



## $\alpha$ Rby : An Embedding of Alloy in Ruby

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4th International ABZ 2014 Conference

June 2 - 6 2014  
Toulouse - France

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- alloy-like **syntax** in ruby (embedded DSL)? ✓

```
abstract sig Person {
  father: lone Man,
  mother: lone Woman
}
sig Man extends Person {
  wife: lone Woman
}
sig Woman extends Person {
  husband: lone Man
}
fact TerminologyBiology {
  wife = ~husband
  no p: Person |
    p in p.^(mother + father)
}
```

```
abstract sig Person [
  father: (lone Man),
  mother: (lone Woman)
]
sig Man extends Person [
  wife: (lone Woman)
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sig Woman extends Person [
  husband: (lone Man)
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fact terminology_biology {
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- retain the same alloy modeling environment
- write and analyze the same old alloy models
- add general-purpose scripting layer around it

# Why Scripting?

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## **practical reasons**

- automate multiple model finding tasks
- pre-processing (e.g., prompt for analysis parameters)
- post-processing (e.g., display the results of the analysis)
- build tools more easily

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```
s = SudokuModel::Sudoku.parse("0,0,1; 0,3,4; 3,1,1; 2,2,3")
s.solve      # invokes Alloy to solve the sudoku embodied in 's'
s.display    # draws some fancy graphical grid displaying the solution
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- quest for a **synergy** between **imperative** and **declarative**
- imperative **generation** of declarative specifications
  - can this change the way we write specifications?
  - can this simplify specification languages?

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**not studied  
as much**



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- recreate the alloy modeling environment in an existing language
  - **challenges**:
    - achieving alloy's relational semantics
    - achieving alloy's "non-standard" operators
    - achieving alloy's complex syntax
    - **reconcile two different paradigms**

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- recreate the alloy modeling environment in an existing language
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$\alpha$ Rby

$\alpha$ Rby by example: **Sudoku**

# Example: Sudoku in $\alpha$ Rby

```
alloy :SudokuModel do

  sig Sudoku [
    # cell coordinate -> cell value
    grid: Int ** Int ** (lone Int)
  ]

  # ...
end
```

# Example: Sudoku in $\alpha$ Rby

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alloy :SudokuModel do

  sig Sudoku [
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  pred solved[s: Sudoku] {
    # each row contains 1..N
    # each column contains 1..N
    # each matrix contains 1..N
  }
end
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    # each matrix contains 1..N
  }
end
```

## 1. translates ruby to classes/methods

```
module SudokuModel

  class Sudoku < Arby::Ast::Sig
    attr_accessor :grid
  end

  def self.solved(s)
    # exactly the same body in the
    # spec as on the left
  end
end
```



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    # each column contains 1..N
    # each matrix contains 1..N
  }
end
```

2. can be used in regular OOP

- monkey patch classes with utility methods

```
class SudokuModel::Sudoku
  def display
    puts grid # or draw fancy grid
  end

  def self.parse(str)
    Sudoku.new grid:
      str.split(/;\s*/).map{ |x|
        x.split(/,/).map(&:to_i) }
  end
end
```

- create objects, get/set fields, call methods

```
s = SudokuModel::Sudoku.new
s.grid = [[0, 0, 1], [1, 3, 2]]
puts s.grid
s = SudokuModel::Sudoku.parse(
  "0,0,1; 0,3,4; 3,1,1; 2,2,3")
s.display
```

# Sudoku in $\alpha$ Rby: Mixed Execution

```
alloy :SudokuModel do

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```

- *goal*: parameterize the spec by sudoku size

# Sudoku in $\alpha$ Rby: Mixed Execution

```
alloy :SudokuModel do
  SudokuModel::N = 9

  sig Sudoku [
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  pred solved[s: Sudoku] {
    # each row contains 1..N
    # each column contains 1..N
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```

- **goal:** parameterize the spec by sudoku size
3. specification **parameterized** by sudoku size

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  SudokuModel::N = 9

  sig Sudoku [
    # cell coordinate -> cell value
    grid: Int ** Int ** (lone Int)
  ]

  pred solved[s: Sudoku] {
    # concrete
    m = Integer(Math.sqrt(N))
    rng = lambda{|i| m*i...m*(i+1)}

    # symbolic
    all(r: 0..N) {
      s.grid[r][Int] == (1..N) and
      s.grid[Int][r] == (1..N)
    } and
    all(c, r: 0..m) {
      s.grid[rng[c]][rng[r]] == (1..N)
    }
  }
end
```

● **goal**: parameterize the spec by sudoku size  
3. specification **parameterized** by sudoku size

4. **mixed** concrete and symbolic **execution**

- the **spec** is the **return value** of the method
- special  $\alpha$ Rby methods return symbolic values (e.g., **all**, overloaded operators, ...)
- everything else executes concretely
- executed **lazily**:

this ruby code

```
SudokuModel.N = 4
puts SudokuModel.to_als
```

and this code

```
SudokuModel.N = 9
puts SudokuModel.to_als
```

produce different alloy specifications.

# Sudoku in $\alpha$ Rby: Partial Instance

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alloy :SudokuModel do
  SudokuModel::N = 9

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```

- **goal:** shrink bounds to enforce the partial solution known upfront (the pre-filled Sudoku cells)

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    }
  }
end
```

## 5. solving with partial instance

```
class SudokuModel::Sudoku
  def pi
    b = Arby::Ast::Bounds.new
    inds = (0..N)**(0..N) -
           self.grid.project(0..1)
    b[Sudoku] = self
    b.lo[Sudoku.grid] = self**self.grid
    b.hi[Sudoku.grid] = self**inds**(1..N)
    b.bound_int(0..N)
  end
  def solve
    # satisfy pred solved given partial inst
    SudokuModel.solve :solved, self.pi
  end
end
```

# Sudoku in $\alpha$ Rby: Partial Instance

## 5. solving with partial instance

```
alloy :SudokuModel do
  SudokuModel::N = 9

  sig Sudoku [
    # cell coord create empty bounds
    grid: Int ** Int ** (lone Int)
  ]

  pred solved[s: Sudoku] {
    # concrete
    m = Integer(Math.sqrt(N))
    rng = lambda{|i| m*i...m*(i+1)}

    # symbolic
    all(r: 0..N) {
      s.grid[r][Int] == (1..N) and
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```
alloy :SudokuModel do
  SudokuModel::N = 9

  sig Sudoku [
    # cell coordinate > cell value
    grid
  ]

  pred solved[s: Sudoku] {
    # concrete
    m = Integer(Math.sqrt(N))
    rng = lambda{|i| m*i...m*(i+1)}

    # symbolic
    all(r: 0...N) {
      s.grid[r][Int] == (1..N) and
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    } and
    all(c, r: 0...m) {
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```

compute indexes of empty cells

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  }
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```

exact bound for Sudoku:  
exactly self

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    } and
    all(c, r: 0..m) {
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    }
  }
end
```

lower bound for grid:  
must include the filled cells

```
class SudokuModel::Sudoku
  def pi
    b = Arby::Ast::Bounds.new
    inds = (0..N)**(0..N) -
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  SudokuModel::N = 9

  sig Sudoku [
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  ]

  pred solved
    upper bound for grid:
    may include (1..N) for all empty cells
    m = Integer(Math.sqrt(N))
    rng = lambda{|i| m*i...m*(i+1)}

    # symbolic
    all(r: 0..N) {
      s.grid[r][Int] == (1..N) and
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only ints from 0 to N

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```

if SAT, automatically updates all "sig class" objects used as part of the partial instance

```
    s.grid[r][Int] == (1..N) and
    s.grid[Int][r] == (1..N)
  } and
  all(c, r: 0...m) {
    s.grid[rng[c]][rng[r]] == (1..N)
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  end
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    SudokuModel.solve :solved, self.pi
  end
end
```

## 6. continue to use as regular OOP

```
s = SudokuModel::Sudoku.parse(
  "0,0,1; 0,3,4; 3,1,1; 2,2,3")
s.solve; s.display
```

## Sudoku in $\alpha$ Rby: Staged Execution

**goal:** generate a minimal sudoku puzzle

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```
def min(sudoku)
  # ...
end
```

```
s = Sudoku.new(); s.solve(); s = min(s);
puts "local minimum: #{s.grid.size}"
```

← start with empty sudoku,  
solve it, then minimize it



# Sudoku in $\alpha$ Rby: Staged Execution

**goal:** generate a minimal sudoku puzzle

```
def dec(s, order=Array(0...s.grid.size).shuffle)
  # ...
end
```

```
def min(sudoku) ←
  (s1 = dec(sudoku)) ? min(s1) : sudoku
end
```

```
s = Sudoku.new(); s.solve(); s = min(s);
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```

try to decrement;  
if successful minimize the result,  
otherwise return the input sudoku

# Sudoku in $\alpha$ Rby: Staged Execution

**goal:** generate a minimal sudoku puzzle

```
def dec(s, order=Array(0...s.grid.size).shuffle)
  return nil if order.empty?
  # remove a cell, then re-solve
  s_dec = Sudoku.new grid:
    s.grid.delete_at(order.first)
  sol = s_dec.clone.solve()
  # check if unique
  if sol.satisfiable? && !sol.next.satisfiable?
    s_dec # return decremented sudoku
  else # try deleting some other cell
    dec(s, order[1..-1])
  end
end

def min(sudoku)
  (s1 = dec(sudoku)) ? min(s1) : sudoku
end

s = Sudoku.new(); s.solve(); s = min(s);
puts "local minimum: #{s.grid.size}"
```

pick a cell to remove and check if  
the new sudoku has a unique solution;  
keep trying until run out of cells;

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**goal:** generate a minimal sudoku puzzle

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  return nil if order.empty?
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def min(sudoku)
  (s1 = dec(sudoku)) ? min(s1) : sudoku
end

s = Sudoku.new(); s.solve(); s = min(s);
puts "local minimum: #{s.grid.size}"
```

pick a cell to remove and check if  
the new sudoku has a unique solution;  
keep trying until run out of cells;

solve next to check for uniqueness

# Sudoku in $\alpha$ Rby: Staged Execution

**goal:** generate a minimal sudoku puzzle

```
def dec(s, order=Array(0...s.grid.size).shuffle)
  return nil if order.empty?
  # remove a cell, then re-solve
  s_dec = Sudoku.new grid:
    s.grid.delete_at(order.first)
  sol = s_dec.clone.solve()
  # check if unique
  if sol.satisfiable? && !sol.next.satisfiable?
    s_dec # return decremented sudoku
  else # try deleting some other cell
    dec(s, order[1..-1])
  end
end

def min(sudoku)
  (s1 = dec(sudoku)) ? min(s1) : sudoku
end

s = Sudoku.new(); s.solve(); s = min(s);
puts "local minimum: #{s.grid.size}"
```

pick a cell to remove and check if  
the new sudoku has a unique solution;  
keep trying until run out of cells;

solve next to check for uniqueness

uses the previous solution to  
search for a new (smaller) one



# $\alpha$ Rby Implementation Tricks

## Reconciling Alloy and Ruby

- **alloy**: declarative, relational, based on FOL
- **ruby**: imperative, non-relational, object-oriented



# Reconciling Alloy and Ruby

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<b>structures</b>	modules sigs fields predicates	modules classes attributes methods

# Reconciling Alloy and Ruby



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<b>syntax</b>	“non-standard” operators	very liberal parser, can accommodate most cases



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<b>syntax</b>	“non-standard” operators	very liberal parser, can accommodate most cases
<b>semantics</b>	everything is a relation	“monkey patch” relevant ruby classes to make them look like relations

# Reconciling Alloy and Ruby: **Syntax**

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fun return type declaration	<code>fun f[s: S]: set S {}</code>	<code>fun f[s: S][set S] {}</code>
set comprehension	<code>{s: S   p1[s]}</code>	<code>S.select{ s  p1(s)}</code>
illegal Ruby operators	<code>x in y, x !in y x !&gt; y x -&gt; y x . y #x x =&gt; y x =&gt; y else z S &lt;: f, f &gt;: Int</code>	<code>x.in?(y), x.not_in?(y) not x &gt; y x ** y x.(y) x.size y if x if x then y else z S.&lt; f, f.&gt; Int</code>
operator arity mismatch	<code>^x, *x</code>	<code>x.closure, x.rclosure</code>

# Achieving Syntax: $\alpha$ Rby Builder Methods

how is this parsed by ruby?

```
abstract sig Person [ father: (lone Man), mother: (lone Woman) ] { <facts> }
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^

Module#const\_missing(:Person)  $\rightarrow$  *builder*

### legend:

- **blue identifiers**: method names implemented or overridden by  $\alpha$ Rby
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abstract sig Person [ father: (lone Man), mother: (lone Woman) ] { <facts> }
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```
^      |  
Module#|onst_missing(:Person) → builder  
      builder.send :[], {father: ...}, &proc{<facts>} → builder
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- run  $\alpha$ Rby programs using the standard ruby interpreter
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pred solved[s: Sudoku] {
  # concrete
  m = Integer(Math.sqrt(N))
  rng = lambda{|i| m*i...m*(i+1)}

  # symbolic
  all(r: 0...N) { s.grid[r][Int] == (1..N) && s.grid[Int][r] == (1..N) } and
  all(c, r: 0...m) { s.grid[rng[c]][rng[r]] == (1..N) }
}
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## benefits

- overload methods in sym classes instead of writing interpreter
- concrete values automatically evaluated

## challenge

- not all ruby operators can be overridden
  - all logic operators: `&&`, `||`, `and`, `or`, ...
  - all branching constructs: `if-then-else` (and all its variants)



# Online Source Instrumentation

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## solution

- use an off-the-shelf ruby parser
- run a simple AST search-replace algorithm
- replace

→ `x if c` with `BinExpr.new(IMPLIES, proc{c}, proc{x})`

→ `a and b` with `BinExpr.new(AND, proc{a}, proc{b})`, etc.

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- optional instrumentation (nicer syntax for some idioms)

→ `s.*f` → `s.join(f.closure)`

→ ...

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### mixed execution

- dynamically generate Alloy **models** (specifications)
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- **example**: pre-filled Sudoku cells

## staged model finding

- **iteratively** run Alloy (e.g., until some fixpoint)
  - at each step use previous solutions as a guide
- **example**: generate a minimal Sudoku puzzle

# Conclusions

## the $\alpha$ Rby approach

- addresses a collection of **practical** problems
- demonstrates an **alternative** to building **classical APIs**



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  - design a clean set of core modeling features
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# Conclusions

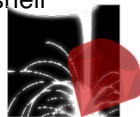
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# Thank You!



$\alpha$ Rby: <http://people.csail.mit.edu/aleks/arby>



# Sudoku: More Reasons for Mixed Execution

# Sudoku: More Reasons for Mixed Execution

- one of the top results for the “alloy sudoku” internet search [1]

[1] <https://gist.github.com/athos/1817230>

```
// Numbers
abstract sig Digit {}
one sig One, Two, Three, Four extends Digit {}

// cells
sig Cell { content: one One+Two+Three+Four }
one sig Ca0, Ca1, Ca2, Ca3,
      Cb0, Cb1, Cb2, Cb3,
      Cc0, Cc1, Cc2, Cc3,
      Cd0, Cd1, Cd2, Cd3 extends Cell {}

// groups
sig Group { cells: set Cell } {
  no disj c,c': cells | c.content=c'.content
}
sig Row, Column, Matrix extends Group {}
one sig Ra, Rb, Rc, Rd extends Row {}
one sig C0, C1, C2, C3 extends Column {}
one sig M0, M1, M2, M3 extends Matrix {}

// assign cells to groups
fact {
  Ra.cells = Ca0+Ca1+Ca2+Ca3
  Rb.cells = Cb0+Cb1+Cb2+Cb3
  Rc.cells = Cc0+Cc1+Cc2+Cc3
  Rd.cells = Cd0+Cd1+Cd2+Cd3

  C0.cells = Ca0+Cb0+Cc0+Cd0
  C1.cells = Ca1+Cb1+Cc1+Cd1
  C2.cells = Ca2+Cb2+Cc2+Cd2
  C3.cells = Ca3+Cb3+Cc3+Cd3

  M0.cells = Ca0+Ca1+Cb0+Cb1
  M1.cells = Ca2+Ca3+Cb2+Cb3
  M2.cells = Cc0+Cc1+Cd0+Cd1
  M3.cells = Cc2+Cc3+Cd2+Cd3
}

run {} for 20 but 16 Cell
```

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**good**

- elegant solution (very simple constraints)
  - doesn't use integer arithmetic
  - possibly more efficient than with integers
- the structure can be encoded as a partial instance

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good

```
// Numbers
```

```
abstract sig Digit {}
```

```
one sig One, Two, Three, Four, Five, Six, Seven, Eight, Nine
```

```
// cells
```

```
sig Cell { content: one Digit }
```

```
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```

```
Cb0, Cb1, Cb2, Cb3,
```

```
Cc0, Cc1, Cc2, Cc3,
```

```
Cd0, Cd1, Cd2, Cd3
```

```
// groups
```

```
sig Group { cells: list Cell }
```

```
no disj c, c' | c.content = c'.content
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- doesn't use integer arithmetic

- possibly more efficient than with integers

→ the structure can be encoded as a partial instance

bad

- hardcoded for size 4

- too much “copy-paste” repetition

- tedious to write for larger sizes

- partial instance encoded as a constraint

```
run {} for 20 but 16 Cell
```

# Sudoku: Mixed Execution in $\alpha$ Rby

```
# Returns an Alloy model formally specifying the Sudoku puzzle for a given size.
# @param n: sudoku size
def self.gen_sudoku_spec(n)
  m = Math.sqrt(n).to_i # precompute sqrt(n) (used below to build the spec)

  # use the aRby DSL to specify Alloy model
  alloy :Sudoku do
    # ...
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  alloy :Sudoku do
    self::N = n # save 'n' as a constant in this Ruby module

    # declare base sigs (independent of sudoku size)
    abstract sig Digit
    abstract sig Cell [ content: (one Digit) ]
    abstract sig Group [ cells: (set Cell) ] {
      no(c1, c2: cells) { c1 != c2 and c1.content == c2.content }
    }
    # ...
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    abstract sig Cell [ content: (one Digit) ]
    abstract sig Group [ cells: (set Cell) ] {
      no(c1, c2: cells) { c1 != c2 and c1.content == c2.content }
    }
    # generate concrete sigs for the given size
    (0...n).each do |i|
      one sig "D#{i}" < Digit
      one sig "R#{i}", "C#{i}", "M#{i}" < Group
      (0...n).each{ |j| one sig "C#{i}#{j}" < Cell }
    end
  end
end
```



# Reconciling Alloy and Ruby: Structures

```
alloy :Grandpa do
  abstract sig Person [
    father: (lone Man),
    mother: (lone Woman)
  ]
  sig Man extends Person [
    wife: (lone Woman)
  ]
  sig Woman extends Person [
    husband: (lone Man)
  ]
  fact terminology_biology {
    wife == ~husband and
    no(p: Person) {
      p.in? p.^(mother + father)
    }
  }
end
```



```
module Grandpa
  class Person < Arby::Ast::Sig
    attr_accessor :father
    attr_accessor :mother
  end
  class Man < Person
    attr_accessor :wife
  end
  class Woman < Person
    attr_accessor :husband
  end
  def fact_terminology_biology
    wife = ~husband and
    no(p: Person) {
      p.in? p.^(father + mother)
    }
  end
end
```

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    mother: (lone Woman)
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  ]
  sig Woman extends Person [
    husband: (lone Man)
  ]
  fact terminology_biology {
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  }
end
```



```
module Grandpa
  class Person < Arby::Ast::Sig
    attr_accessor :father
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  end
  class Man < Person
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  end
  class Woman < Person
    attr_accessor :husband
  end
  def fact_terminology_biology
    wife = ~husband and
    no(p: Person) {
      p.in? p.^(father + mother)
    }
  end
end
```

- generated on the fly, automatically and transparently
- type (and other) info saved in `Grandpa.meta`

document