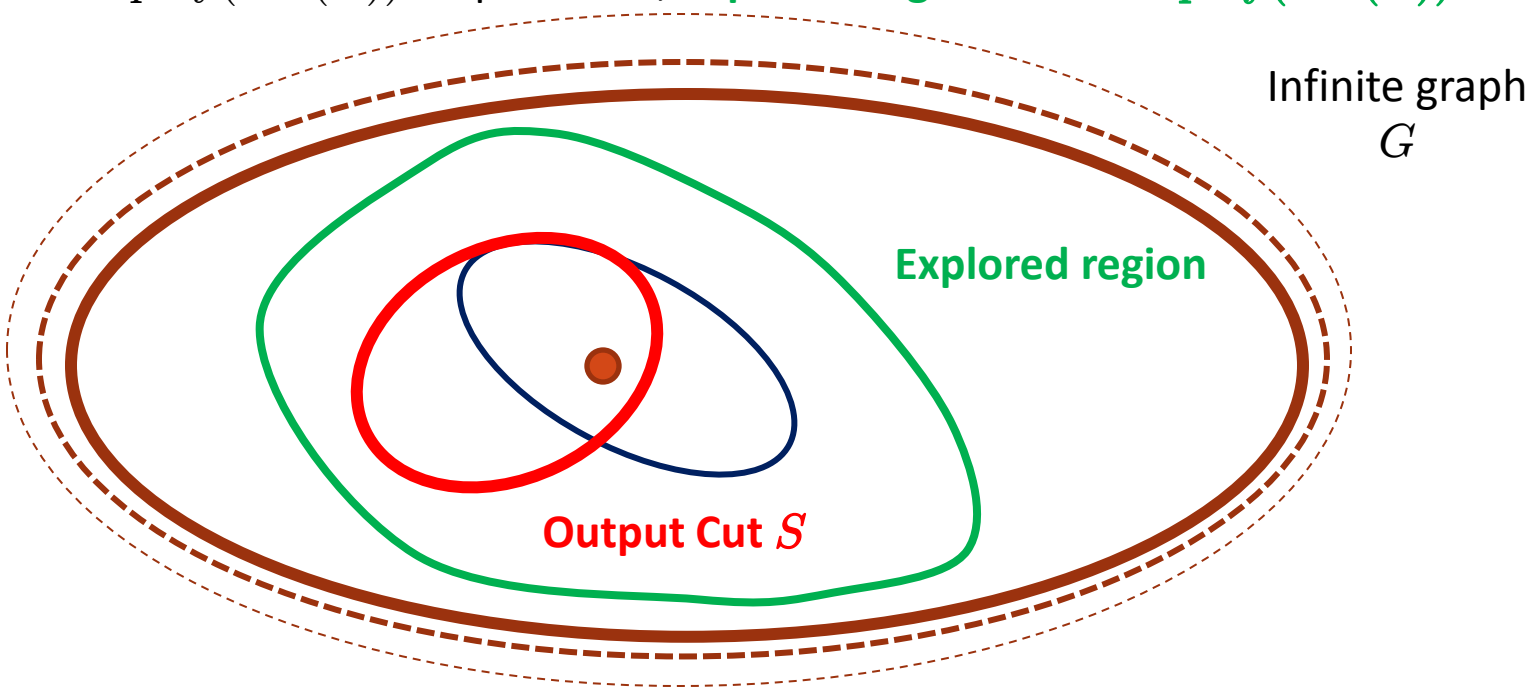
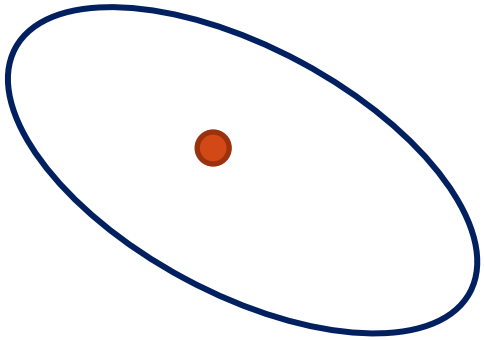


Random-Walk-Based Algorithms

Compute marginals of **random walk started at seed**



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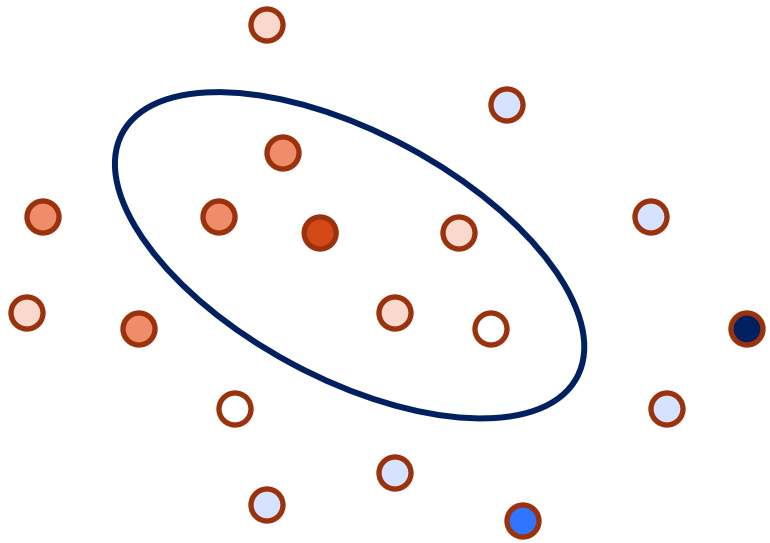

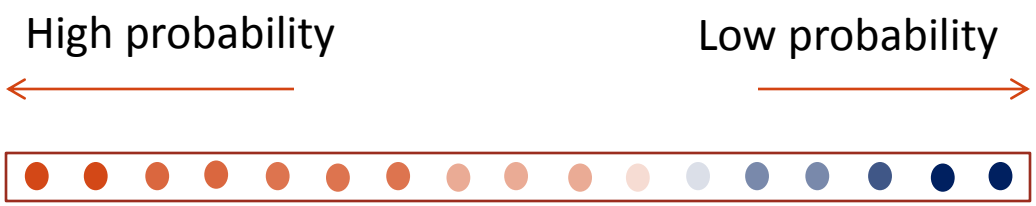

Vector of marginals

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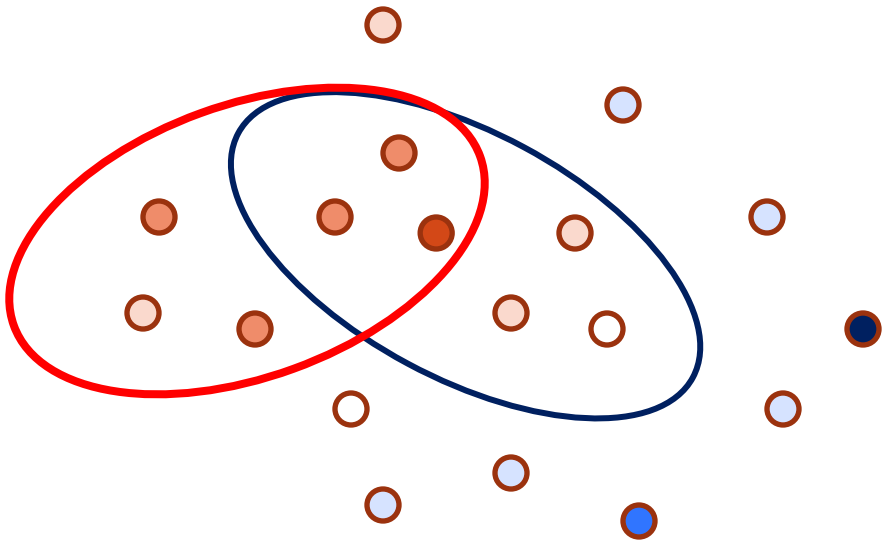

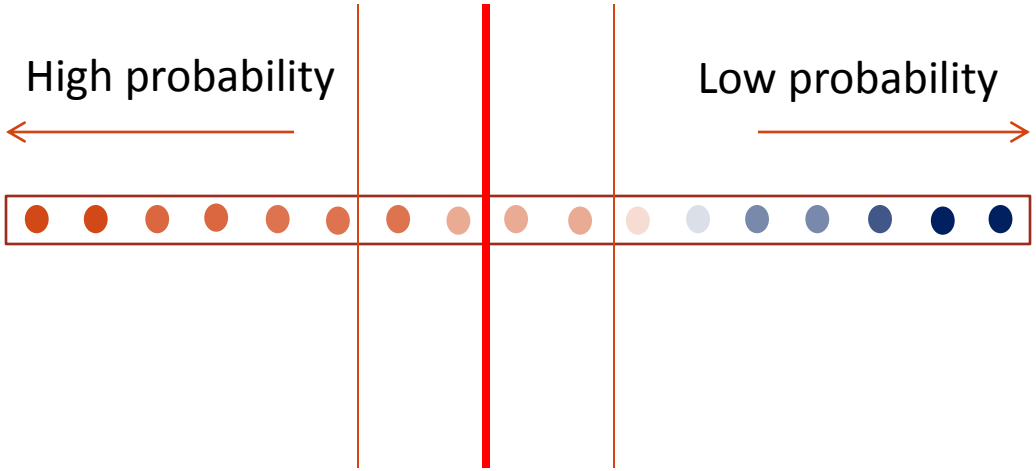

Output sweep cut of minimum conductance

Random-Walk-Based Algorithms

Algorithm	Output Conductance	Support Size	Running Time
[Spielman, Teng'04]	$\sqrt{\phi(A)\log^3 \mathrm{vol}(A)}$	$O(\mathrm{Vol}(A))$	$\tilde{O}\left(\frac{\mathrm{Vol}(A)}{\phi(A)^2}\right)$
[Andersen, Chung, Lang '07]	$\sqrt{\phi(A)\log \mathrm{vol}(A)}$	$O(\mathrm{Vol}(A))$	$\tilde{O}\left(\frac{\mathrm{Vol}(A)}{\phi(A)}\right)$
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Essentially optimal for Random-Walk approach

Well-Connected Local Clustering

OBSERVATION:
Spectral, random-walk-based algorithms **in practice work better than theoretical bounds**

RECENT WORK (both in global and local graph clustering):
Strengthen assumptions to explain behavior of spectral algorithms:

NEW ASSUMPTION: "A good ground-truth clustering exists"

GLOBAL SETTING: bounds on higher eigenvalues.

LOCAL SETTING: mixing time τ_A of graph induced by target cut A.

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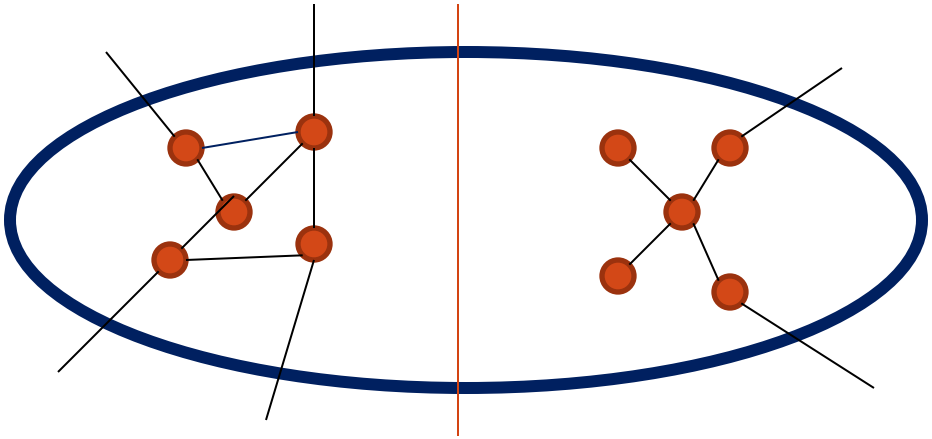

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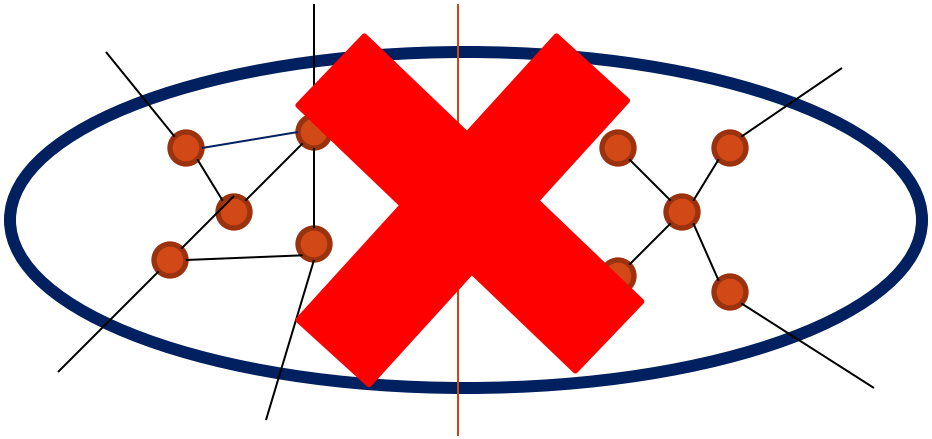

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[Zhu, Lattanzi, Mirrokni '13] Under the well-connectedness assumption

$$\tau_A \cdot O\left(\frac{1}{\phi(A)}\right)$$

MIXING TIME WITHIN SET A
SMALLER THAN TO THE OUTSIDE

The random-walk-based algorithm outputs cut S with

$$\phi(S) \cdot \underbrace{O(\sqrt{\tau_A})}_{\text{APPROXIMATION RATIO}} \cdot \phi(A) \quad \text{and} \quad \mathrm{vol}(\mathrm{Explored})) \cdot O(\mathrm{vol}(A))$$

LOCALITY GUARANTEE

Limitations of Random-Walks

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LIMITATIONS:

- This result is also essentially tight for the random-walk approach
- Only weak pseudo-approximation, when $\phi(A)$ is small

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$$\tau_A = \Theta\left(\frac{1}{\phi(A)}\right) \longrightarrow \phi(S) \cdot O(\sqrt{\phi(A)})$$

Typical Cheeger-like spectral guarantee

Our Result

OUR RESULT [Orecchia, Zhu'14]:
Under the same well-connectedness assumption,

$$\tau_A \cdot \quad O\left(\frac{1}{\phi(A)}\right)$$

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our algorithm runs in time $\tilde{O}\left(\frac{\mathrm{Vol}(A)}{\phi(A)}\right)$ and yields

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$$\underbrace{\phi(S) \cdot O(\phi(A))}_{\text{CONSTANT APPROXIMATION}} \quad \text{and} \quad \underbrace{\mathrm{vol}(\mathrm{Explored})) \cdot O(\mathrm{vol}(A))}_{\text{CONSTANT SIZE BLOW-UP}}$$

ALGORITHMIC IDEA:
Post-processes output of random-walk algorithm by **localized flow computation**

Overlap Property

ADDITIONAL PROPERTY OF RANDOM-WALK –BASED ALGORITHM:

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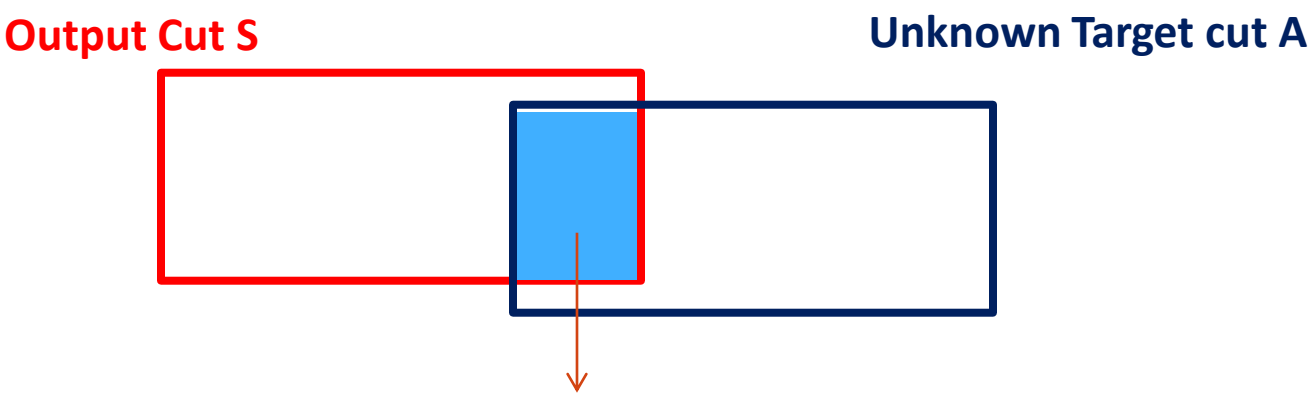

Intersection is constant fraction of volume of target cut

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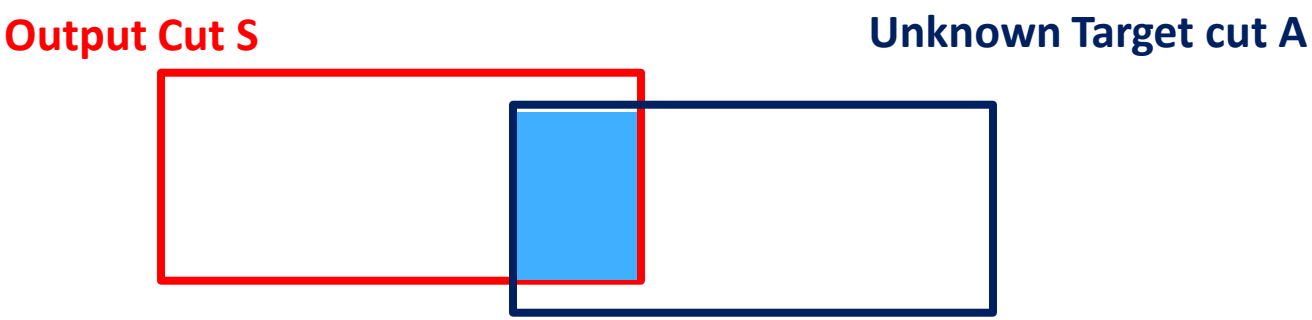

GOAL: Exploit overlap to produce better approximation to ϕ(A)

Cut-Improvement Algorithms

[Lang, Rao '93][Andersen, Lang '08]

OVERLAP GUARANTEE → APPROXIMATION GUARANTEE

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OVERLAP GUARANTEE $\longrightarrow$ APPROXIMATION GUARANTEE

On input cut S, this algorithm outputs cut C such that, for any target cut A with

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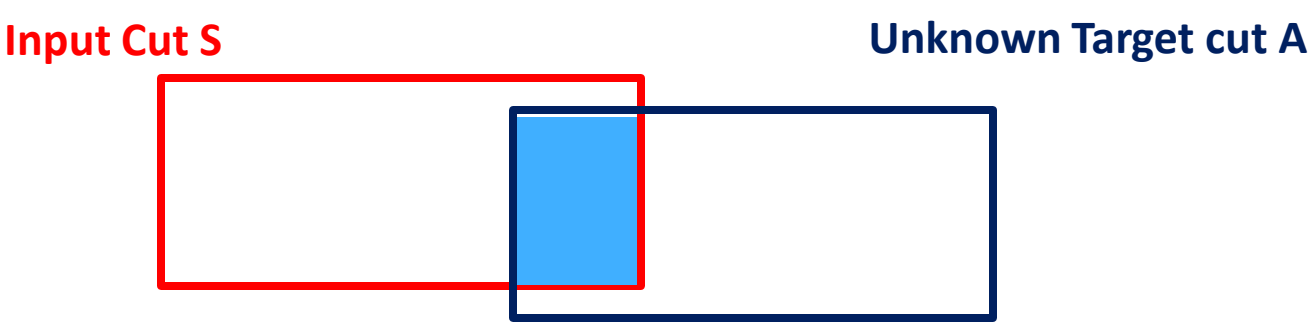

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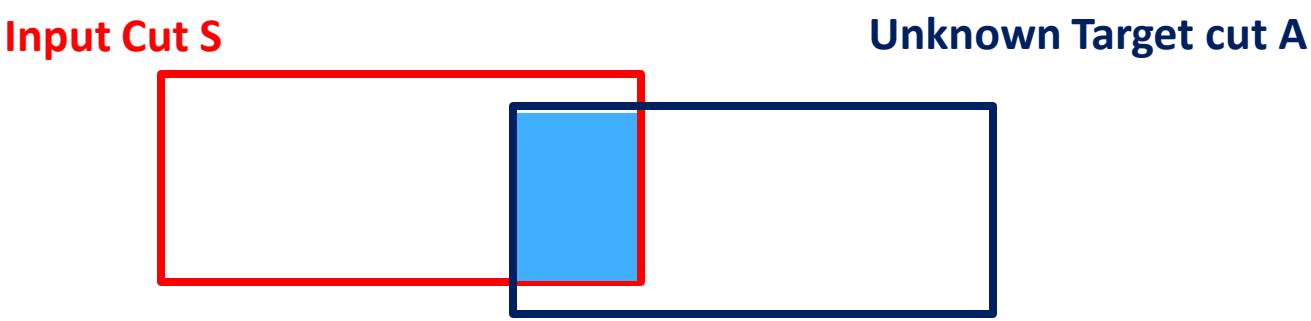

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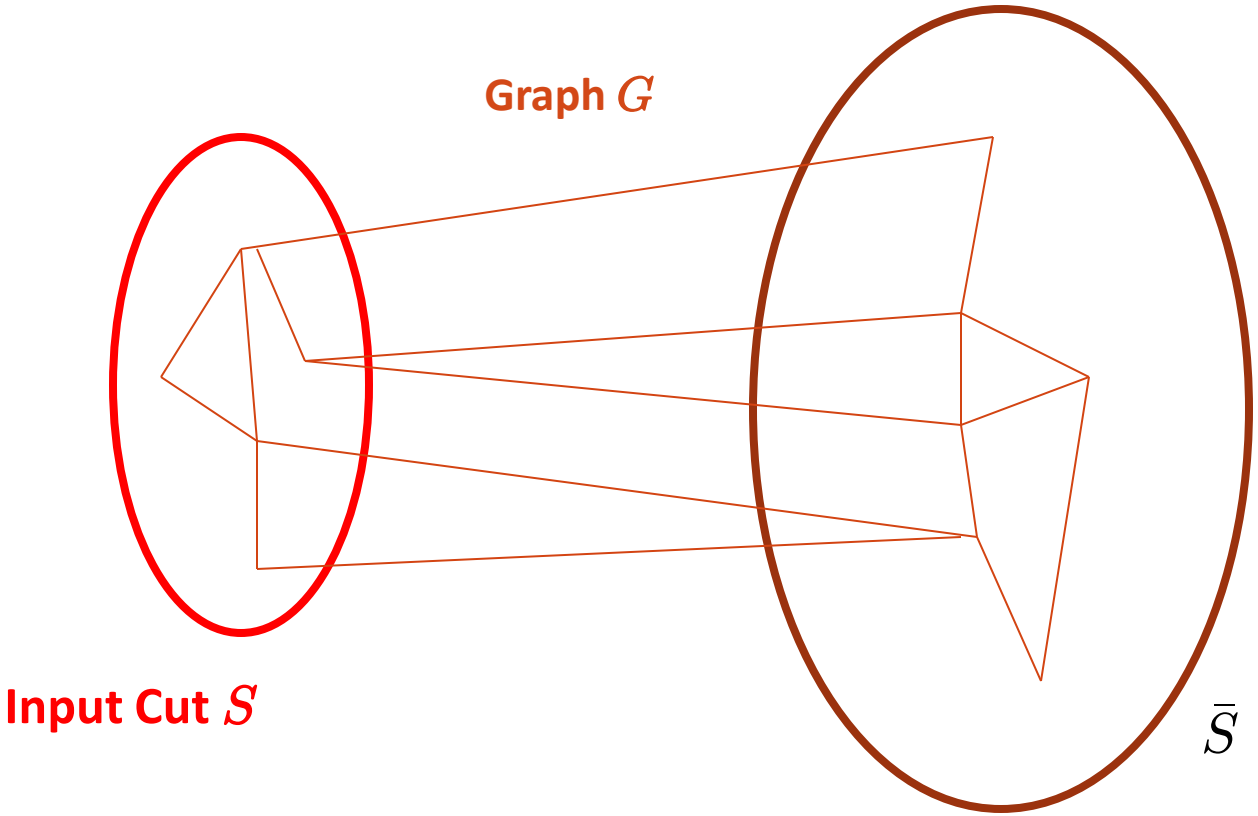
$$\phi(C) \cdot \frac{1}{\delta}\phi(A) \boxed{\cdot \quad O(\phi(A))}$$

The algorithm runs a small number of **global s-t maxflow computations**.

IN OUR CASE: $\mathrm{vol}(V), \mathrm{vol}(\bar{S}) \to \infty$

Global Solution to Cut-Improvement



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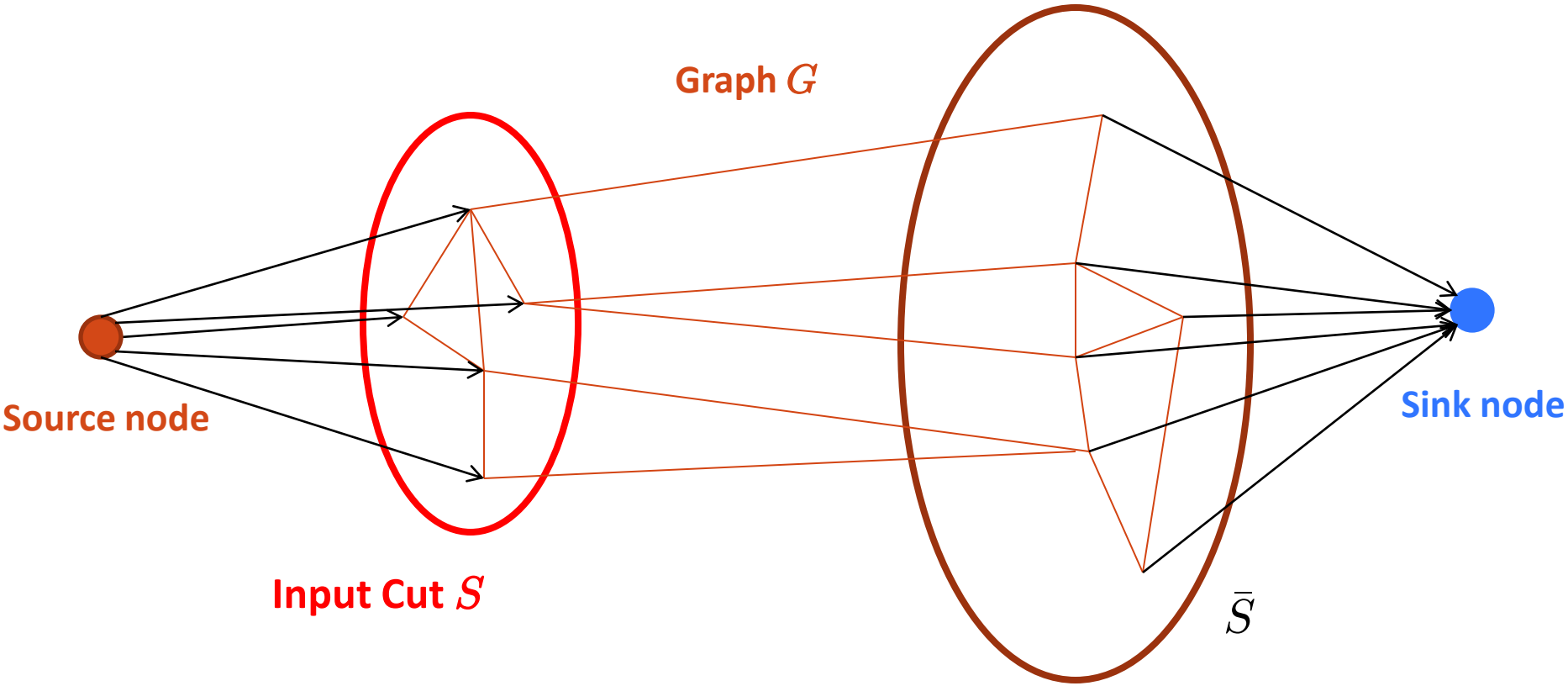

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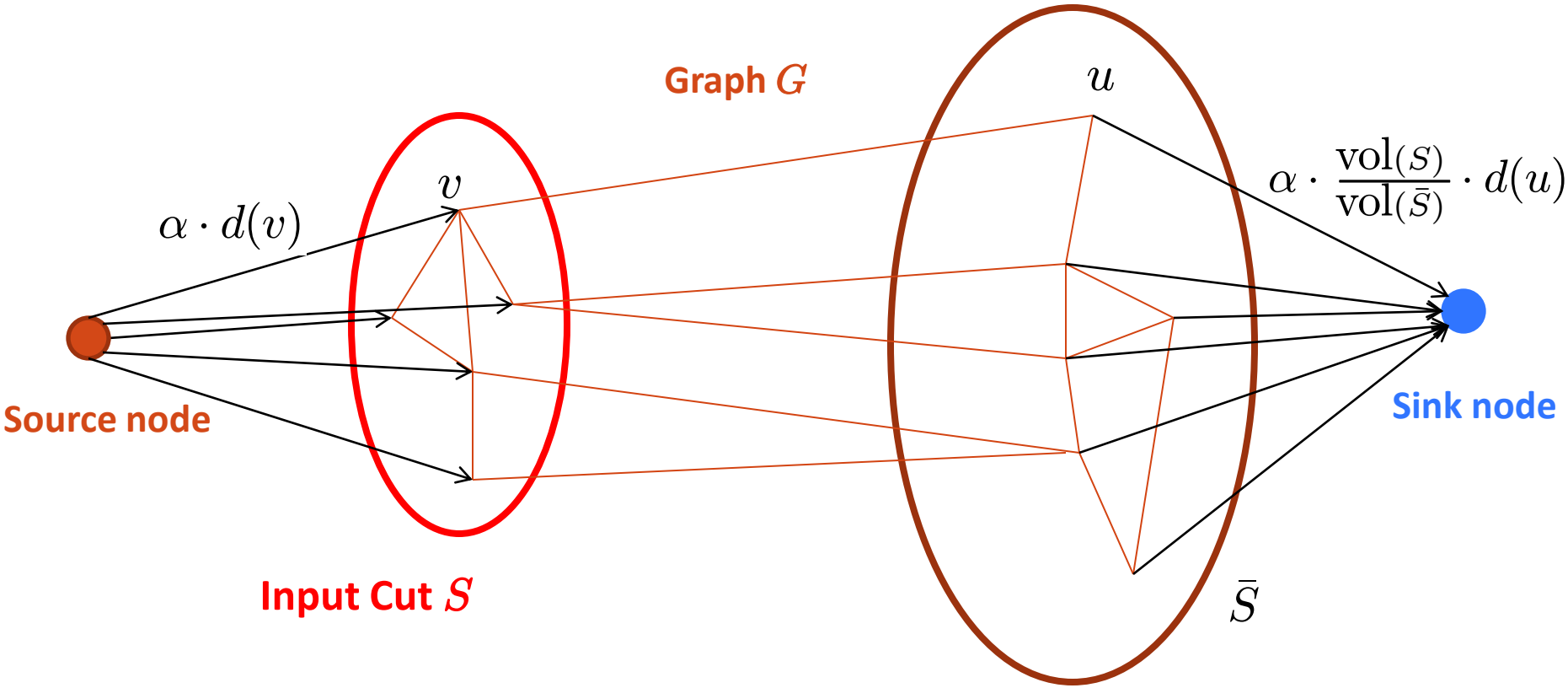

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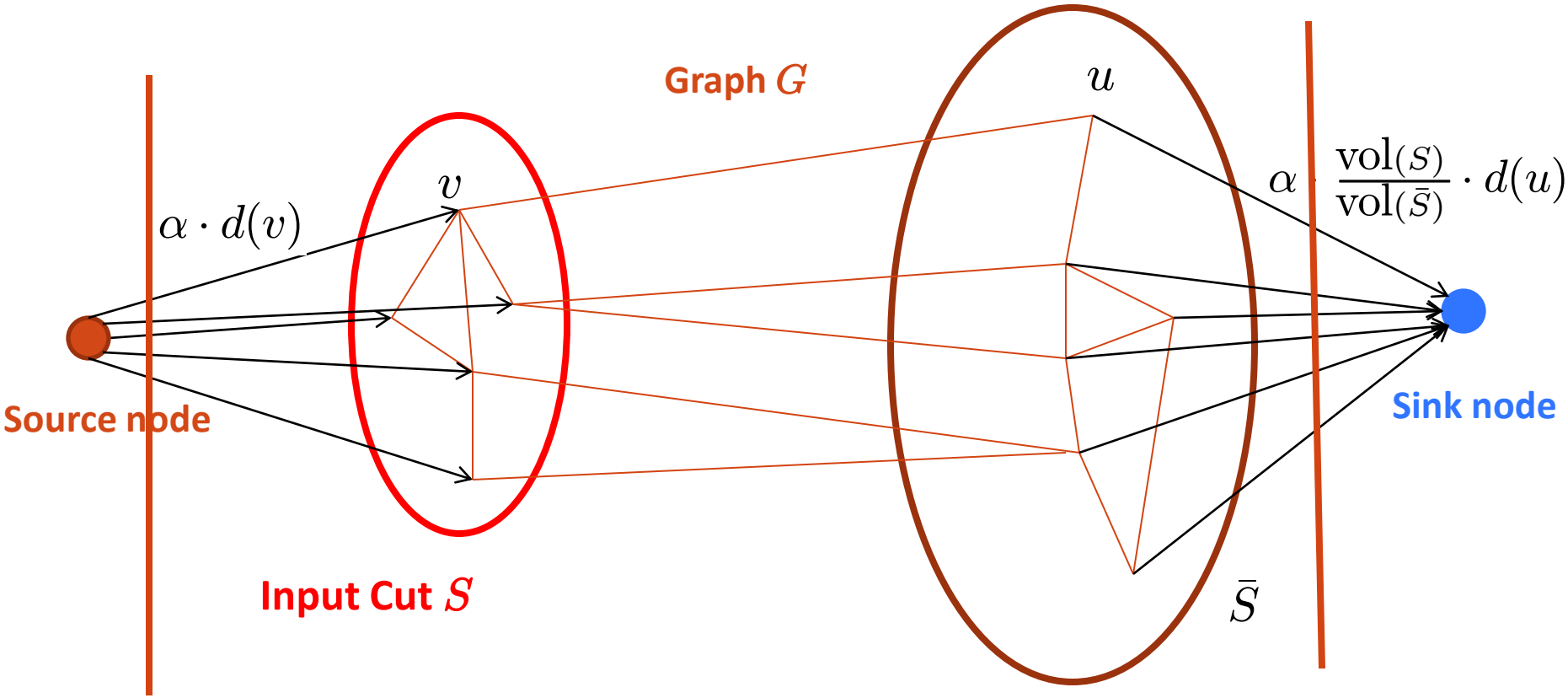

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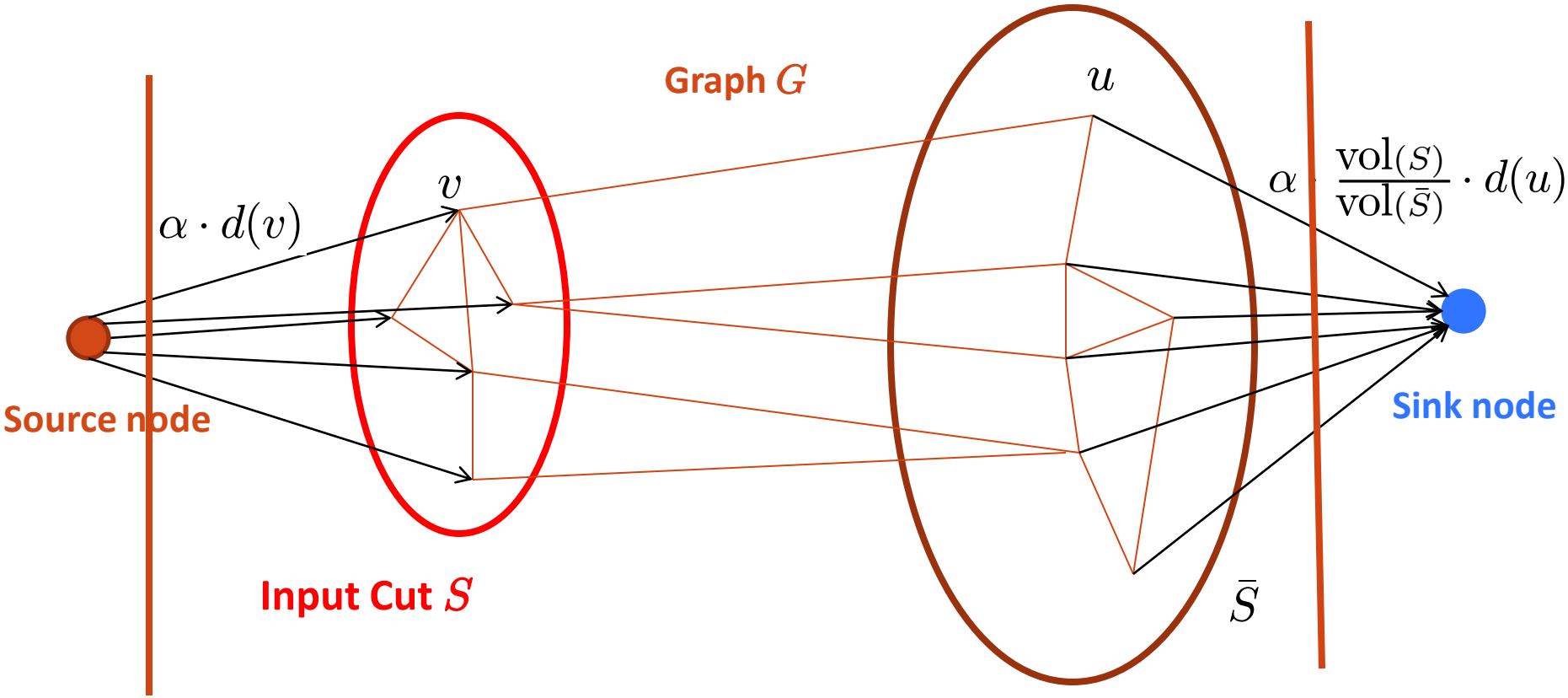

Cut capacity = $\alpha \ \mathrm{vol}(S)$

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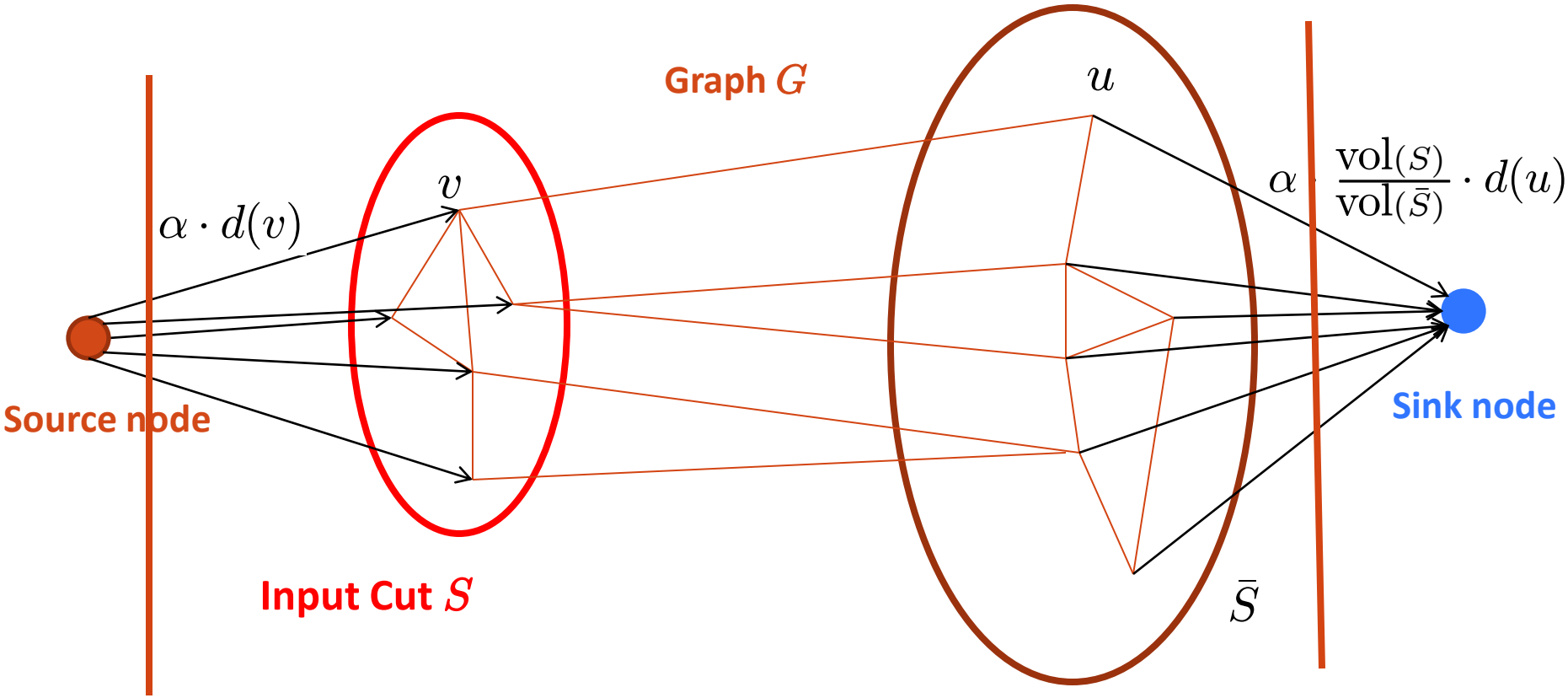
Solve parametric flow problem:

$$\max \ \alpha$$

s.t. total source/sink capacity can be routed

Global Solution to Cut-Improvement



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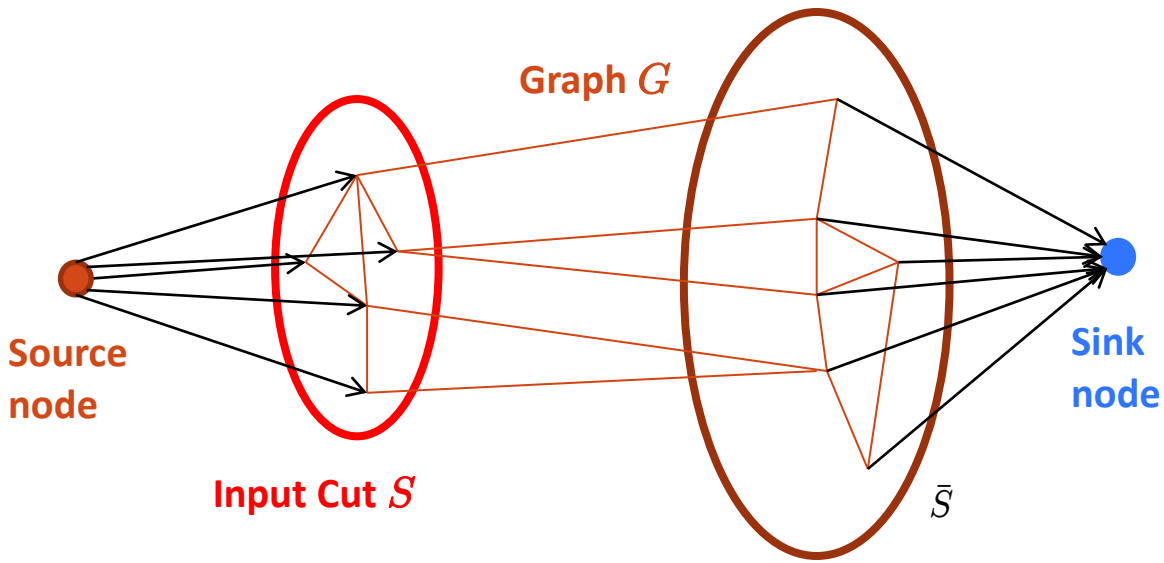
PROBLEM: optimal (and nearly-optimal) flow solutions are **inherently global**

Localization of Cut-Improvement



QUESTION: Can the cut-improvement algorithm be localized?

PESSIMISTIC CONJECTURE: **NO.**

Localization is an exclusive feature of random-walk-based algorithms.

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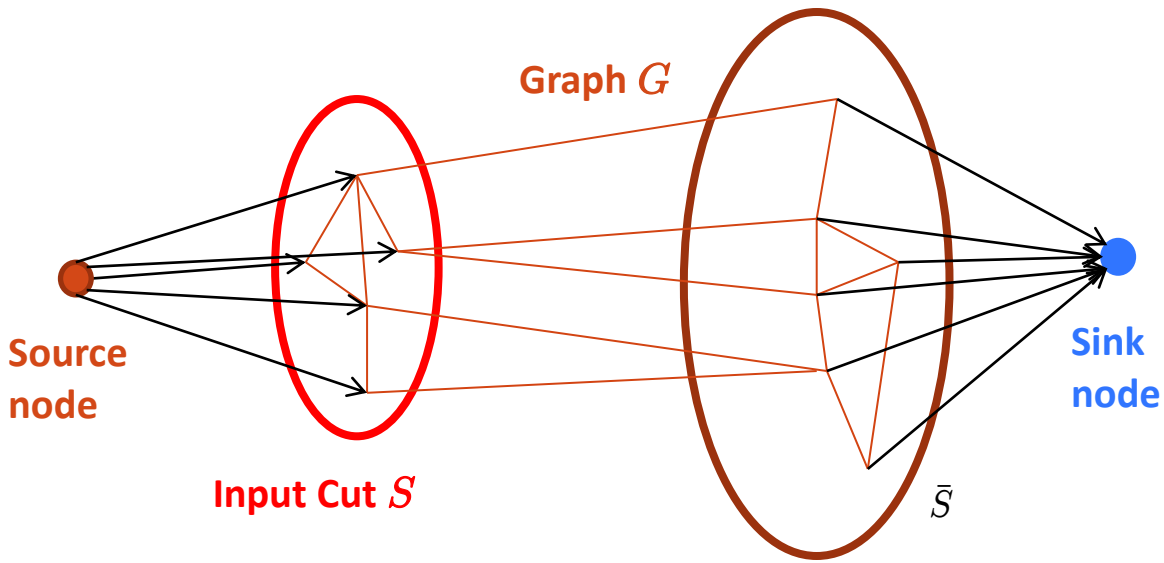

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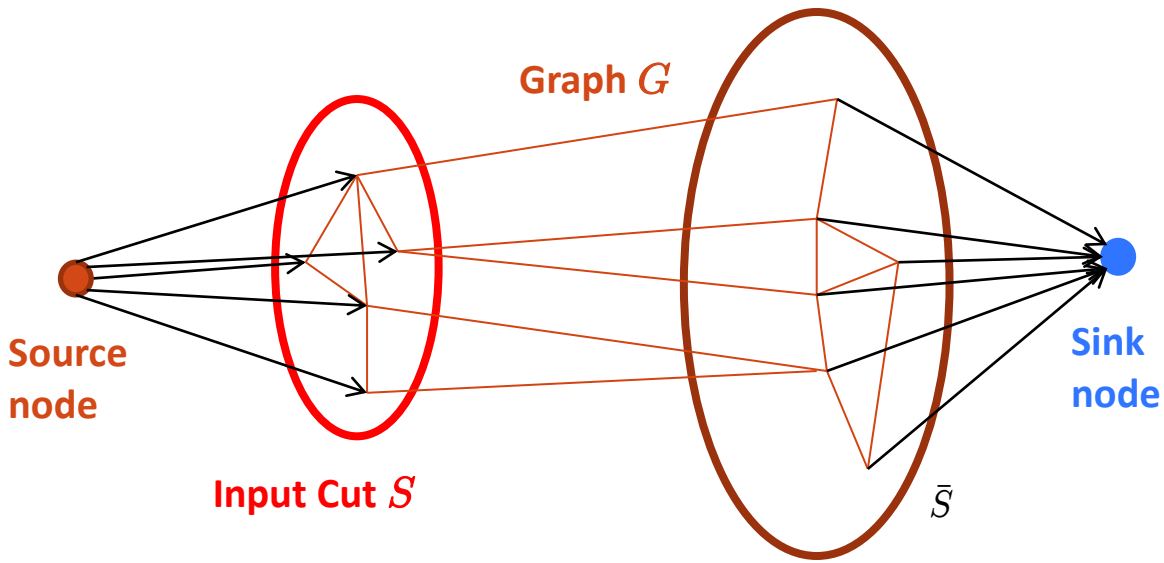

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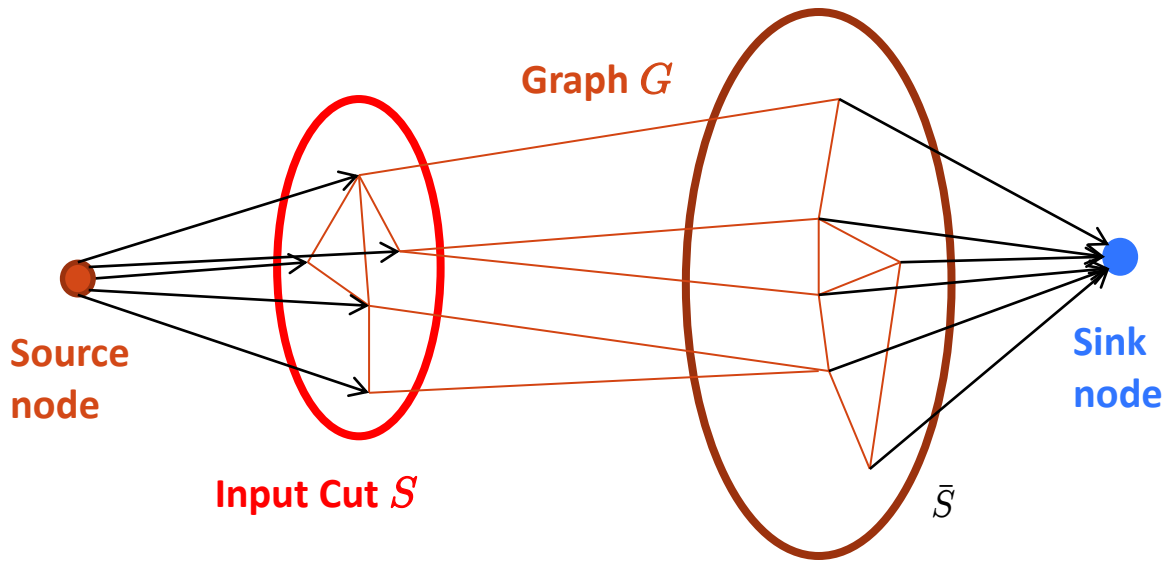
HOW DOES RANDOM WALK APPROACH ACHIEVE LOCALIZATION?

Localization of Cut-Improvement



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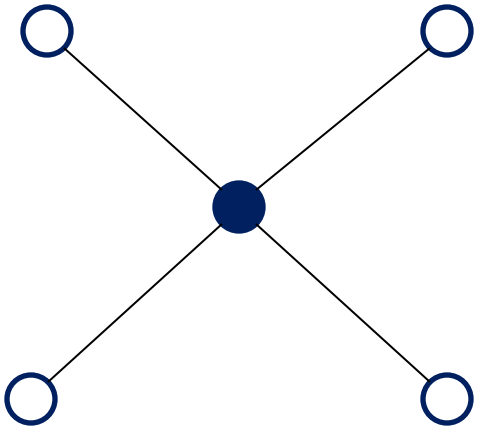
Localization of Random-Walk

Marginals are computed by ``**pushing'' probability mass** from vertices:



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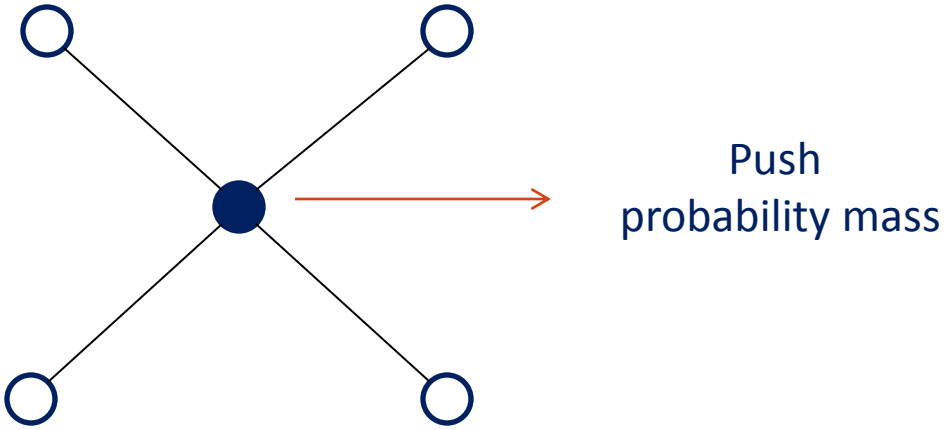

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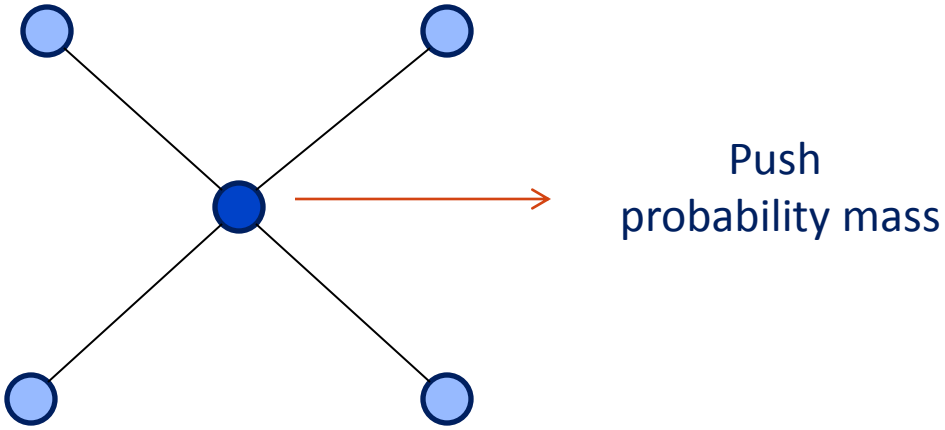

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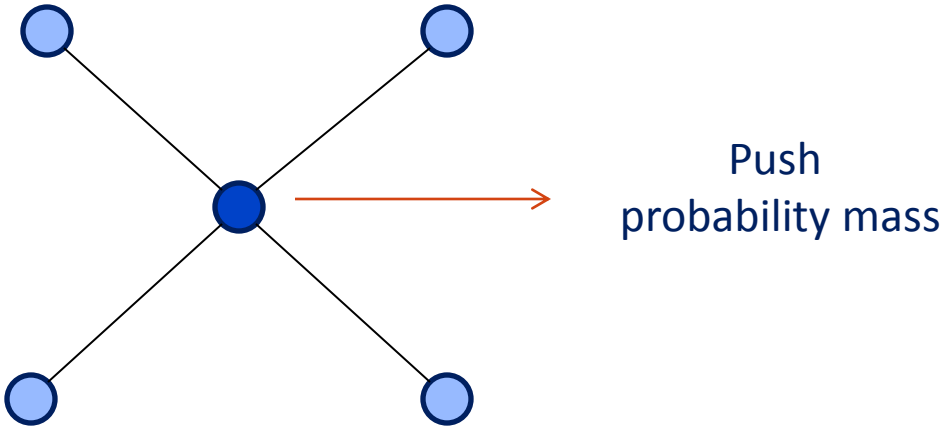
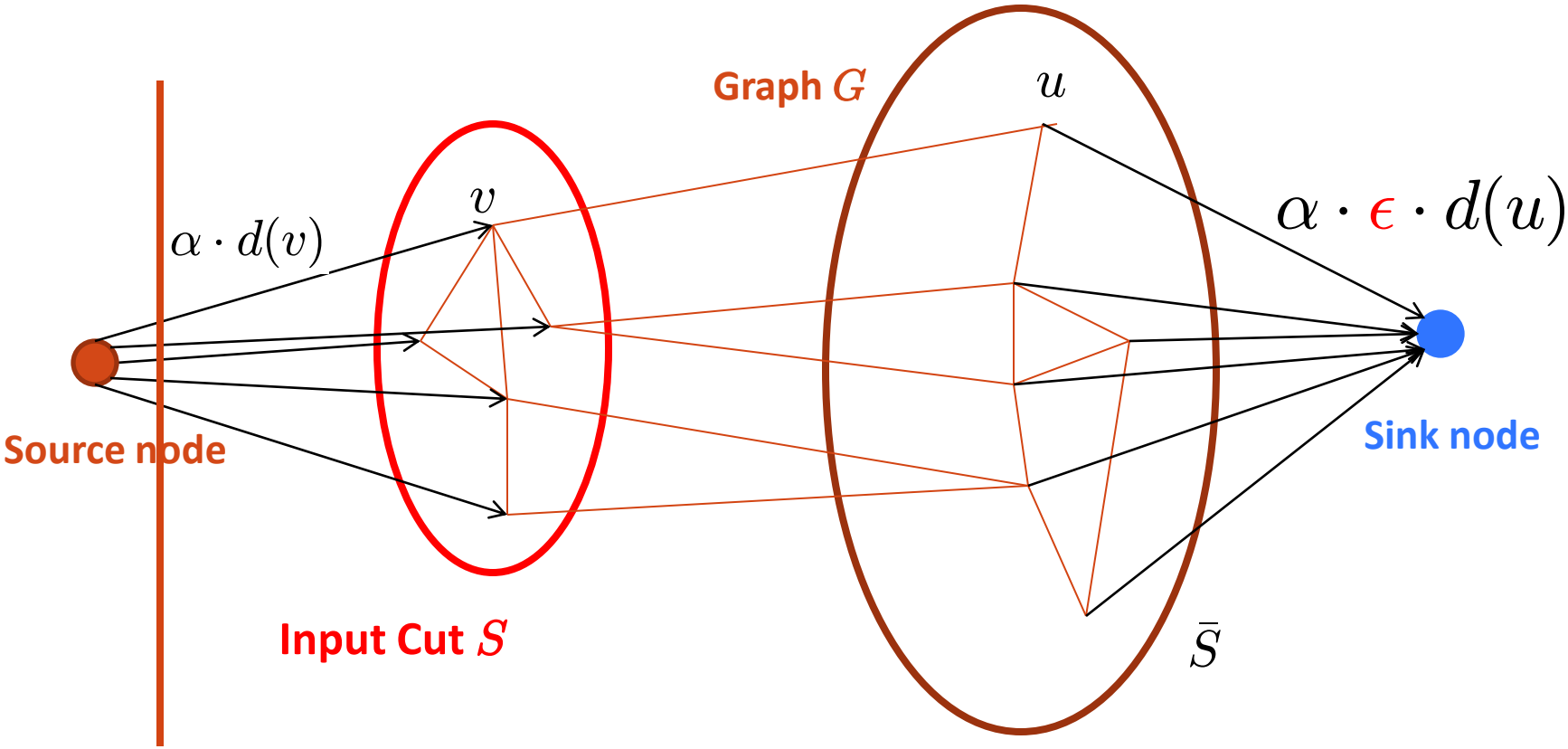


Localization is achieved by pushing mass
only if current mass at vertex is larger than a certain threshold

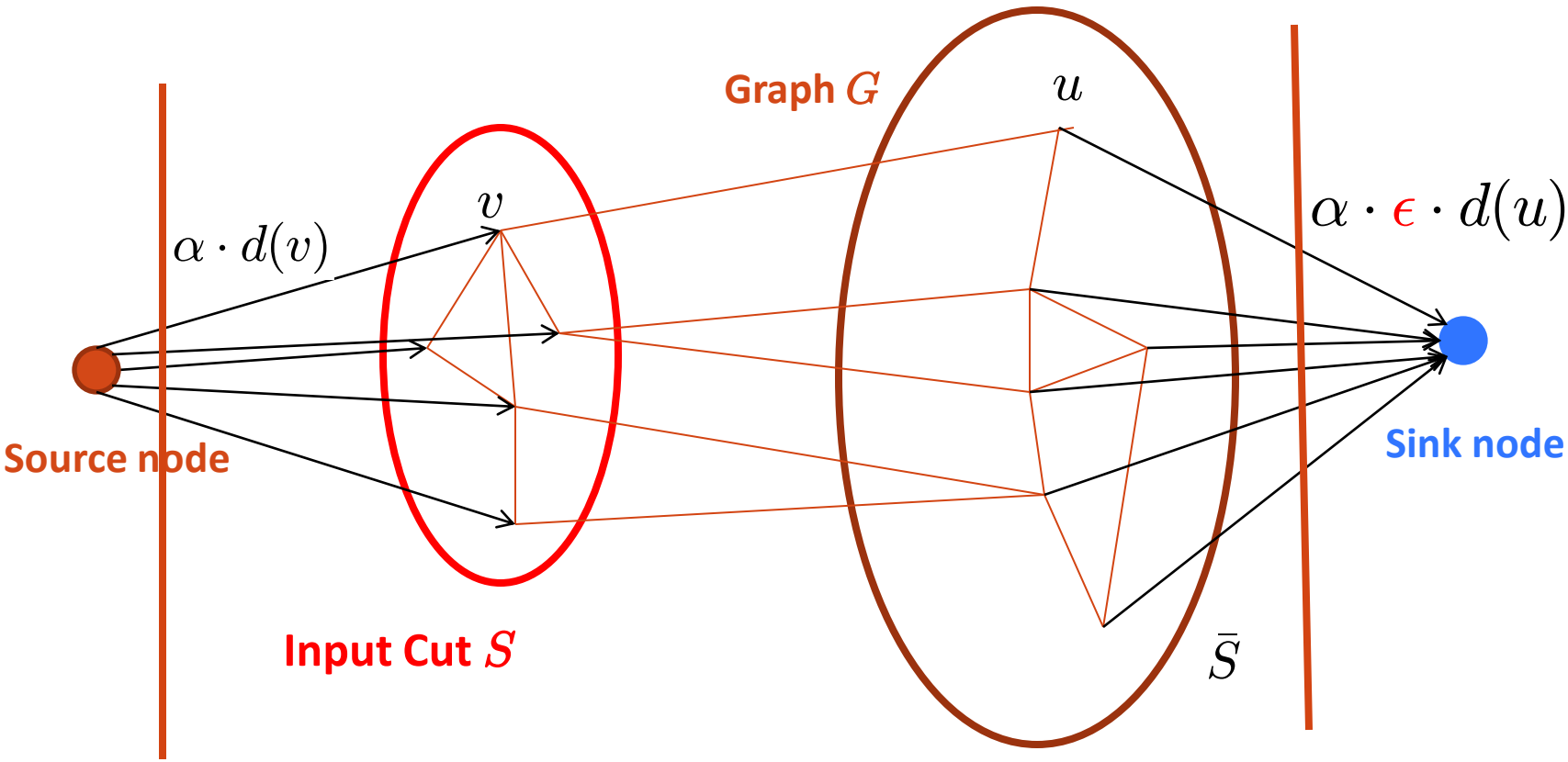
Modification of Flow Problem Achieves Localization



Cut capacity = $\alpha \, \mathrm{vol}(S)$

Every sink-side node has a large capacity to the sink: $\epsilon = \square(1)$

Modification of Flow Problem Achieves Localization



Cut capacity = $\alpha \, \mathrm{vol}(S)$ $\qquad \epsilon = \square(1)$ $\qquad$ Cut capacity = $\alpha \cdot \epsilon \cdot \mathrm{vol}(\bar{S})$

CONSEQUENCE: There exist local optimal solutions:

$$\mathrm{vol}(\mathrm{Explored}) \cdot \frac{\alpha \mathrm{vol}(S)}{\alpha \epsilon} = \frac{\mathrm{vol}(S)}{\epsilon}$$

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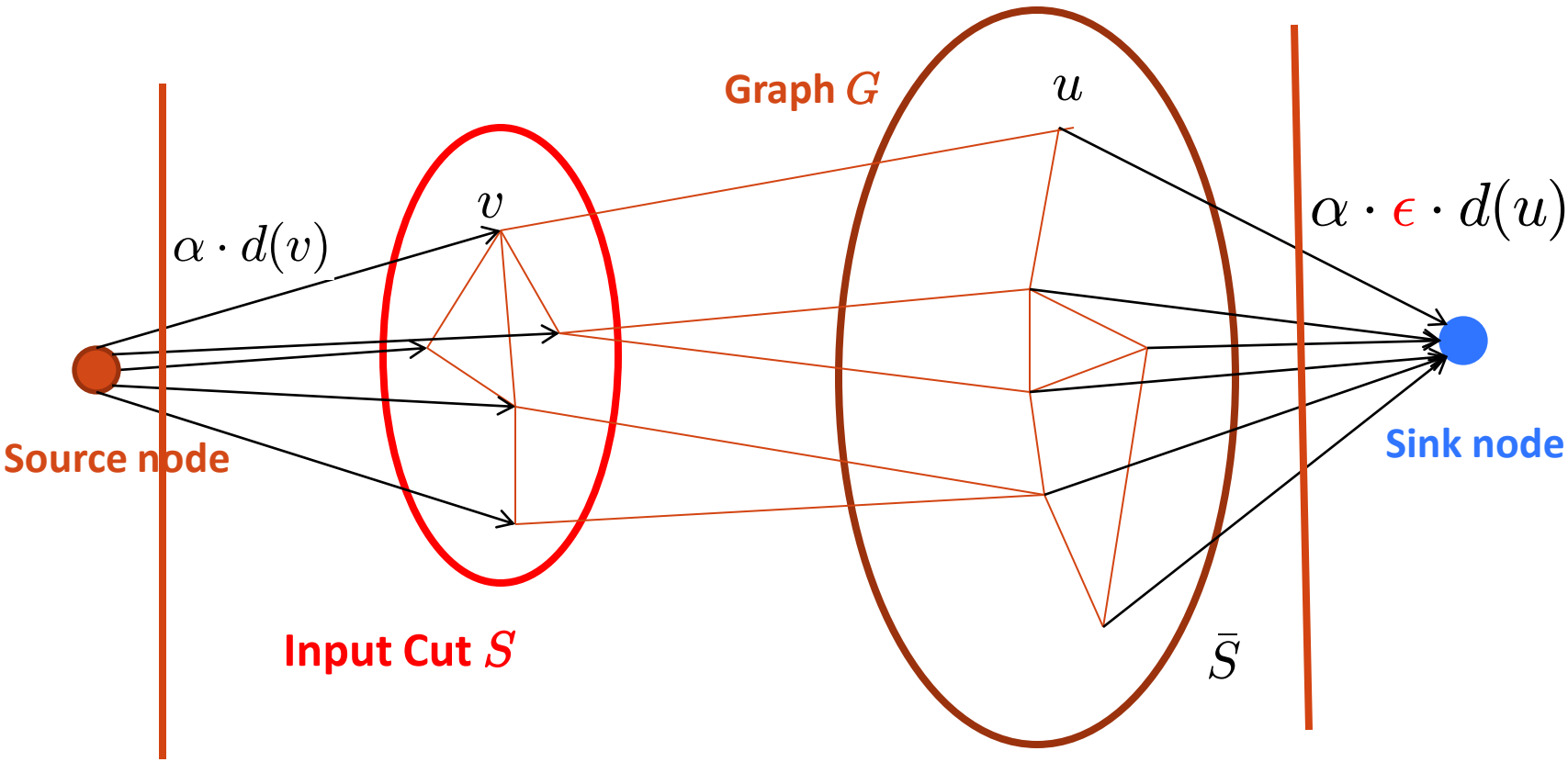

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LAST OBSTACLE: Is this modification still able to solve cut-improvement problem?

Optimization Interpretation

GLOBAL CUT-IMPROVEMENT on input S:

$$\min_{C \subseteq V} \frac{\phi(C)}{\frac{\mathrm{vol}(C \cap S)}{\mathrm{vol}(S)} - \frac{\mathrm{vol}(C \cap \bar{S})}{\mathrm{vol}(\bar{S})}}$$

NB: can be solved by maxflow thanks to maxflow-mincut theorem.

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CORRELATION BETWEEN C AND S

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OVERLAP LOWER BOUND

Optimization Interpretation

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OVERLAP LOWER BOUND

RESULT:

This problem is solved exactly by the localized parametric flow problem.

Future Directions

- Combine local flow and random-walk algorithms in stronger ways

OPEN QUESTION: Does there exist a **local version of cut-matching game**?

It would yield **unconditional polylog approximation** to local graph clustering.

- Exploit optimization interpretation to design more local algorithms.

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THE END – THANK YOU!