Model Checking Using SMT and Theory of Lists

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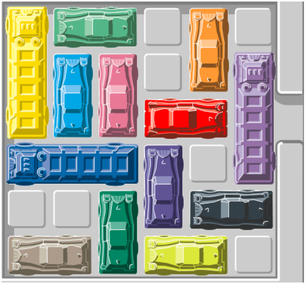
April 18, 2011

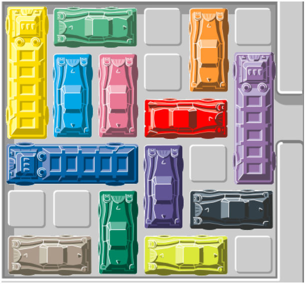
Solving Planning Problems



Rush Hour puzzle

Goal: drive the red car out of the jam

Solving Planning Problems



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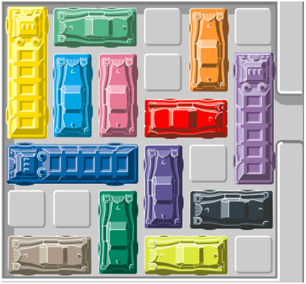

Rush Hour puzzle

Goal: drive the red car out of the jam

- solve using a satisfiability solver
- problem: number of necessary steps is not known

Software Model Checking without Loop Unrolling

```
void selectSort(int[] a, int N) {
 for (int j=0; j<N-1; j++) {
  int min = j;
  for (int i=j+1; i < N; i++)
   if (a[min] > a[i]) min = i;
  int t = a[j];
  a[j] = a[min];
  a[min] = t;
 }
 for (int j=0; j<N-1; j++)
  assert a[j] <= a[j+1];
}
```

Selection Sort algorithm

Goal: verify for all `int` arrays of size up to N

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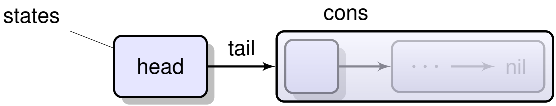
Selection Sort algorithm

Goal: verify for all `int` arrays of size up to N

- verify using model checking with satisfiability solving
- problem: number of necessary loop unrollings is not known
- moreover, the number of loop unrollings is not independent of N

Use Lists to Model State Transitions



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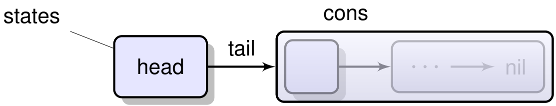

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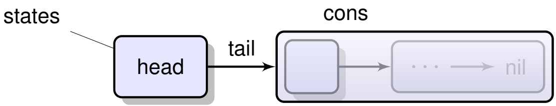

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Specify what the list should look like, not how long it should be.

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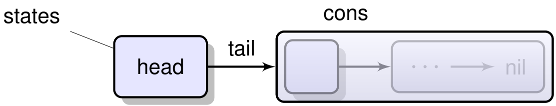

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To solve the rush hour puzzle:

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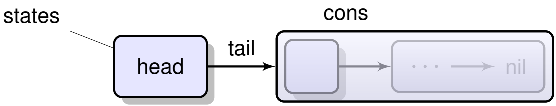

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Specify what the list should look like, not how long it should be.

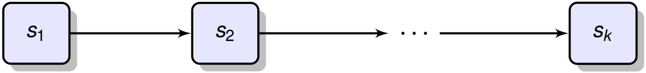
To solve the rush hour puzzle:

- use a list to model a sequence of car movements
- don't have to specify the number of steps

To solve a software model-checking problem:

- use a list to model a program trace
- don't have to specify the number of loop unrollings

Background: Bounded Model Checking



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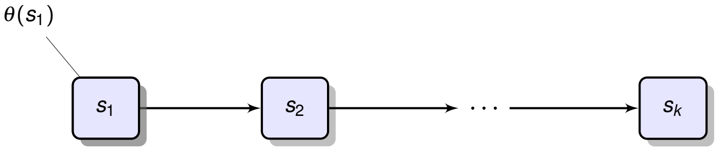

Initial state constraint:

$$\theta(s_1)$$

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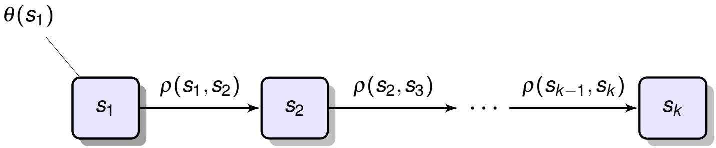

Initial state constraint:

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Transition constraint:

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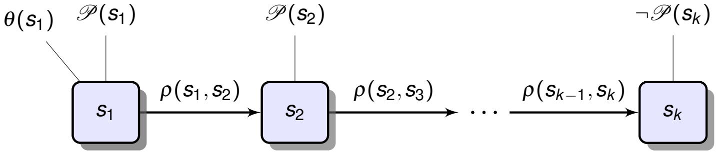

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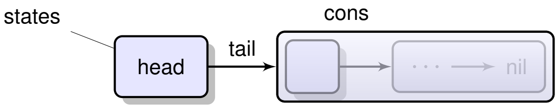
Safety Property constraint:

$$\mathscr{P}(s_1) \wedge \mathscr{P}(s_2) \wedge \cdots \wedge \mathscr{P}(s_{k-1}) \wedge \neg\mathscr{P}(s_k)$$

Translation to SMT



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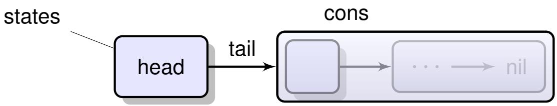

Available operations:
- is_nil(lst)
- is_cons(lst)
- head(lst)
- tail(lst)

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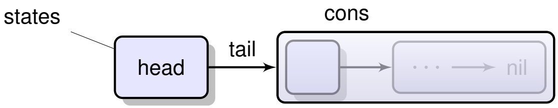

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- is_nil(lst)
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datatype StateList = nil | cons(head: State, tail: StateList)
def states: StateList

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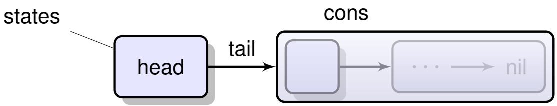

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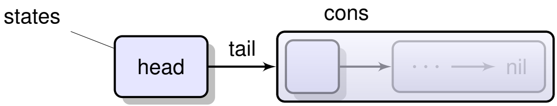

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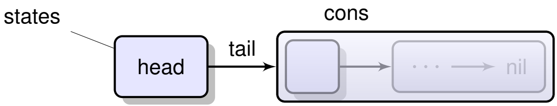
def states: StateList

def check_tr: StateList → bool

assert forall lst: StateList
  if (is_cons(lst) **and** is_cons(tail(lst))) **then**
    ρ(head(lst), head(tail(lst))) **and** check_tr(tail(lst)) **and**

Translation to SMT



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      is_nil(tail(tail(lst)))
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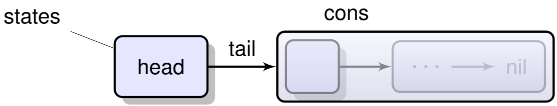

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  :pat {check_tr}

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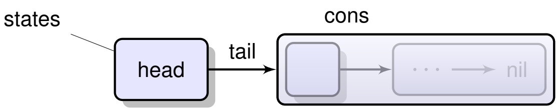

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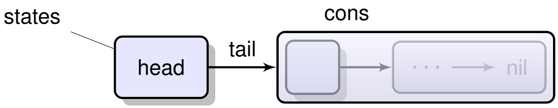
assert is_cons(states) **and** θ(head(states)) **and** check_tr(states)

Translation to SMT



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state declaration

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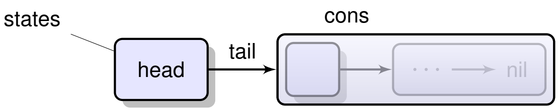
state transition and **safety property** enforced with an *uninterpreted function* and an *axiom*

assert is_cons(states) **and** θ(head(states)) **and** check_tr(states)

formula to check

Translation to SMT



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formula to check

Application to Software Model Checking

```
void simpleWhile(int N) {
 int x = 0, i = 0;
 while (i < N) {
  if (i % 2 == 0)
    x += 2;
  i++;
 }
 assert x == N ||
        x == N + 1;
}
```

Application to Software Model Checking

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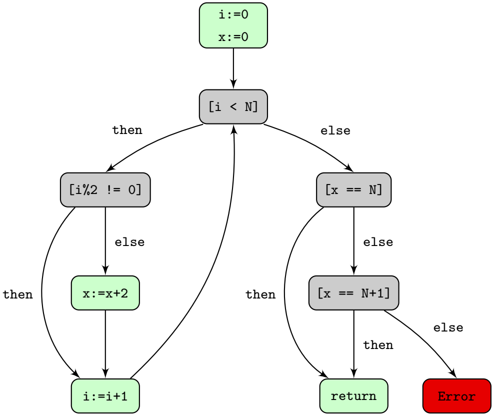

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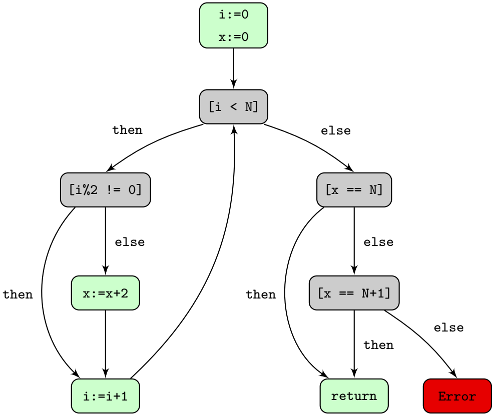

- goal: find a feasible path from start to an error node

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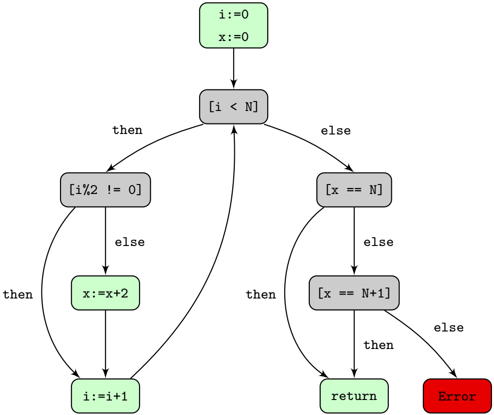
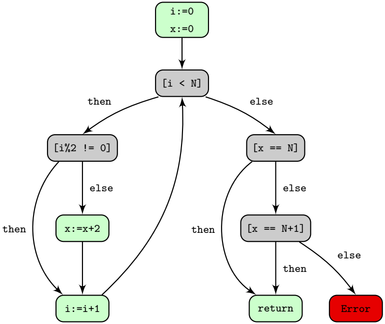


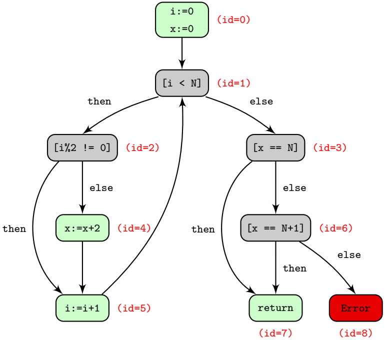
- **goal**: find a feasible path from start to an error node
- **idea**: use a list to represent a path in the graph

From CFG to θ, ρ, $\mathscr{P}$



From CFG to θ, ρ, $\mathscr{P}$

1. assign IDs to basic blocks



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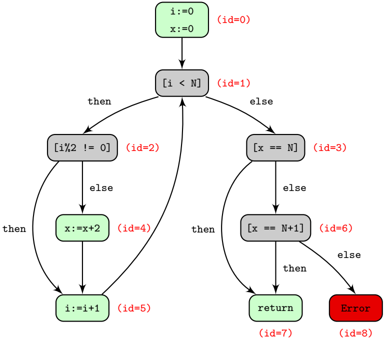

From CFG to θ, ρ, $\mathscr{P}$

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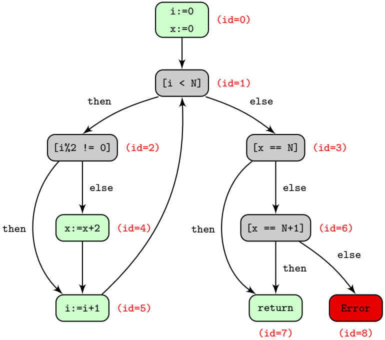
2. state tuple: [id, i, x]
3. initial state constraint θ:
   head(states).i=0 $\wedge$ head(states).x=0



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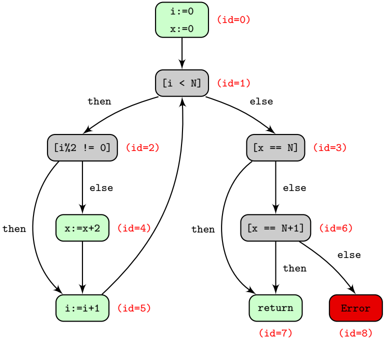

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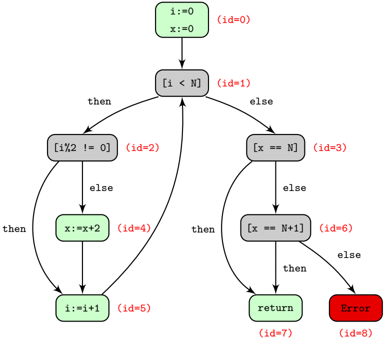
5. transition constraint $\rho(curr, next)$:
   if head(curr).id=0 **then**
   head(next).id=1 $\wedge$ head(next).i=0 $\wedge$ head(next).x=0
   else if head(curr).id=1 **then**
   if head(curr).i $<$ N **then** head(next).id=2 **else** head(next).id=3
   ...
   else
   false



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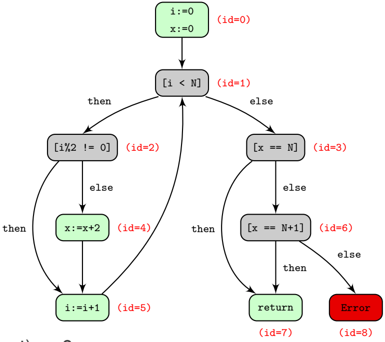

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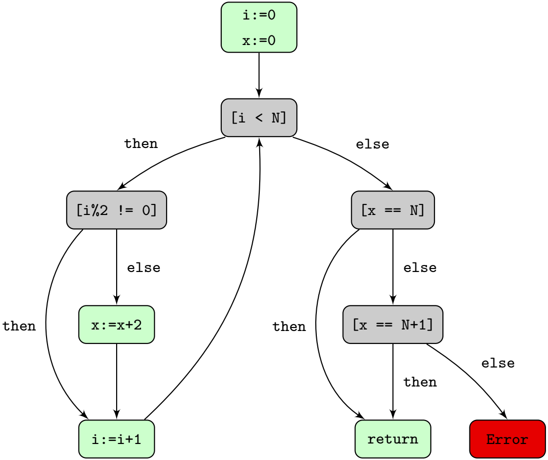
6. bounds on some pieces of data: N $>$ 0 $\wedge$ N $<$ 10

Optimization: Removing Empty Nodes



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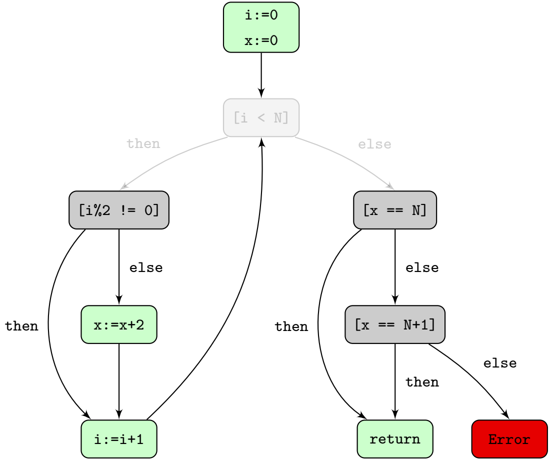

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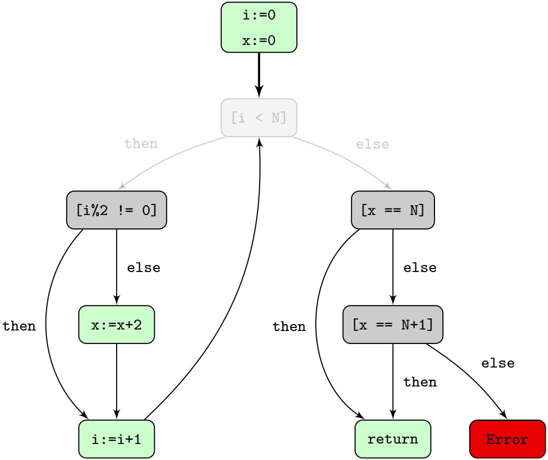

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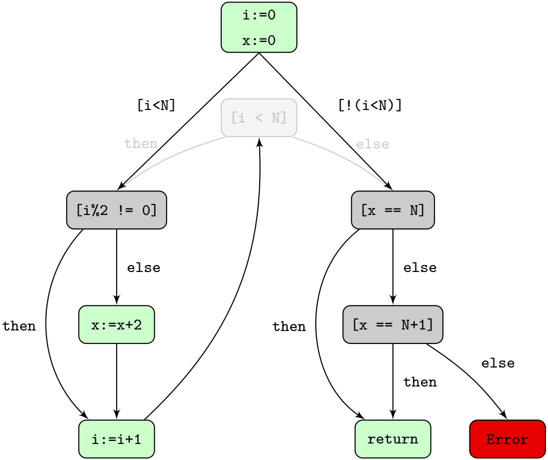

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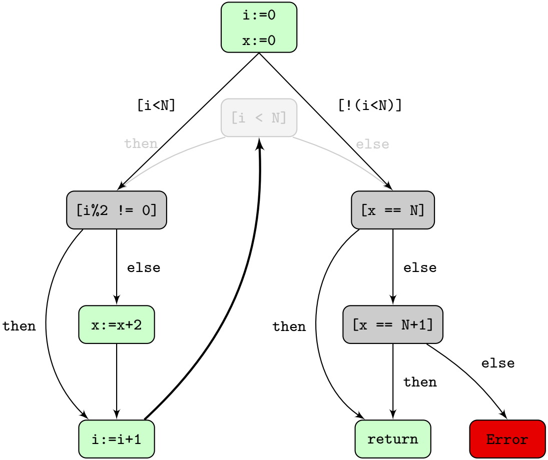
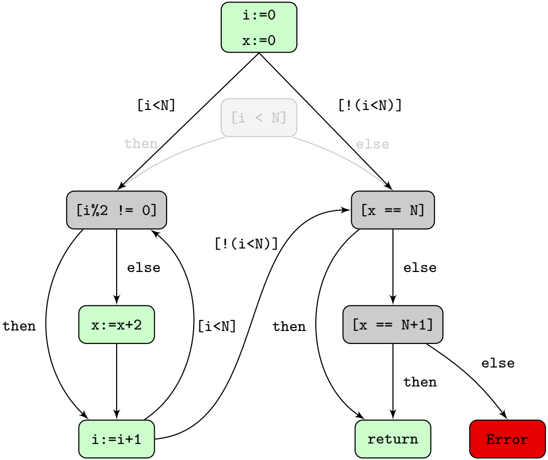


Optimization: Removing Empty Nodes



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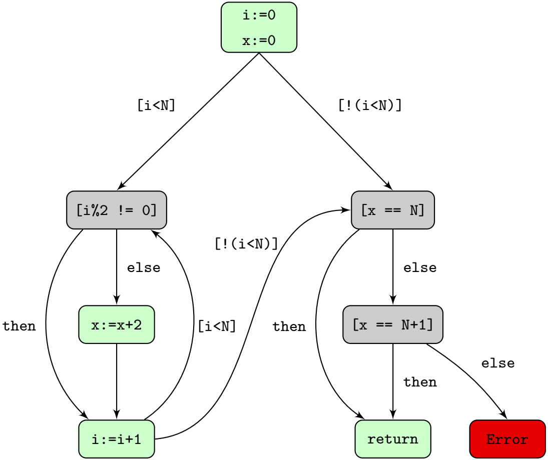
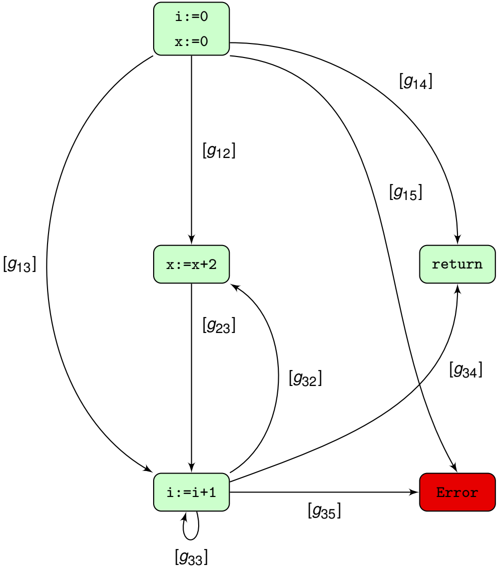


Optimization: Removing Non-Looping Nodes



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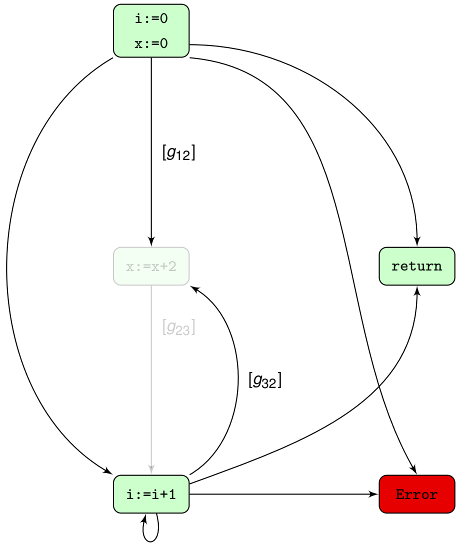

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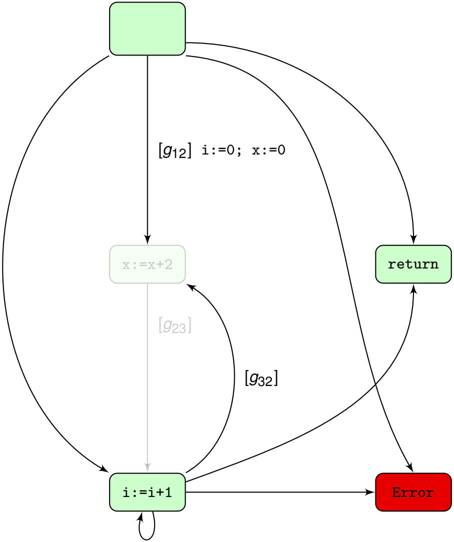

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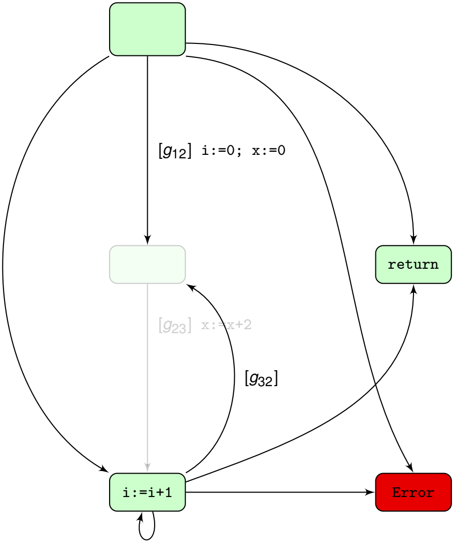

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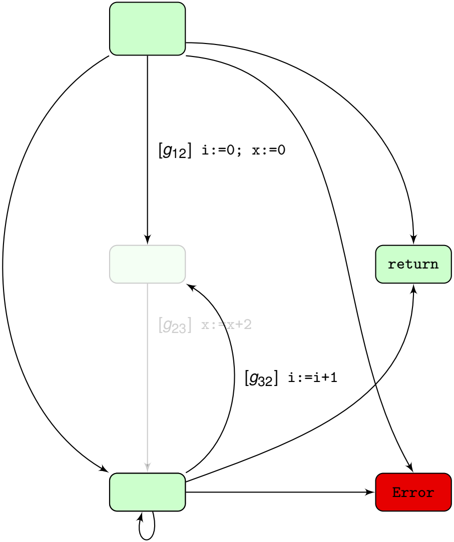

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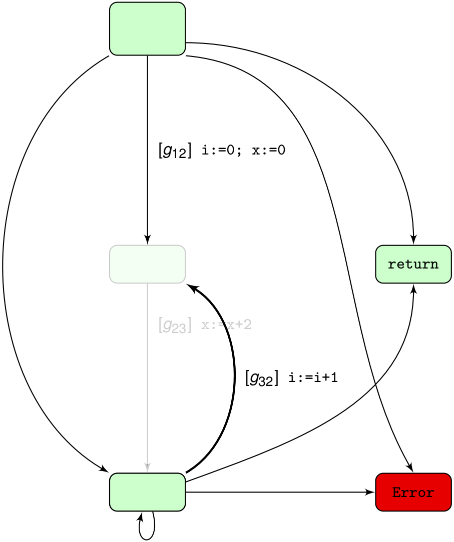

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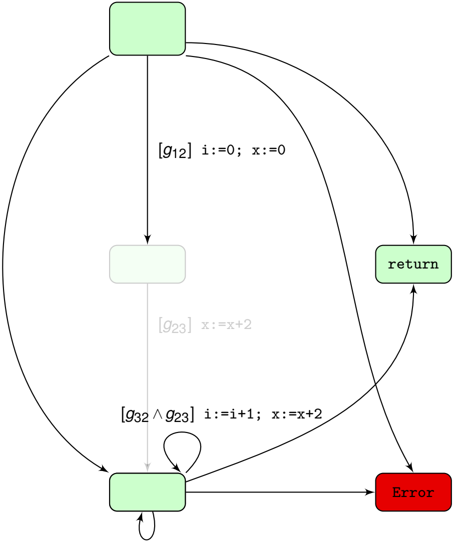

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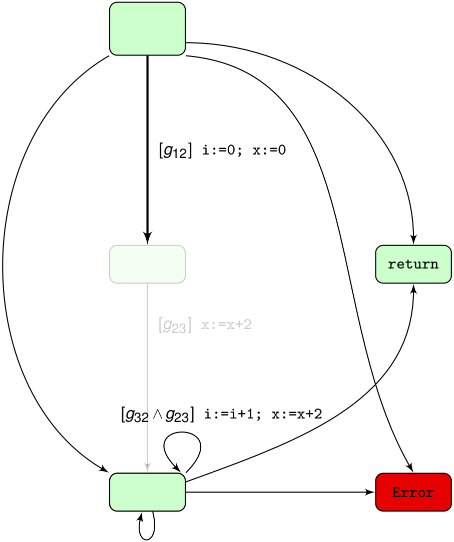

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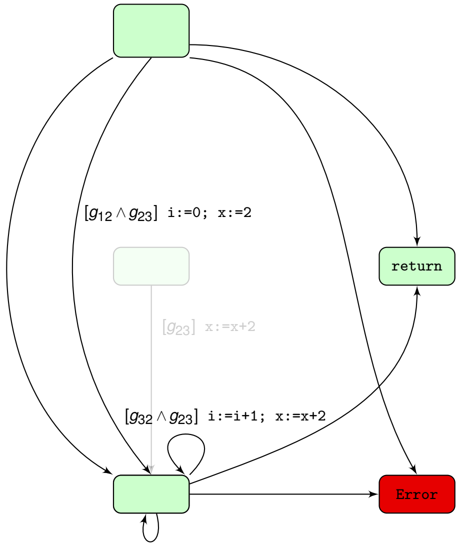

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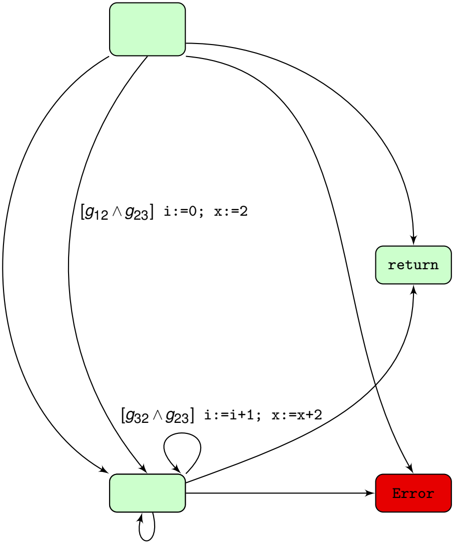


Evaluation: Verifying Simple Algorithms

Simple While Loop

```
void simpleWhile(int N) {
 int x = 0, i = 0;
 while (i < N) {
  if (i % 2 == 0)
    x += 2;
  i++;
 }
 assert x == N ||
        x == N + 1;
}
```

Selection Sort Algorithm

```
void selectSort(int[] a, int N) {
 for (int j=0; j<N-1; j++) {
  int min = j;
  for (int i=j+1; i < N; i++)
   if (a[min] > a[i]) min = i;
  int t = a[j]; a[j] = a[min]; a[min] = t;
 }
 for (int j=0; j<N-1; j++)
  assert a[j] <= a[j+1];
}
```

Integer Square Root Algorithm

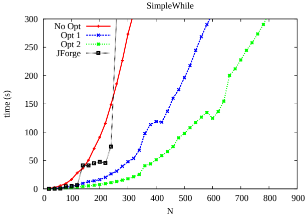
```
int intSqRoot(int N) {
  int r = 1, q = N;
  while (r+1 < q) {
    int p = (r+q) / 2;
    if (N < p*p) q = p;
    else r = p;
  }
  assert r*r <= N &&
         (r+1)*(r+1)>N;
  return r;
}
```

Bubble Sort Algorithm

```
void bubbleSort(int[] a, int N) {
 for (int j=0; j<N-1; j++)
  for (int i=0; i<N-j-1; i++)
   if (a[i] > a[i+1]) {
     int t = a[i];
     a[i] = a[i+1];
     a[i+1] = t;
   }
 for (int j=0; j<N-1; j++)
  assert a[j] <= a[j+1];
}
```

Evaluation: Verifying Simple Algorithms

Simple While Loop



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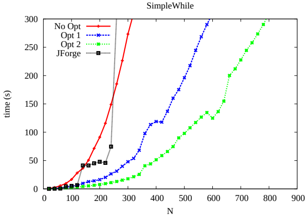
Evaluation: Verifying Simple Algorithms

Simple While Loop



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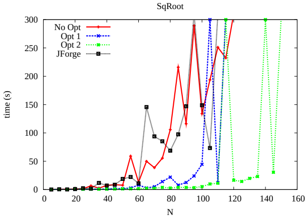
Integer Square Root Algorithm



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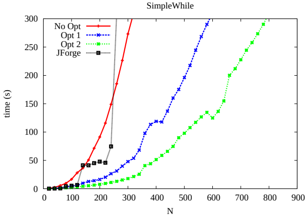
Evaluation: Verifying Simple Algorithms

Simple While Loop



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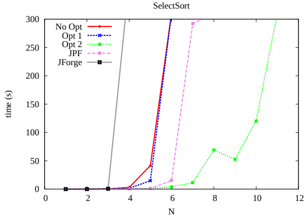
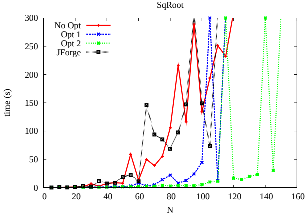


Integer Square Root Algorithm



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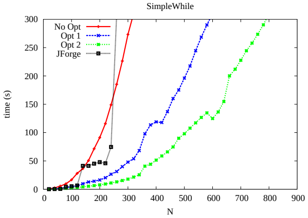
Evaluation: Verifying Simple Algorithms

Simple While Loop



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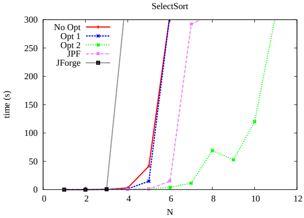

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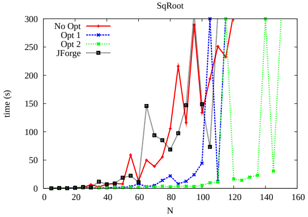
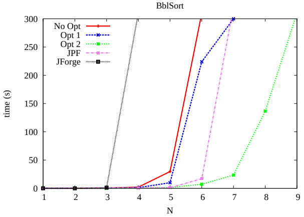
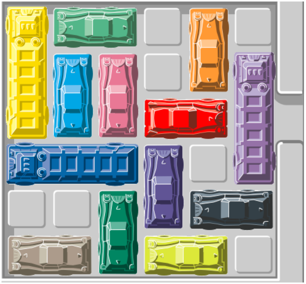


Bubble Sort Algorithm



Evaluation: Solving the Rush Hour Puzzle



Source:
http://www.puzzles.com/products/rushhour.htm

	Bounded	Using Lists	#steps
Jam 25	1.20s	1.88s	16
Jam 30	1.21s	2.17s	22
Jam 38	4.47s	36.6s	35
Jam 39	1.90s	14.66s	40
Jam 40	6.31s	17.89s	36

bounded: single flat formula, number of steps given up front
using lists: our approach with lists
#steps: min number of steps needed to solve the puzzle

- able to solve all puzzles from • in less than 40 seconds
- limitation: doesn't terminate if puzzle can't be solved
  - possible solution: optimize the solver not to explore same states

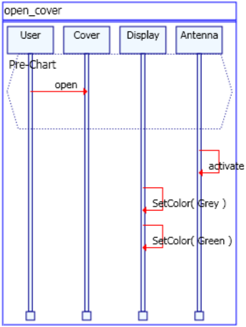
Case Study: Execution of Live Sequence Charts

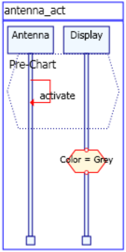
Goal: find valid single and super steps

- single step - a single message that doesn't cause a violation
- super step - a sequence of messages that closes all charts

Approach: formulate as a model-checking problem

- use a list to represent sent messages, and the state after each message
- transition constraint: messages don't cause violations
- safety property: not all charts are closed





Result: incorporated in the *Synthesizing Biological Theories* (SBT) tool [CAV'11]

Summary

- Model-checking technique using SMT Theory of Lists
- Theory of Lists lets you:
  - model unbounded state sequences
  - perform bounded model checking without explicitly bounding the length of counter examples
  - perform software model checking without loop unrolling

Thank You!

Microsoft Research

CSAIL

Related Work

Bounded Model Checking with SMT
(explicit loop unrolling required)

- Armando et al. (STTT 2009)

Unbounded Model Checking with SAT
(multiple invocation of the solver required)

- Kang et al. (DAC 2003), McMillan et al.(CAV 2002)

Bounded Model Checking with SAT
(explicit loop unrolling required)

- CBMC (Clarke04), JForge (Dennis09), Alloy Analyzer (Jackson06)

Explicit State Model Checking

- Java PathFinder (Visser00)

Future Work

Comparison with other tools/approaches

- planning problems: SMT Lists vs Alloy event paradigm
- software model checking: SMT Lists vs unbounded SAT

Optimization of SMT heuristics for theory of lists

- explore implementing fixpoint search inside SMT

Synthesizing Biological Theories

- Try out on more models of biological systems