

Folding Wrong with Filtered Maps	Drawing Swords	Long Live fold-right!	Substituting Regents	TIARA Is A Recursive Acronym
<u>100</u>	<u>100</u>	<u>100</u>	<u>100</u>	<u>200</u>
<u>200</u>	<u>200</u>	<u>200</u>	<u>200</u>	<u>400</u>
<u>300</u>	<u>300</u>	<u>300</u>	<u>300</u>	<u>600</u>
<u>400</u>	<u>400</u>	<u>400</u>	<u>400</u>	<u>800</u>
<u>500</u>	<u>500</u>	<u>500</u>	<u>500</u>	<u>1000</u>

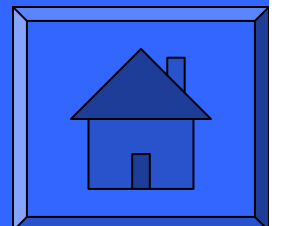
Suppose **x** is bound to the list  
**(1 2 3 4 5 6 7)**. Using **map**, **filter**,  
and/or **fold-right**, write an  
expression involving **x** that  
returns:

**(1 4 9 16 25 36 49)**

Suppose **x** is bound to the list  
**(1 2 3 4 5 6 7)**. Using `map`, `filter`, and/or  
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**(1 4 9 16 25 36 49)**

```
(map (lambda (x) (* x x)) x)
```



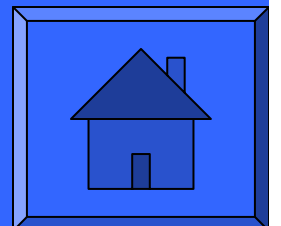
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**((1 1) (3 3) (5 5) (7 7))**

Suppose **x** is bound to the list  
**(1 2 3 4 5 6 7)**. Using `map`, `filter`, and/or `fold-right`, write an expression involving **x** that returns:

**((1 1) (3 3) (5 5) (7 7))**

```
(map (lambda (x) (list x x))  
     (filter odd? x))
```



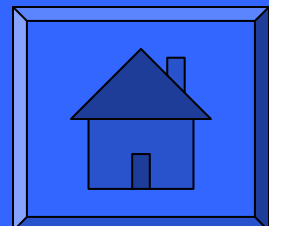
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**((2) ((4) ((6) #f)))**

Suppose **x** is bound to the list  
**(1 2 3 4 5 6 7)**. Using `map`, `filter`, and/or `fold-right`, write an expression involving **x** that returns:

**((2) ((4) ((6) #f)))**

```
(fold-right  
  (lambda (x accum)  
    (cons (list x) (list accum)))  
  #f  
  (filter even? x))
```



Suppose **x** is bound to the list  
**(1 2 3 4 5 6 7)**. Using **map**, **filter**,  
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expression involving **x** that  
returns:

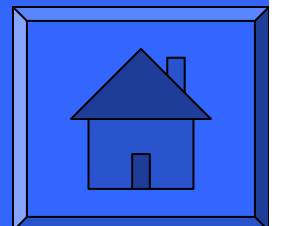
**The maximum element of x: 7**



Suppose **x** is bound to the list  
**(1 2 3 4 5 6 7)**. Using **map**, **filter**, and/or  
**fold-right**, write an expression involving  
**x** that returns:

**The maximum element of x: 7**

```
(fold-right  
  max  
  (car x)  
  (cdr x))
```



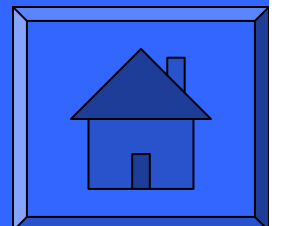
Suppose **x** is bound to the list  
**(1 2 3 4 5 6 7)**. Using **map**, **filter**,  
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returns:

**The last pair of x: (7)**

Suppose **x** is bound to the list  
**(1 2 3 4 5 6 7)**. Using map, filter, and/or fold-  
right, write an expression involving **x** that  
returns:

**The last pair of x: (7)**

**Answer:** trick question! It's not possible.  
Map, filter, and fold-right do not give you  
access to the original list's backbone, they  
only let you see the values.

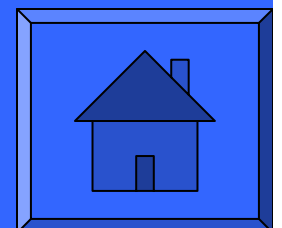
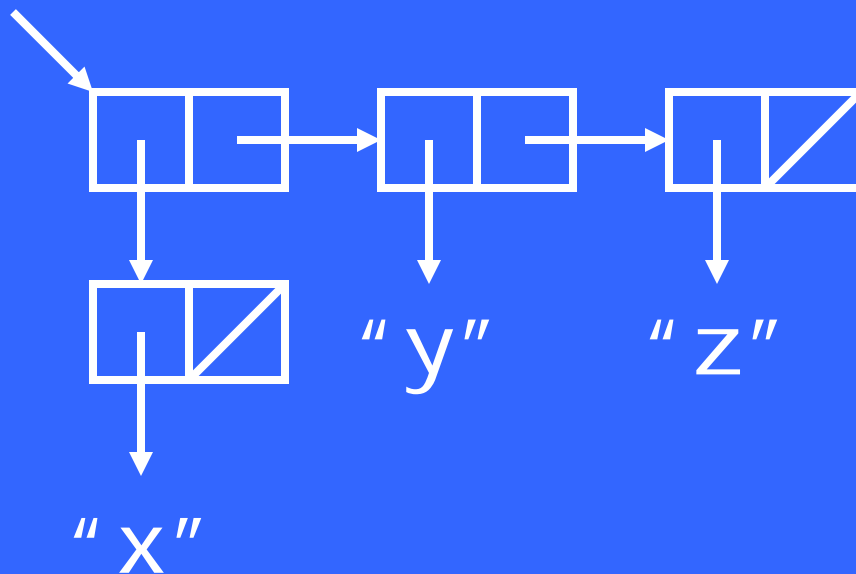


Draw a box-and-pointer diagram for the value produced by the following expression:

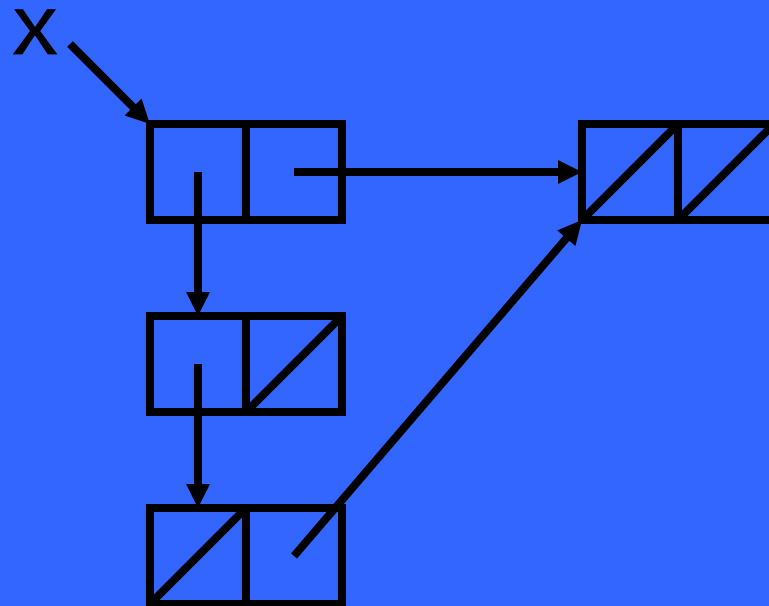
```
(cons (cons "x" nil)
      (cons "y" (cons "z" nil)))
```

Draw a box-and-pointer diagram for the value produced by the following expression:

```
(cons (cons "x" nil)
      (cons "y" (cons "z" nil)))
```



What code will produce the following box-and-pointer diagram?





Write code that will cause the following to be printed:

```
(1 2 (3 (((4))) 5))
```

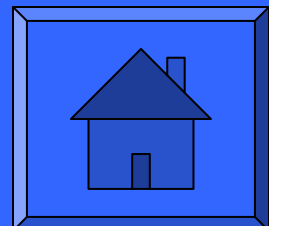


Write code that will cause the following to be printed:

```
(1 2 (3 (((4))) 5))
```

```
(list 1 2
      (list 3
            (list (list (list 4)))
            5))
```

```
(cons 1
      (cons 2
            (cons (cons 3
                      (cons (cons (cons 4 '())
                                  '()) '()) (cons 5 '()))
                    '()))))
```

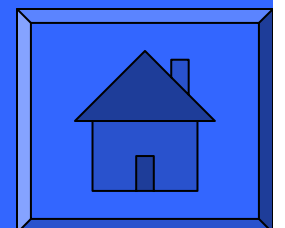
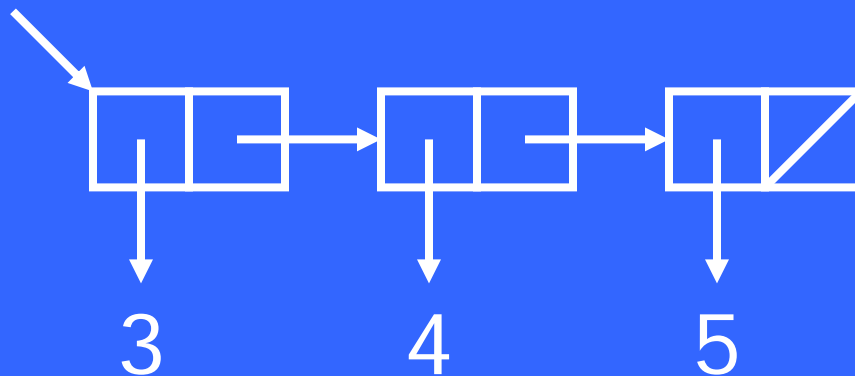


Draw a box-and-pointer diagram for the value produced by the following expression:

```
(map car  
  (list (list 3)  
        (list 4)  
        (list 5)))
```

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```
(map car  
  (list (list 3)  
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        (list 5)))
```

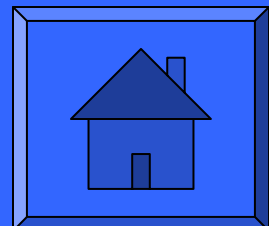
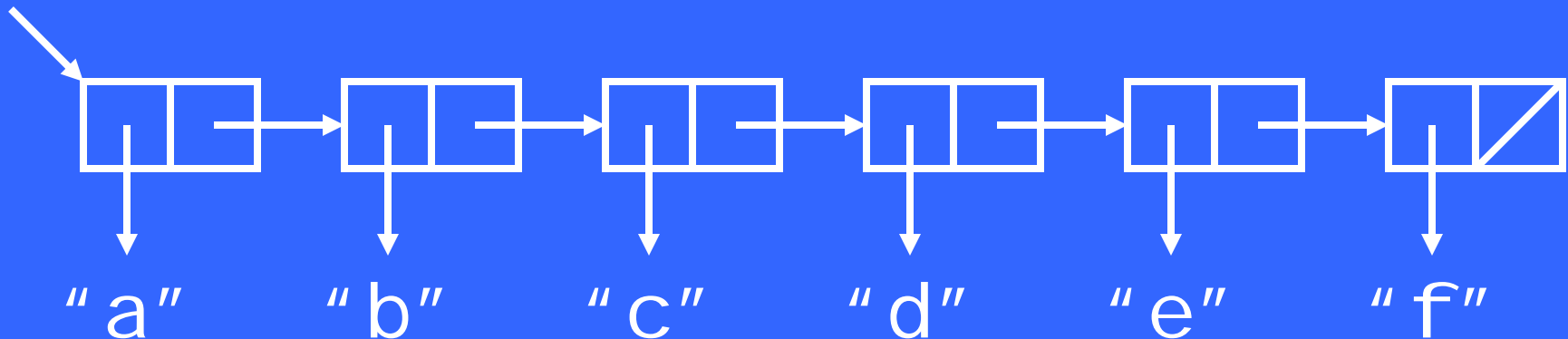


Draw the box-and-pointers diagram for the value of the following expression:

```
(fold-right  
  append  
  '()  
  (list (list "a" "b")  
        (list "c")  
        (list "d" "e" "f"))))
```

Draw the box-and-pointers diagram for the value of the following expression:

```
(fold-right  
  append  
  '()  
  (list (list "a" "b")  
        (list "c")  
        (list "d" "e" "f"))))
```



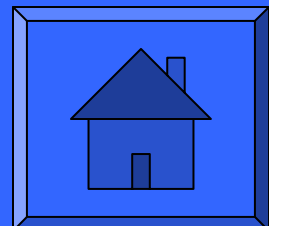
Write the following procedure using fold-right:

```
; Creates a new list with  
; the same elements as lst  
(define (copy-list lst)
```

```
)
```

Write the following procedure using fold-right:

```
; Creates a new list with  
; the same elements as lst  
(define (copy-list lst)  
  (fold-right cons '() lst))
```



Write the following procedure using fold-right:

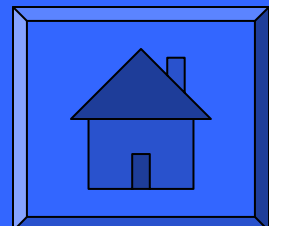
```
(define (append list1 list2)
```

```
)
```



Write the following procedure using fold-right:

```
(define (append list1 list2)
  (fold-right cons list2 list1))
```



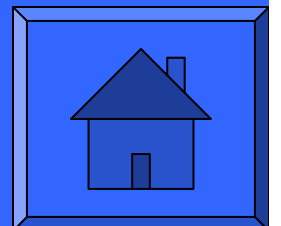
Write a procedure to reverse a list using fold-right (you may also use length, append, list, and/or cons):

```
(define (reverse lst)
```

```
)
```

Write a procedure to reverse a list using fold-right (you may also use length, append, list, and/or cons):

```
(define (reverse lst)
  (fold-right
    (lambda (new accum)
      (append accum
                (list new)))
    ()
    lst))
```

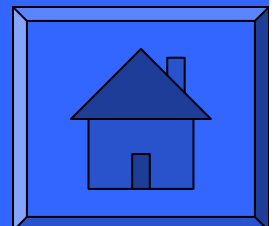


Write the **for-all?** procedure using **fold-right**. It should return **#t** if applying the procedure **pred** to each element of **lst** evaluates to **#t**.

```
;; for-all? :  
;;   list<A>, (A->boolean) -> boolean  
;; Examples:  
;;   (for-all? (list 1 3 5 7) odd?) => #t  
;;   (for-all? (list 1 3 5 6) odd?) => #f  
(define (for-all? lst pred) ... )
```

Write the **for-all?** procedure using **fold-right**.  
It should return **#t** if applying the procedure **pred** to each element of **lst** evaluates to **#t**.

```
;; for-all? :  
;;   list<A>, (A->boolean) -> boolean  
;; Examples:  
;;   (for-all? (list 1 3 5 7) odd?) => #t  
;;   (for-all? (list 1 3 5 6) odd?) => #f  
(define (for-all? lst pred)  
  (fold-right  
    (lambda (x accum)  
      (and accum (pred x)))  
    #t  
    lst))
```

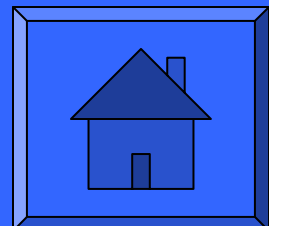


Write the procedure **map** in terms of **fold-right**.

```
(define (map pred lst) ... )
```

Write the procedure **map** in terms of **fold-right**.

```
(define (map pred lst)
  (fold-right
    (lambda (x accum)
      (cons (pred x) accum))
    '()
    lst))
```



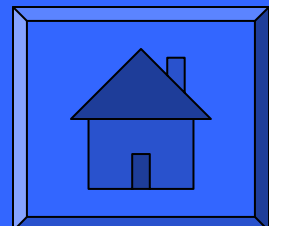
Write the value of the final Scheme expression. Assume the expressions are evaluated in order. Use **unspecified**, **error**, or **procedure** where appropriate..

```
((lambda (x) (+ x x)) 5)
```



Write the value of the final Scheme expression. Assume the expressions are evaluated in order. Use **unspecified**, **error**, or **procedure** where appropriate..

`((lambda (x) (+ x x)) 5) => 10`

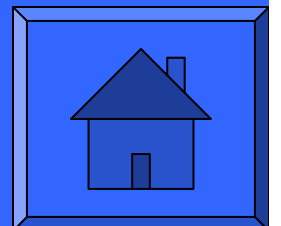


Write the value of the final Scheme expression. Assume the expressions are evaluated in order. Use **unspecified**, **error**, or **procedure** where appropriate..

```
(define x 5)
(define y 6)
(let ((x 7)
      (y x))
  (+ x y))
```

Write the value of the final Scheme expression. Assume the expressions are evaluated in order. Use **unspecified**, **error**, or **procedure** where appropriate..

```
(define x 5)
(define y 6)
(let ((x 7)
      (y x))
  (+ x y)) => 12
```

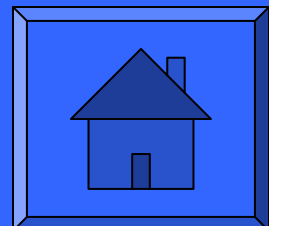


Write the value of the final Scheme expression. Assume the expressions are evaluated in order. Use **unspecified**, **error**, or **procedure** where appropriate..

```
(define x 10)
(define y 20)
(define (foo x)
  (lambda (y) (- x y)))
((foo y) x)
```

Write the value of the final Scheme expression. Assume the expressions are evaluated in order. Use **unspecified**, **error**, or **procedure** where appropriate..

```
(define x 10)
(define y 20)
(define (foo x)
  (lambda (y) (- x y)))
((foo y) x) => 10
```

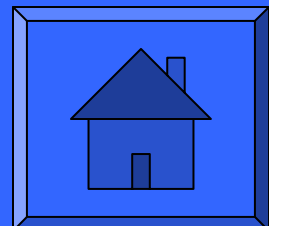


Write the value of the final Scheme expression. Assume the expressions are evaluated in order. Use **unspecified**, **error**, or **procedure** where appropriate..

```
(define (inc x)
  (lambda (y) (+ y 1)))
(inc 1)
```

Write the value of the final Scheme expression. Assume the expressions are evaluated in order. Use **unspecified**, **error**, or **procedure** where appropriate..

```
(define (inc x)
  (lambda (y) (+ y 1)))
(inc 1) => compound procedure
```



Write the value of this Scheme expression. Assume the expressions are evaluated in order. Use **unspecified**, **error**, or **procedure** where appropriate..

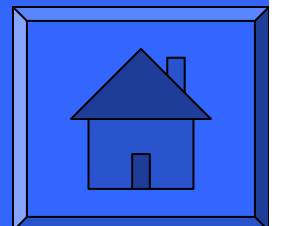
```
((lambda (x y) (x y))  
 (lambda (z)  
   (lambda (a)  
     (+ a z)))) *)
```



Write the value of this Scheme expression. Assume the expressions are evaluated in order. Use **unspecified**, **error**, or **procedure** where appropriate..

```
((lambda (x y) (x y))  
 (lambda (z)  
   (lambda (a)  
     (+ a z)))) *)
```

=> **compound procedure (note: the procedure will generate an error if evaluated)**



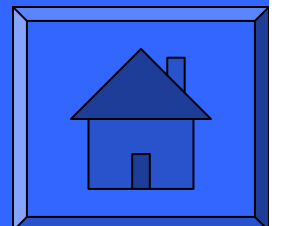
What is the time order of growth of the following procedure? You may assume that  $x$  and  $y$  are non-negative integers.

```
(define (bar x y)
  (if (< x 0)
      #t
      (bar (+ x 1) (+ y y))))
```

What is the time order of growth of the following procedure? You may assume that  $x$  and  $y$  are non-negative integers.

```
(define (bar x y)
  (if (< x 0)
      #t
      (bar (+ x 1) (+ y y))))
```

=> infinite (bad test condition)



# What is the time order of growth of **set-difference**?

```
; set-contains? : set<A>,A → boolean   Theta(log n)
; set->list      : set<A>   → list<A>   Theta(n)
; list->set      : list<A>   → set<A>   Theta(n log n)
;; Returns the set containing all elements in a that are not in b
(define (set-difference a b)
  (let ((a-list (set->list a)))
    (list->set
      (filter
        (lambda (x)
          (not (set-contains? b x)))
        a-list))))
; example:
(define a (list 1 2 3 4 5))
(define b (list 3 4 5 6))
(set-difference a b); -> (1 2)
```

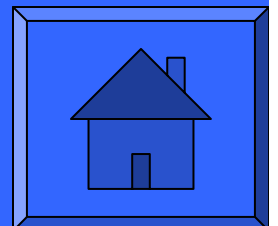
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; set->list      : set<A>   → list<A>   Theta(n)
; list->set      : list<A>   → set<A>   Theta(n log n)
;; Returns the set containing all elements in a that are not in b
```

```
(define (set-difference a b)
  (let ((a-list (set->list a)))
    (list->set
      (filter
        (lambda (x)
          (not (set-contains? b x)))
        a))))
```

Time  $O(n \log n) \Rightarrow \Theta(n \log n)$

Note:  $n + (n \log n) + (n \log n)$



*What's the longest time it will take to guess the number?*

```
(define (make-adversary number)
  (lambda (x)
    (cond ((< x number) "bigger")
          ((= x number) "found it")
          (> x number) "smaller"))))

(define (guess-number adversary min max)
  (let* ((mid (quotient (+ min max) 2))
         (reply (adversary mid)))

    (cond ((equal? reply "smaller" )
           (guess-number adversary min mid))

          ((equal? reply "found it") mid)

          ((equal? reply "bigger" )
           (guess-number adversary mid max)))))
```

;; Usage example:

```
(guess-number (make-adversary 7) 1 100)
```

*What's the longest time it will take to guess the number?*

```
(define (make-adversary number) ...)
```

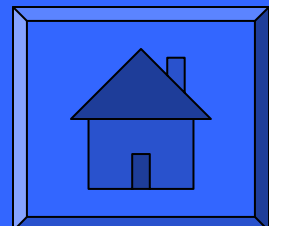
```
(define (guess-number adversary min max)
  (let* ((mid (quotient (+ min max) 2))
        (reply (adversary mid)))
```

```
    (cond ((equal? reply "smaller" )
           (guess-number adversary min mid))
```

```
          ((equal? reply "found it") mid)
```

```
          ((equal? reply "bigger" )
           (guess-number adversary mid max))))))
```

**Answer:  $\Theta(\log \text{max-min})$**



Write a procedure, **fold-left**, that works like **fold-right**, but processes elements of the list in left-to-right order and is iterative.

```
(define (fold-right op init lst)
  (if (null? lst)
      init
      (op (car lst)
          (fold-right op init (cdr lst)))))
```

```
(define (fold-left op init lst)
  (if (null? lst)
      init
      (fold-left op (op (car lst) init) (cdr lst))))
```

```
(fold-right cons '() (list 1 2 3 4 5))
; => (1 2 3 4 5)
```

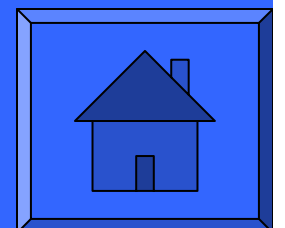
```
(fold-left cons '() (list 1 2 3 4 5))
; => (5 4 3 2 1)
```



Write a procedure, **fold-left**, that works like **fold-right**, but processes elements of the list in left-to-right order and is iterative.

```
(define (fold-right op init lst)
  (if (null? lst)
      init
      (op (car lst)
          (fold-right op init (cdr lst)))))
```

```
(define (fold-left op
                  (op (car lst) init)
                  (cdr lst)))
```



What is the order-of-growth in time and space for **unknown-costs**?

```
(define (costs-n-n n)
  (if (<= n 0)
      0
      (+ n (costs-n-n (- n 1)))))
```

```
(define (costs-n-1 n)
  (if (<= n 0)
      0
      (costs-n-1 (- n 1))))
```

```
(define (unknown-costs n)
  (define (helper n1 n2)
    (if (>= n1 (* n2 n2 n2))
        (costs-n-n (costs-n-1 n1))
        (helper (+ n1 2) n2)))
  (helper 1 n))
```

What is the order-of-growth in time and space for **unknown-costs**?

```
(define (costs-n-n n)
  (if (<= n 0)
      0
      (+ n (costs-n-n (- n 1)))))
```

```
(define (costs-n-1 n)
  (if (<= n 0)
      0
      (costs-n-1 (- n 1))))
```

```
(define (unknown-costs n)
  (define (helper n1 n2)
    (if (>= n1 (* n2 n2 n2))
        (costs-n-n (costs-n-1 n1))
        (helper (+ n1 2) n2)))
  (helper 1 n))
```

```
:: OOG time : n^3      (Notes: n^3 + n^3 + 1)
:: OOG space: 1       (Notes: 1 + 1 + 1 - final call to
costs-n-n is passed 0)
```

