Alin Tomescu | Week 1, Wednesday, February 5th, 2014 | Recitation 1
6.006 Intro to Algorithms | Prof. Srinivas Devadas | Prof. Nancy Lynch | Prof. Vinod Vaikuntanathan

"Big Oh" notation in terms of limits

Notation	**Limit definition**	**Examples**
$f(n) \in \Omega(g(n))$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} \in (0, \infty]$	$n^2 + 1 = \Omega(n) \Leftrightarrow \lim_{n\to\infty} \frac{n^2+1}{n} = \infty$
$f(n) \in \Theta(g(n))$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} \in (0, \infty)$	$n^2 + 3n + 4 = \Theta(n^2) \Leftrightarrow \lim_{n\to\infty} \frac{n^2+3n+4}{n^2} = \lim_{n\to\infty} \left(1 + \frac{3}{n} + \frac{4}{n^2}\right) = 1$
$f(n) \in O(g(n))$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} \in [0, \infty)$	$n^2 - 2n + 5 = O(n^3) \Leftrightarrow \lim_{n\to\infty} \frac{n^2-2n+5}{n^3} = \lim_{n\to\infty} \frac{1}{n} + \frac{2}{n^2} + \frac{5}{n^3} = 0$

Easy way of comparing functions

Little "o"

Used to indicate that $f < g$:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \Rightarrow f(n) = o(g(n))$$

Note: $f(n) = o(g(n)) \Rightarrow f(n) = O(g(n))$, because *Big O* is used to indicate that $f \leq g$

Little omega: ω

Used to indicate that $f > g$:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty \Rightarrow f(n) = \omega(g(n))$$

Note: $f(n) = \omega(g(n)) \Rightarrow f(n) = \Omega(n)$, because *Big Omega* is used to indicate that $f \geq g$

Theta

Used to indicate that $f = g$:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = c \in \mathbb{R} \Rightarrow f(n) = \Theta(g(n))$$

Note: $f(n) = \Theta(g(n)) \Rightarrow \begin{cases} f(n) = O(g(n)) \\ f(n) = \Omega(g(n)) \end{cases}$, because $f = g \Leftrightarrow f \geq g \wedge f \leq g$