Quiz

Wednesday 7:30 to 9:30 in 4-2/370
One sheet of notes double-sided
Quiz review... Wednesday recitation
Conflict exam on Friday:
Let Donna know right now!
dkauf@mit.edu
Because she will be away most of next week
LA quiz review: Sun Oct 2 at 7pm in 4-149
Mon Oct 3 in 4-159 at 7pm
Old quiz subjects are on the web

DrScheme upgrade

Get version 299.400 which, I believe, fixes the save as .scm issue.
In general be careful, verify with another text editor before closing DrScheme.

(compose f g)

Returns the function f(g(x))
(assume f and g take one variable)
((compose sqrt square) -4) \rightarrow 4
(define (compose f g)
    (lambda(x) (f (g x))))
Type?
(B \rightarrow C), (A \rightarrow B) \rightarrow (A \rightarrow C)

((compose square square) 2)
16
Analyze by substitution
((lambda(x) (square (square x))) 2)
(square (square 2))
(square 4)
16

Repeated composition:
(repeated f n)

Returns the function x\rightarrow f(f(\ldots (x)))
((repeated square 4) 2) \rightarrow 65536
Use compose
(define (identity x) x)
(define (repeated f n)
    (if (= n 0) identity
        (compose f (repeated f (- n 1)))))

Repeated composition: (repeated f n)

(define (repeated f n)
    (if (= n 0) identity
        (compose f (repeated f (- n 1)))))
Analyze by substitution (be lazy, use 3)
((repeated square 3) 2)
((if (= 3 0) identity (compose square (repeated square 2))) 2)
((compose square (compose square (compose square identity))) 2)
((compose square (compose square identity)) 2)
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65536
(Actually, identity and square should be replaced by the actual procedure.)
Iterative version!

```scheme
(define (repeated f n)
  (define (iter n ans)
    (if (= n 0) ans
     (iter (- n 1) (compose f ans))))
  (iter n (lambda (x) x))
)
```

Generalized version

Note that repeated composition is similar to the definition of multiplication with multiple additions and the definition of exponentiation with multiple multiplications. Define a higher-order generalized-repeated such that you can do:

```scheme
(define mult (generalized-repeated + 0))
(define exp (generalized-repeated * 1))
(define repeated (generalized-repeated compose identity))
```

And here we go:

```scheme
(define (generalized-repeated op neutral)
  (define helper (lambda(x n)
    (if (= n 0) neutral
     (op x (helper x (- n 1))))))
)
```

Swap: takes procedure of two arguments, returns procedure where arguments are swapped

```scheme
((swap /) 3 6) → 2
(define swap
  (lambda f
    (lambda (x y) (f y x))))
Type?
(A B → C) → (B, A) → C
```

Define (specialize f arg)

takes a function of two variable and returns a function of one variables by always using arg as the second variable

```scheme
(define half (specialize / 2))
(half 6) → 3
(define inverse (specialize (swap /) 1))
Type of specialize?
(A, B → C), B → (A → C)
```

Using specialize

Define positive? that checks if a number is greater than 0

```scheme
(define positive? (specialize > 0))
```

Define inverse by specializing /

```scheme
(define inverse
  (specialize (swap /) 1))
```

Careful, we want to specialize the first argument, and specialize only knows how to specialize the second one. Hence the use of swap

```scheme
(define (specialize-to-0 proc)
  It takes a procedure of two arguments and returns a procedure of one argument where proc is always used with 0 as the second argument
  (define specialize-to-0
    (specialize specialize 0))
  Cool isn’t it?
  If we substitute
  (define specialize-to-0 (specialize specialize 0))
  (define specialize-to-0 (lambda (x)(specialize x 0)))
)```