Équipe de
Recherche en
Physique de
l'Information
Quantique

Belief propagation in the quantum world

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Counting, Inference, and Optimization on Graphs
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Quantum mechanics

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Random variable X

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Quantum particle

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State space Ω (finite dim)

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Product state

$\rho_{AB} = \rho_A \otimes \rho_B$

Motivation

Why should we care about QM?

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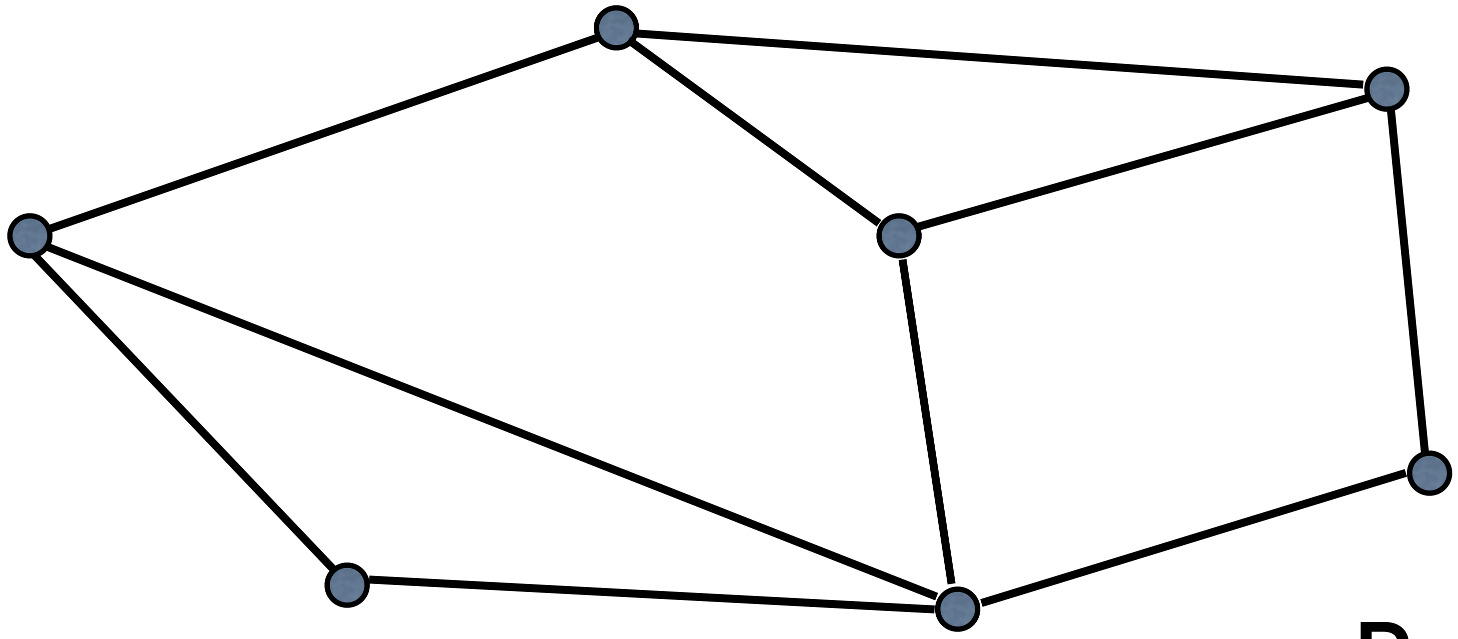
Why should we care about QM?

Because this is how nature really behaves.

Graphical models

$G = (V, E)$



Positive functions

$$p(x_1, x_2, \dots) = \prod_{x \in V} \nu(x) \prod_{(x,y) \in E} \mu(x, y)$$

Graphical models

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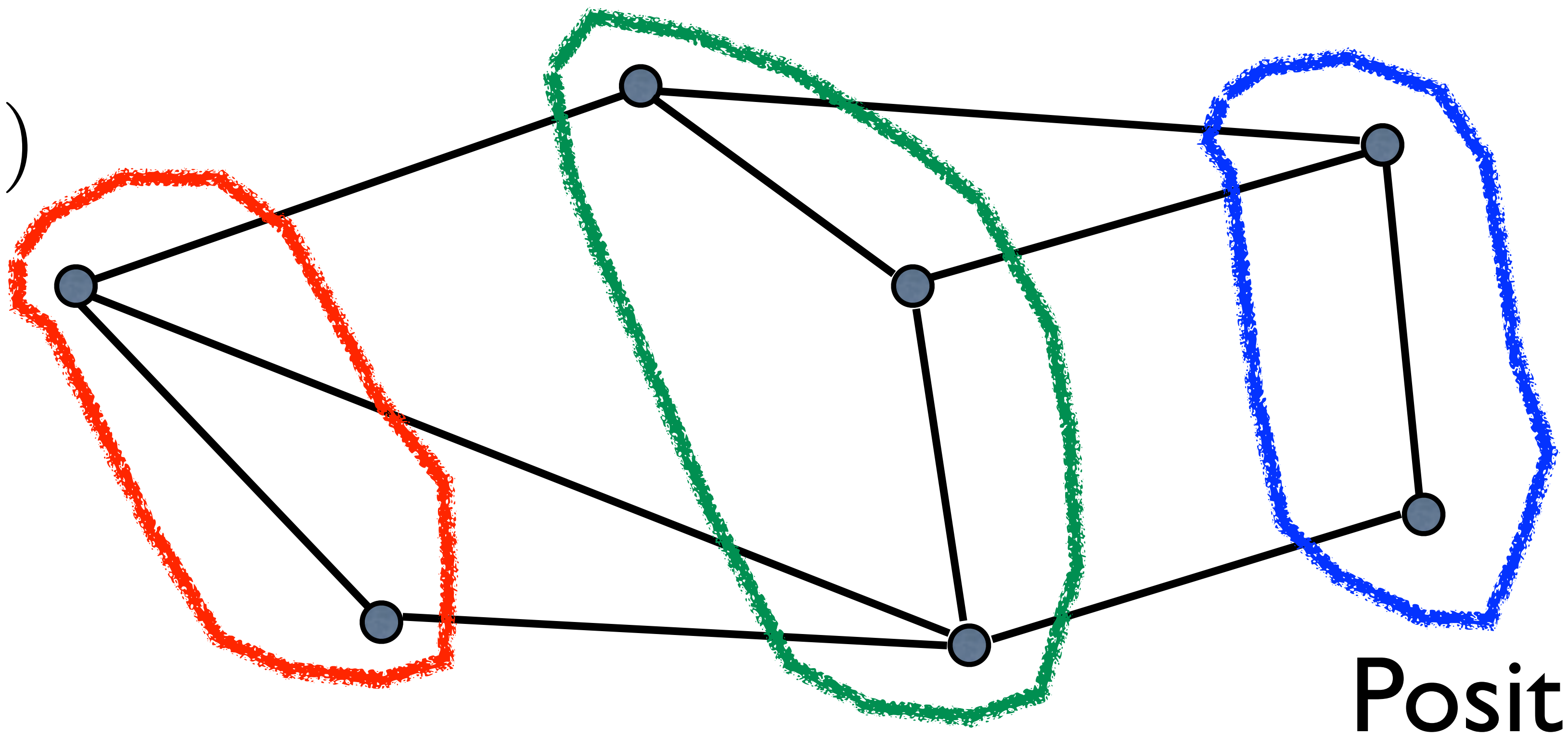

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$$\Downarrow$$

I(A:B|C) = 0 Markov network

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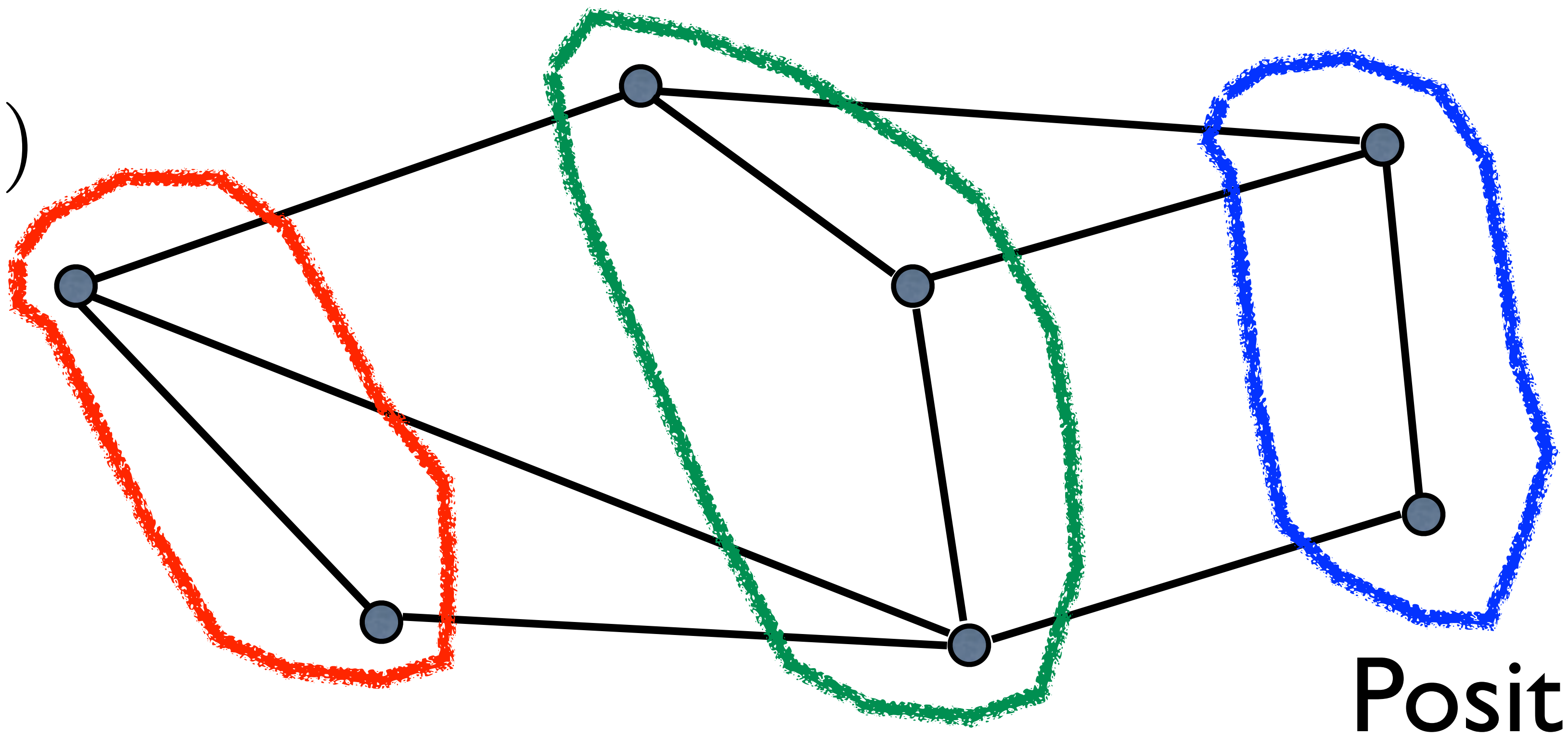

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Hammersley-Clifford $\Downarrow$

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Quantum graphical models?

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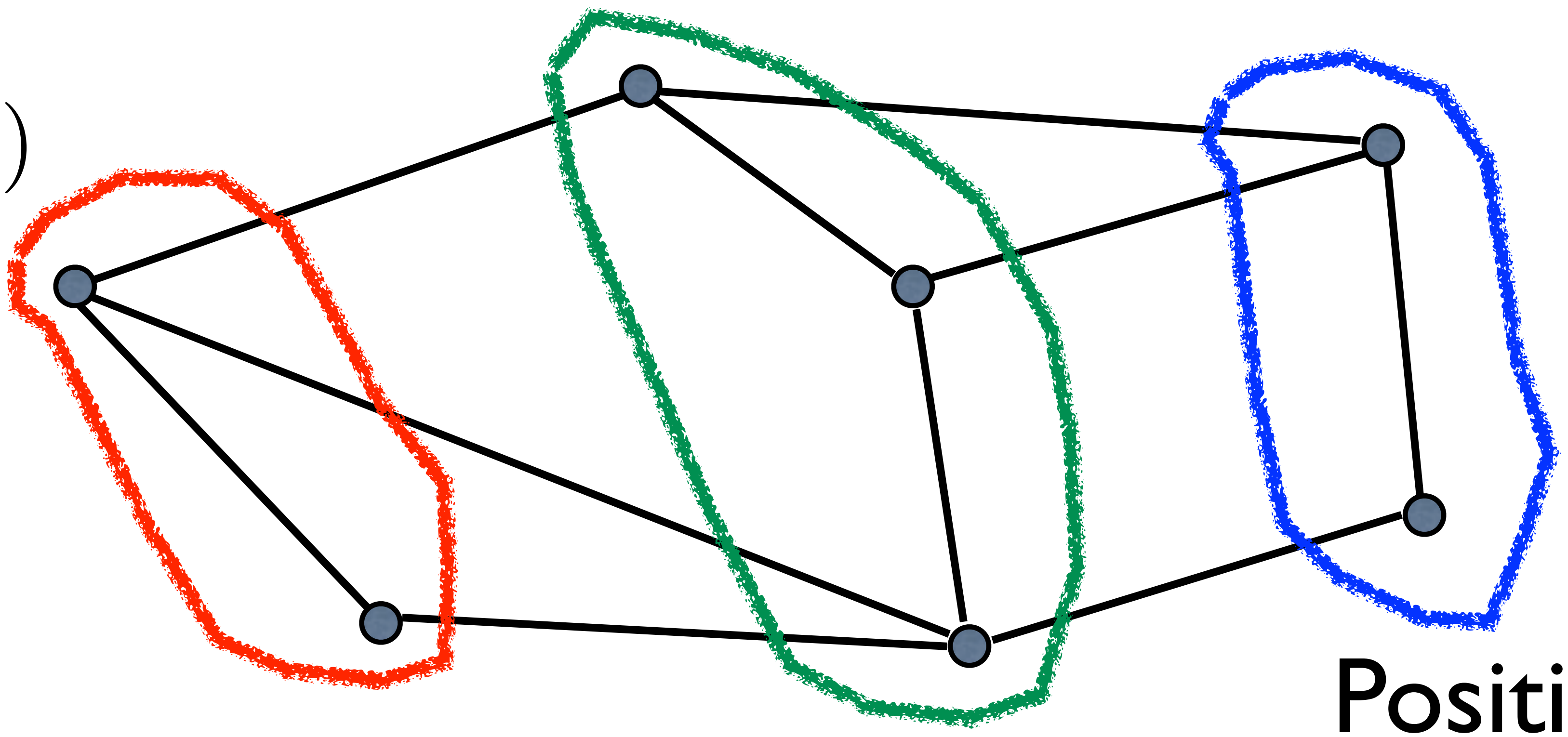

Positive operators

$$\rho_{ABCD\ldots} = \prod_{X\in V} \nu_X \prod_{(X,Y)\in E} \mu_{XY}$$

Quantum graphical models?

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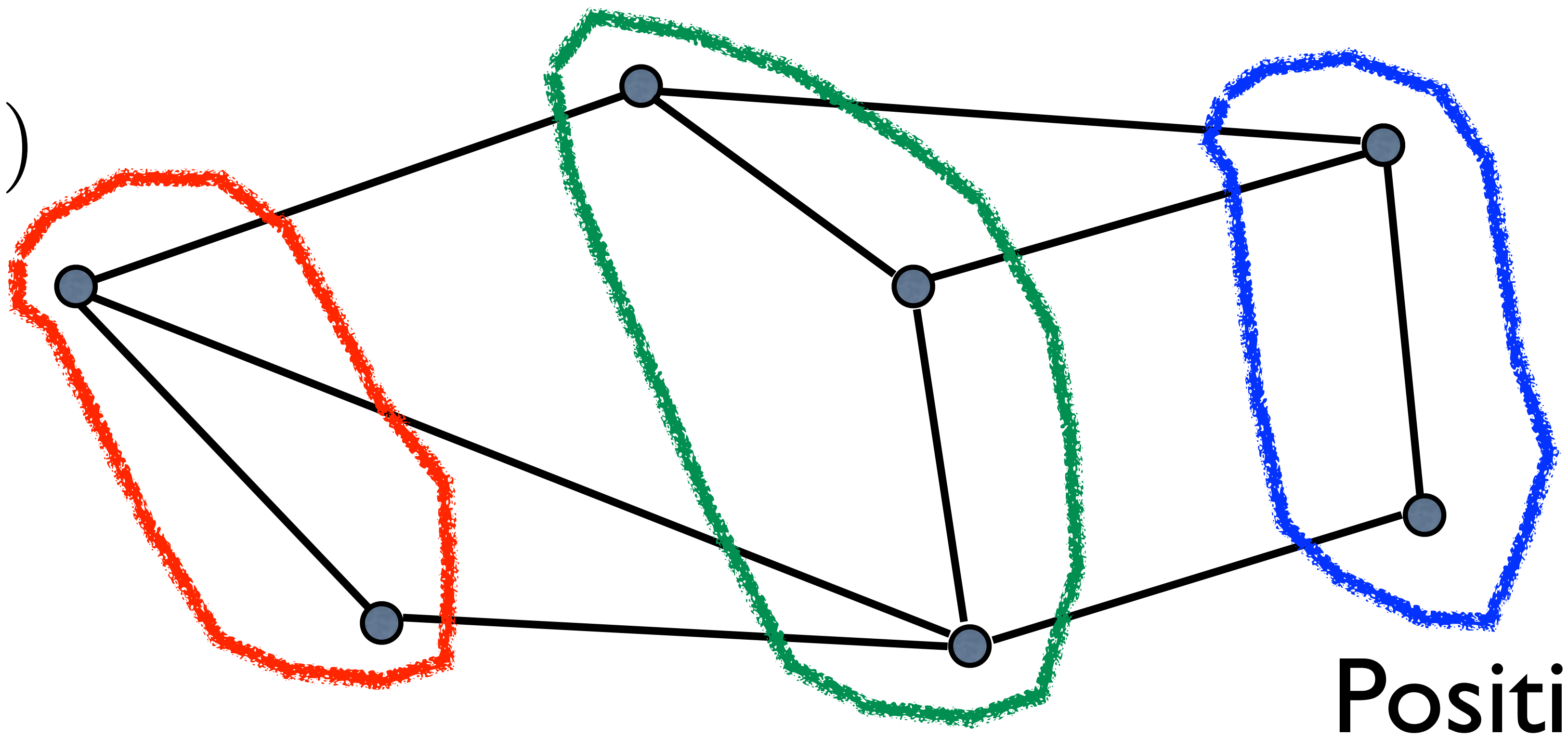

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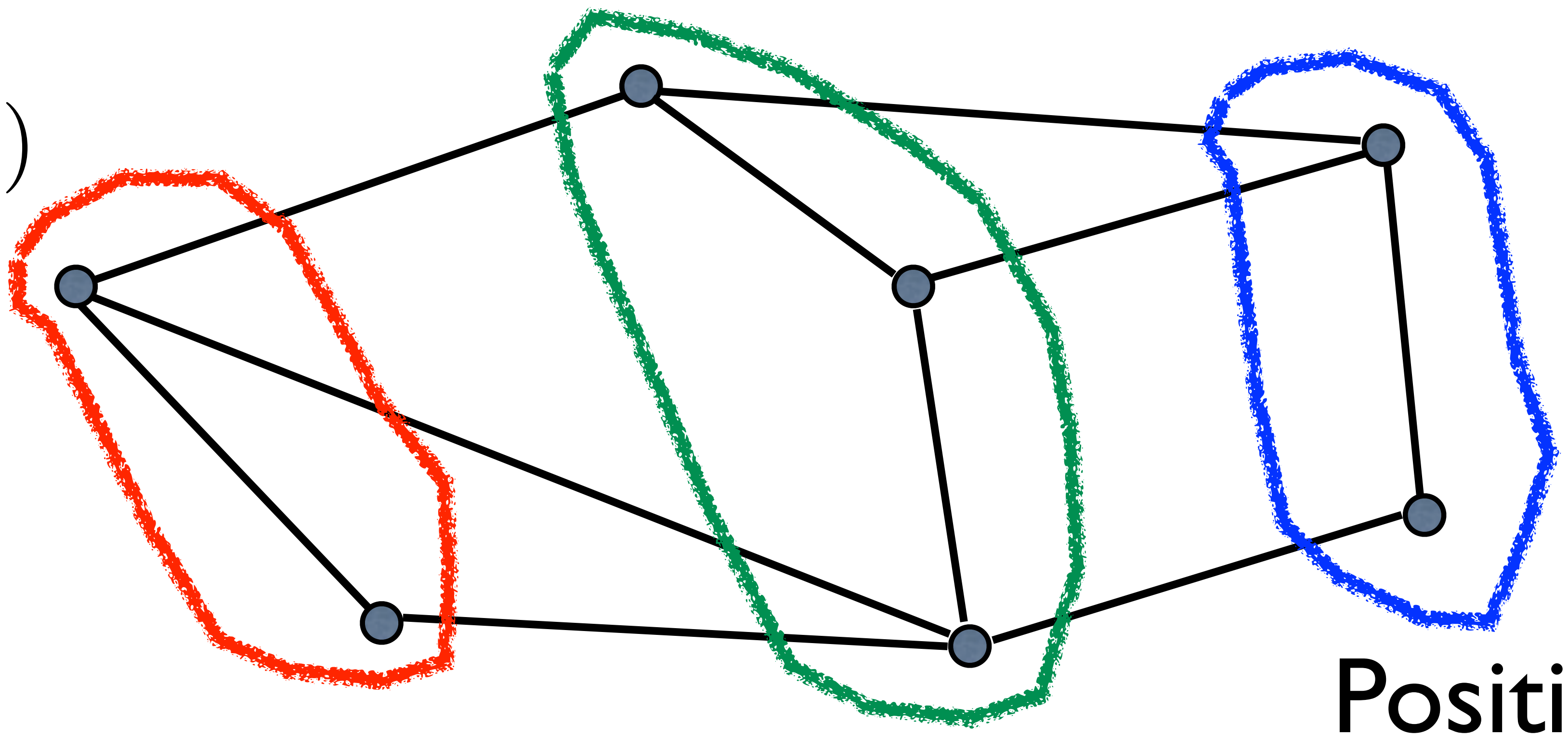

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- Product of positive operators not necessarily positive

Quantum graphical models

Many distinct ways to fixe these problems

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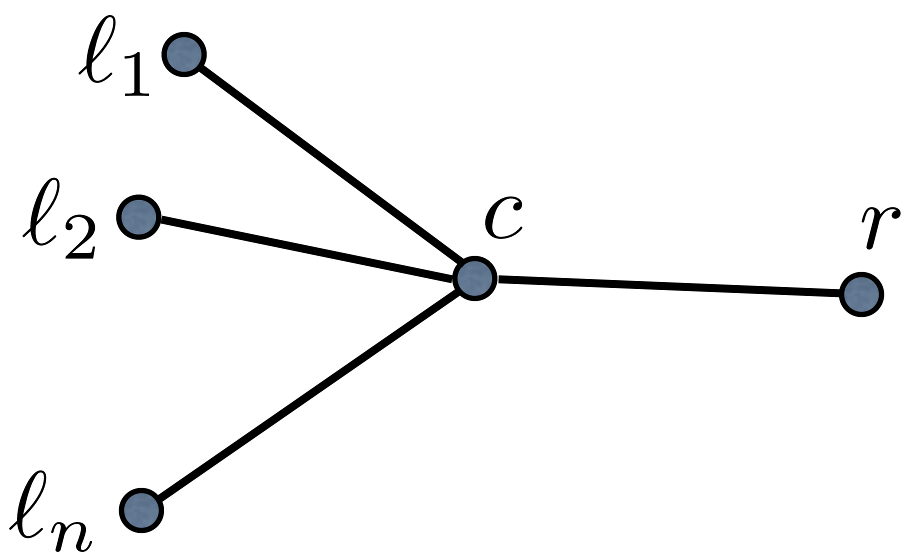
<u>★ product</u>

$$A \star^n B = (A^{\frac{1}{2n}} B^{\frac{1}{n}} B^{\frac{1}{2n}})^n$$

all equal when AB = BA

Quantum belief propagation



$$m_{c\to r}(x_r) = \frac{1}{Z}\sum_{x_c} \nu(x_c, x_r)\left(\prod_i m_{\ell_i\to c}(x_c)\right)\mu(x_c)$$

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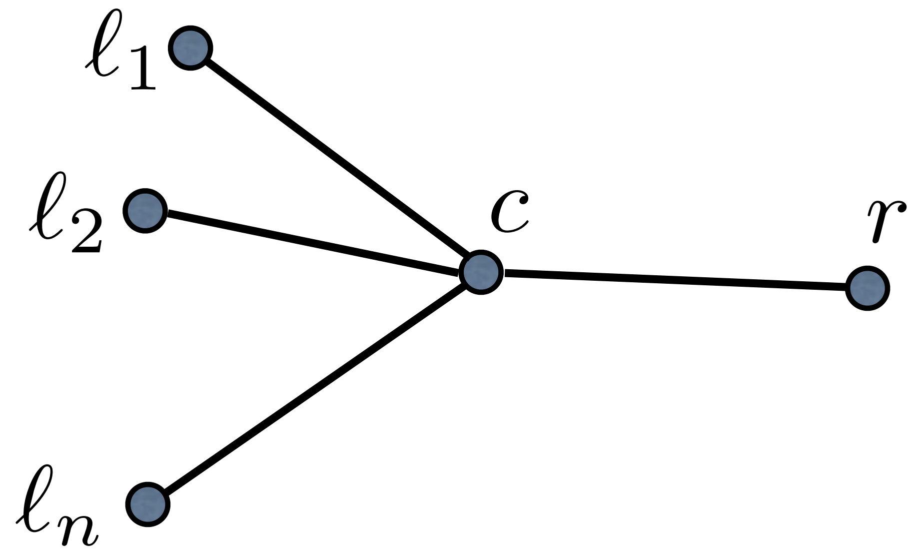

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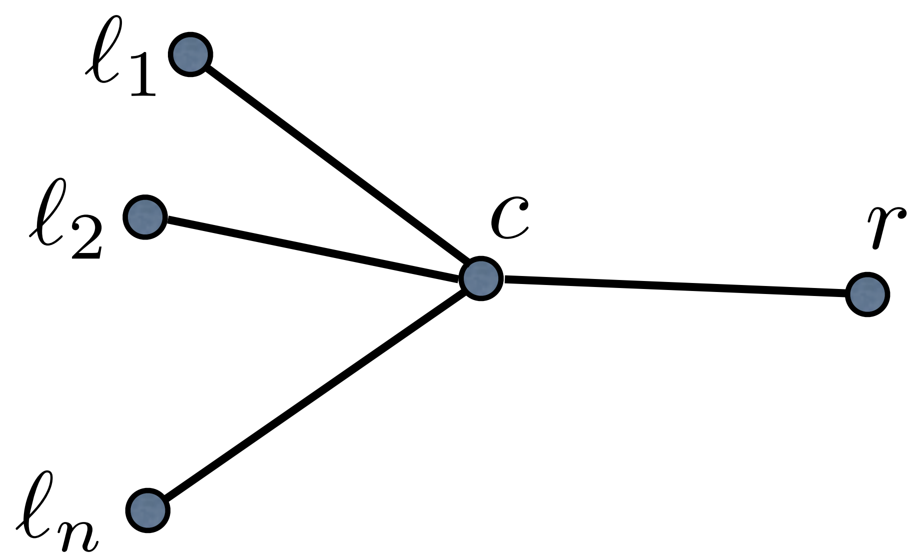

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<u>Quantum Belief Propagation</u>

- Replace sums by traces
- Replace product by ★ product

Quantum error correction

✢Assuming iid quantum channel

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Code state $p(Q_i) = \mathrm{Tr}(Q_i \rho) = 1$

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Given error syndrome $\frac{1}{Z} Q_1 \star \bar{Q}_2 \star \ldots Q_n \star \rho'$ is a quantum graphical model✢

✢Assuming iid quantum channel

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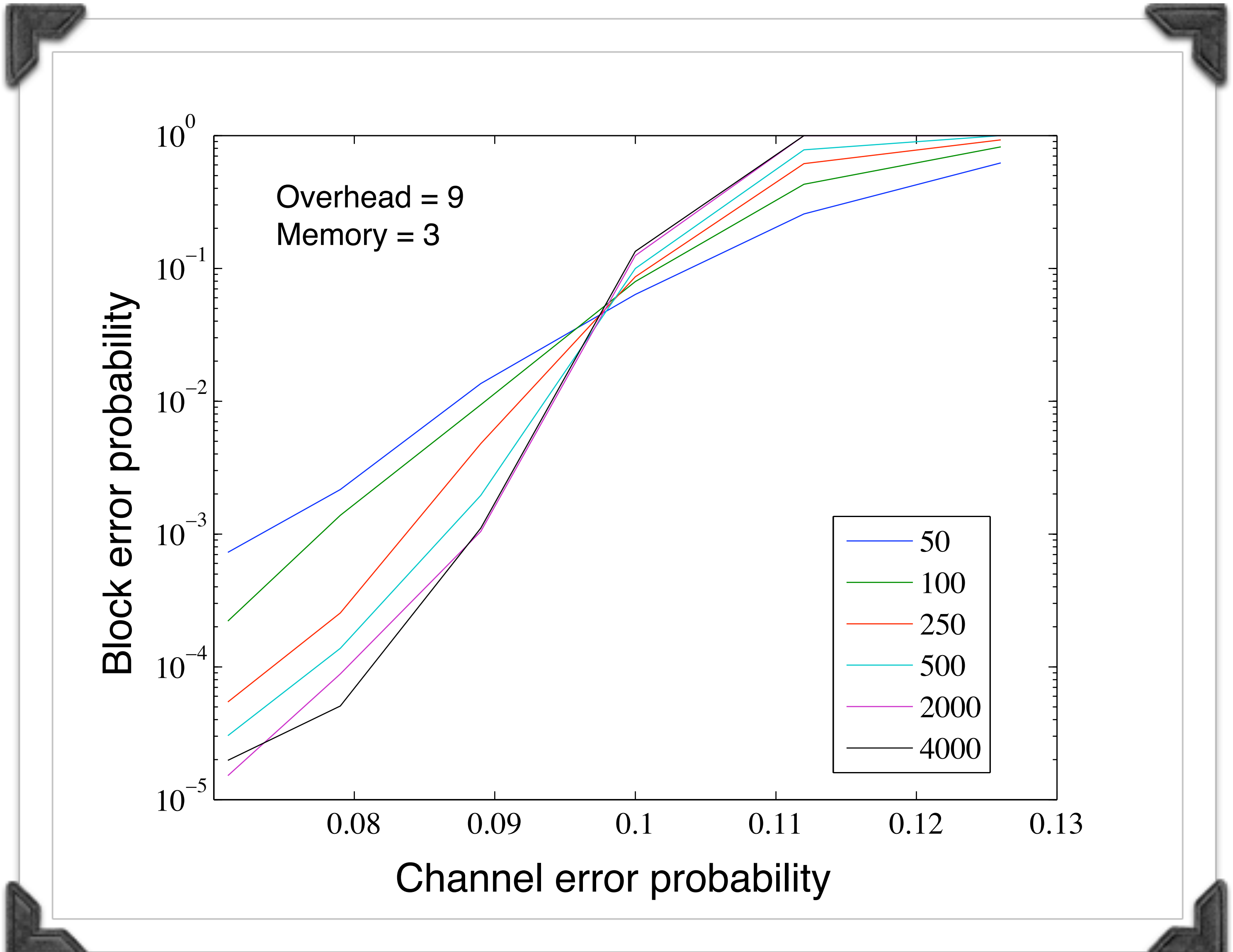

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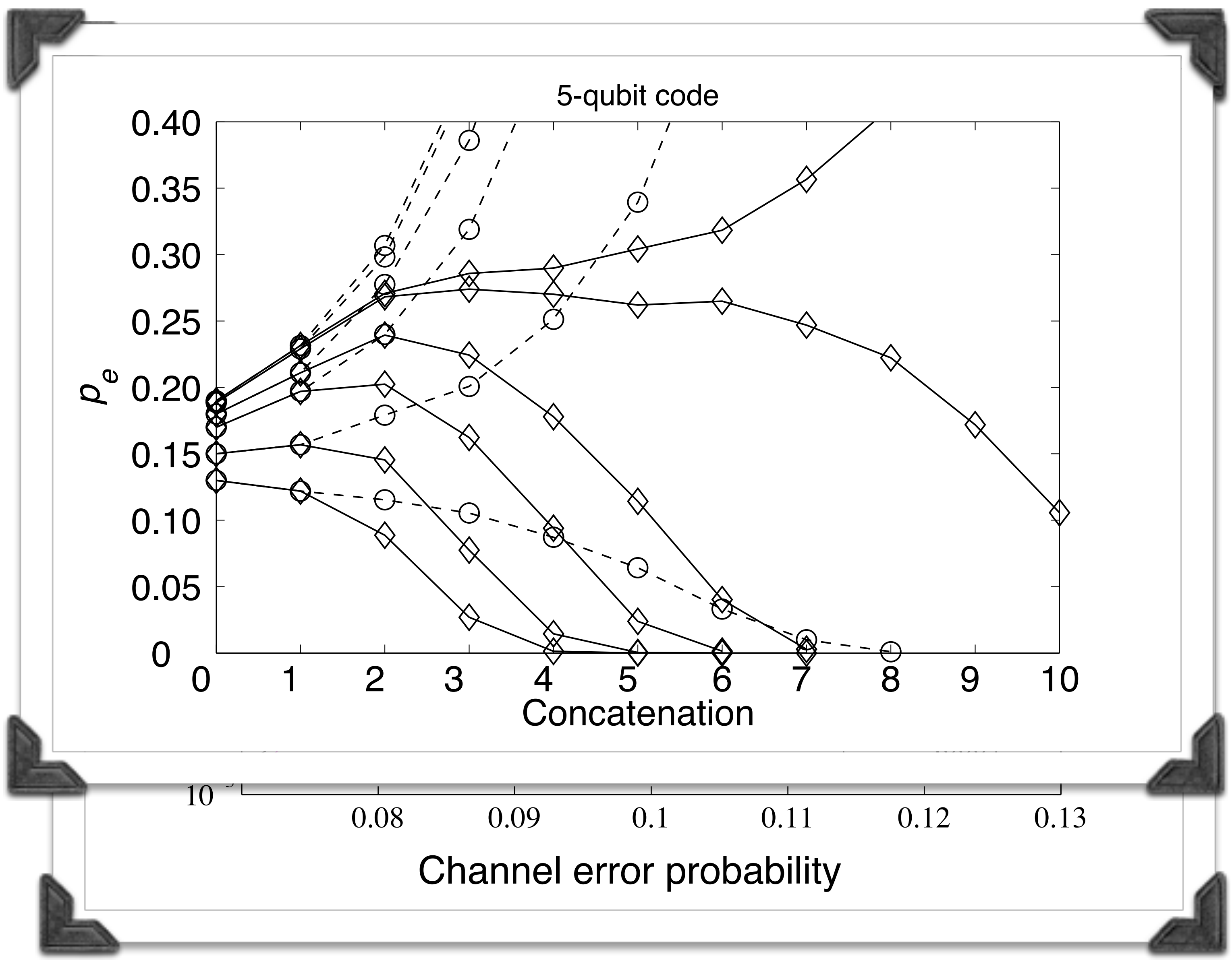

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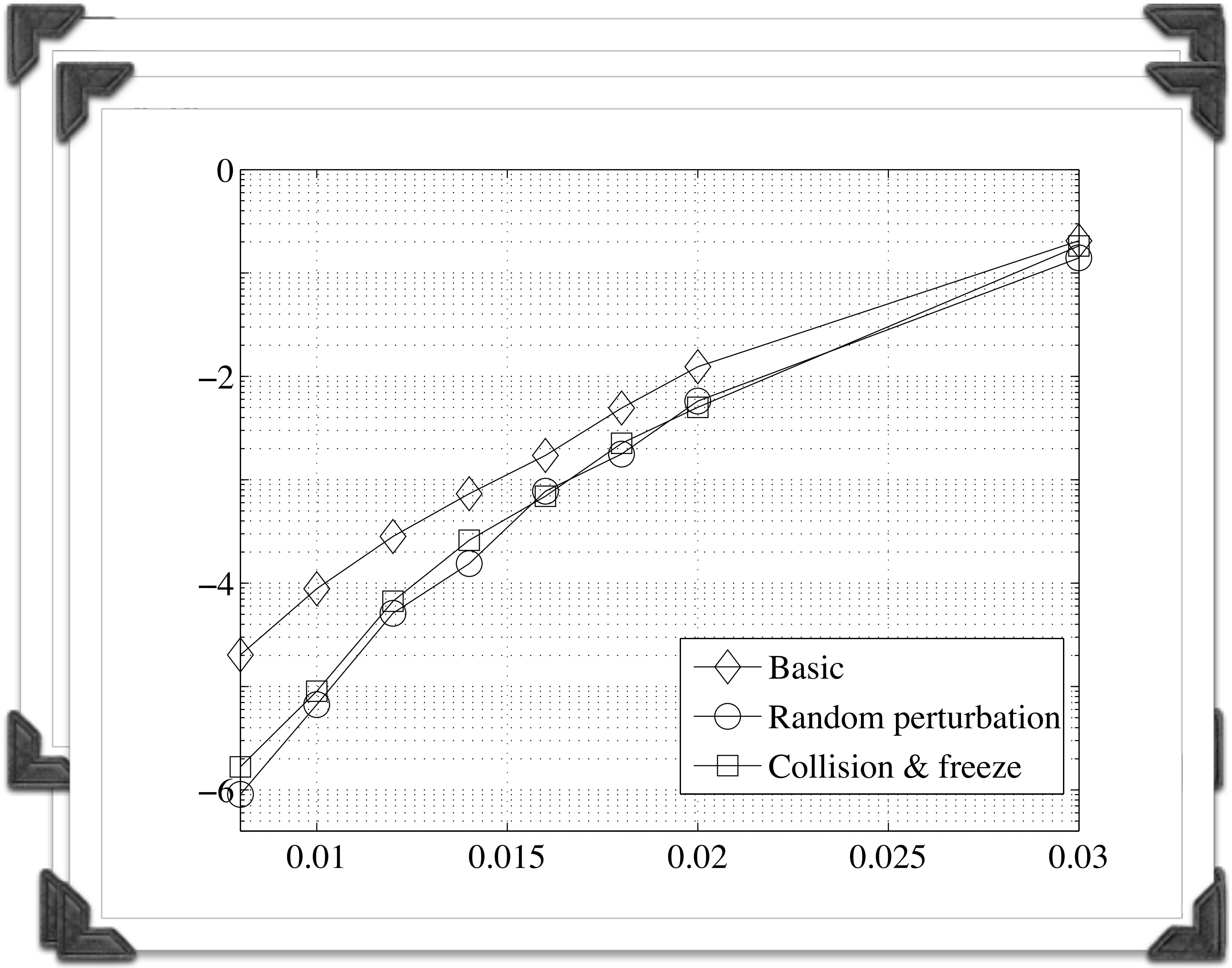
Q LDPC codes



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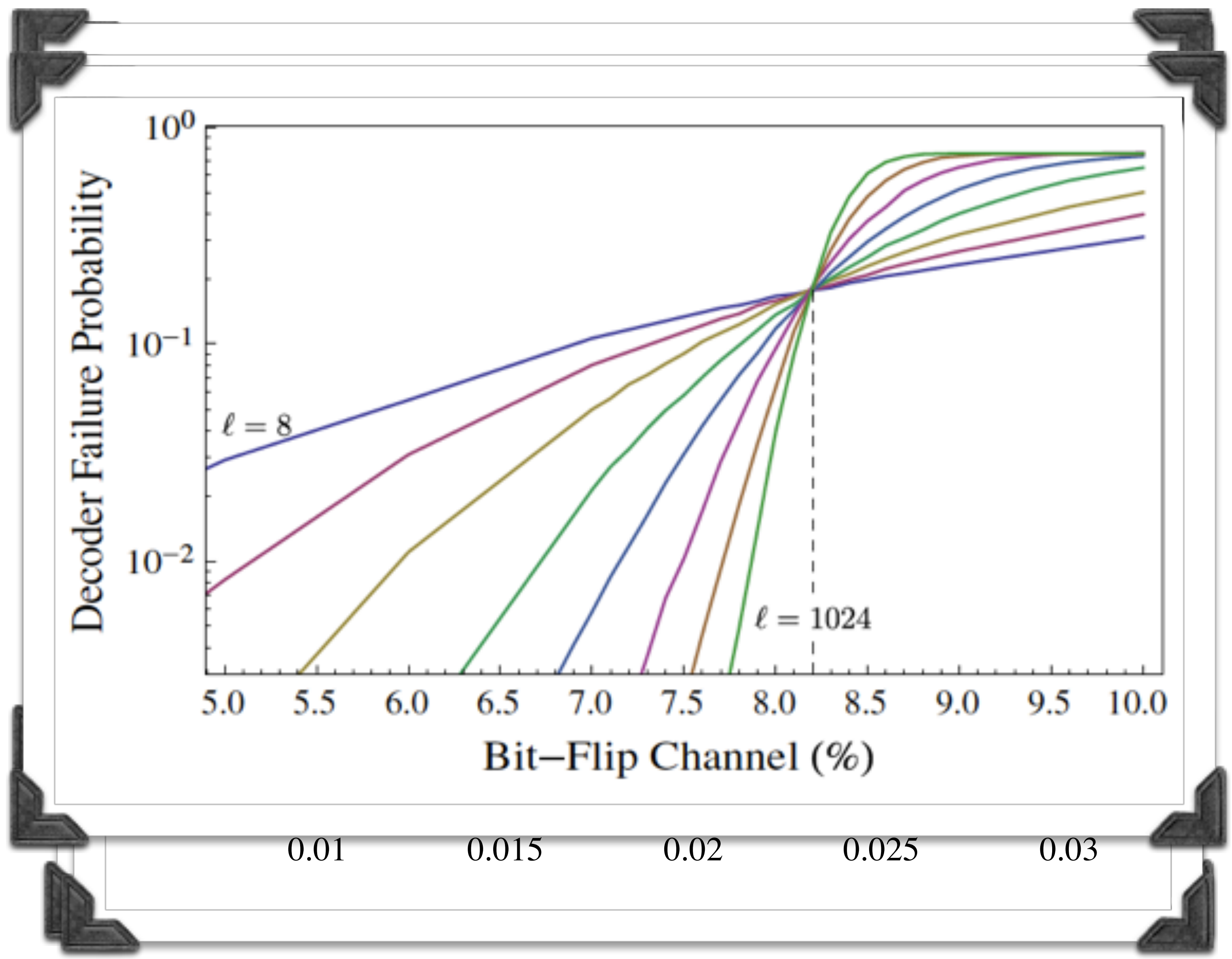
Q LDPC codes

Q topological codes



Does it always work?

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YES, when underlying graph is a tree

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(Works well on graphs with only large loops too...)

Does it always work?

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This is only true for the product $\bigstar^n$ with $n=1$

(e.g. quantum error correction)

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The general case requires a Markov condition

Stat mech

$$H(x_1, x_2, ...) = h(x_1, x_2) + h(x_2, x_3) + ...$$

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$$\begin{aligned} Z &= \sum_{x_1, x_2, \ldots} e^{-H(x_1, x_2, \ldots)} \\ &= \sum_{x_2, x_3, \ldots} \Big(\sum_{x_1} e^{-h(x_1, x_2)} \Big) e^{-h(x_2, x_3) - h(x_3, x_4) - \ldots} \end{aligned}$$

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Distributive law

$$= \sum_{x_2, x_3, \ldots} \left(\sum_{x_1} e^{-h(x_1, x_2)} \right) e^{-h(x_2, x_3) - h(x_3, x_4) - \ldots}$$

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...

$$= \sum_{x_N} m_{N-1\to N}(x_N) e^{-h(x_{N-1}, x_N)}$$

Quantum stat mech

$$H_{ABC...} = h_{AB} + h_{BC} + ...$$

Quantum stat mech

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Two-body hermitian operator

$$h_{BC} = I_A \otimes h_{BC} \otimes I_D \otimes ...$$

Quantum stat mech

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Distributive law only holds with commuting terms

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Distributive law only holds with commuting terms
$h_{AB}h_{BC} = h_{BC}h_{AB}$

More generally, it holds on Markov networks
I(A:CD...|B) = 0

Cavity field

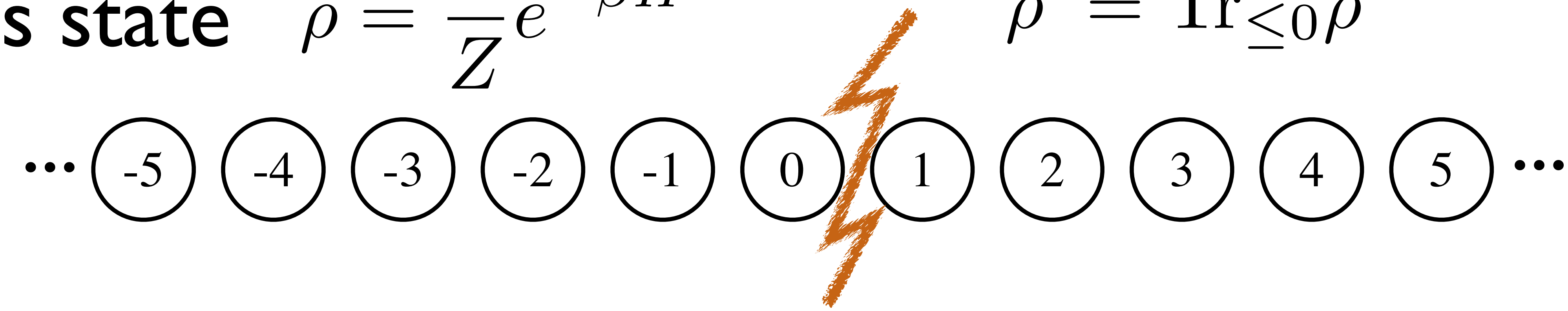
Gibbs state $\rho = \frac{1}{Z} e^{-\beta H}$

$\cdots$ -5 -4 -3 -2 -1 0 1 2 3 4 5 $\cdots$

Cavity field

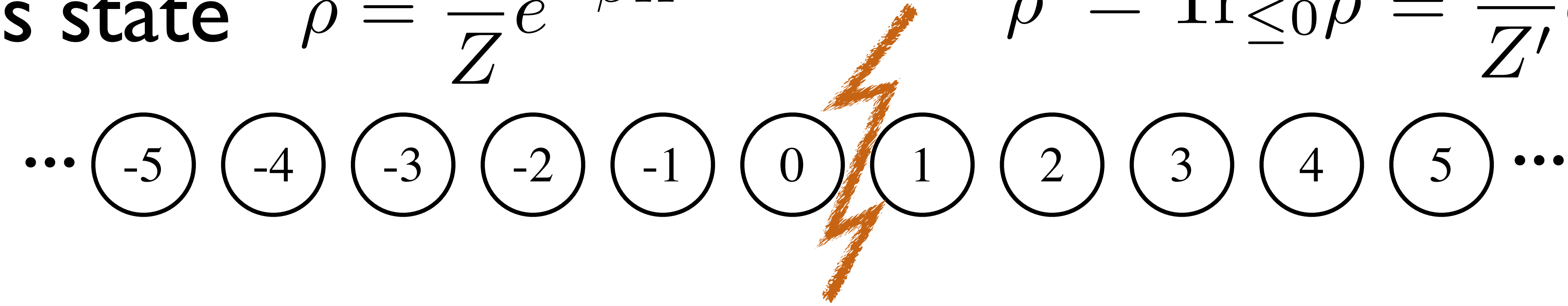
Gibbs state $\rho = \frac{1}{Z} e^{-\beta H}$

$\rho' = \mathrm{Tr}_{\leq 0} \rho$



Cavity field

Gibbs state $\rho = \frac{1}{Z} e^{-\beta H}$ $\rho' = \mathrm{Tr}_{\leq 0} \rho = \frac{1}{Z'} e^{-\beta H'}$



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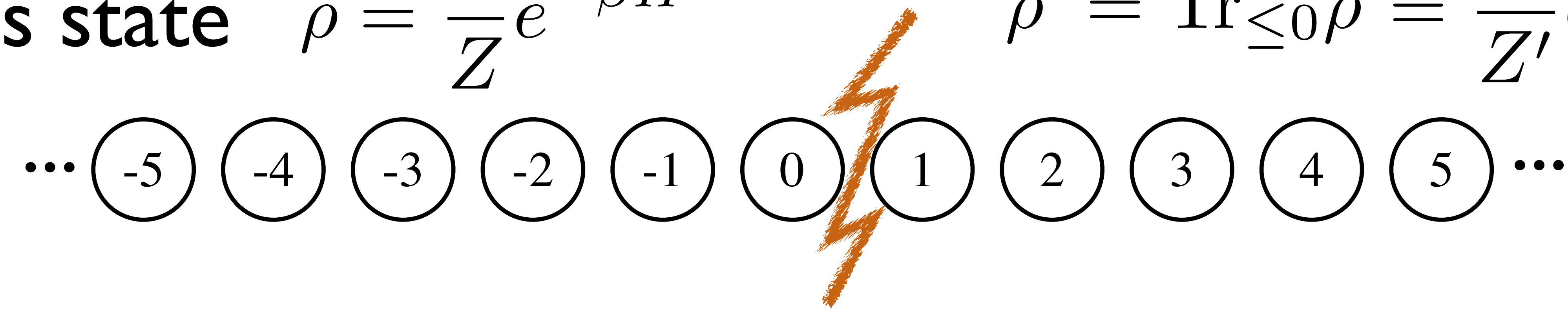

What is H'?

Cavity field

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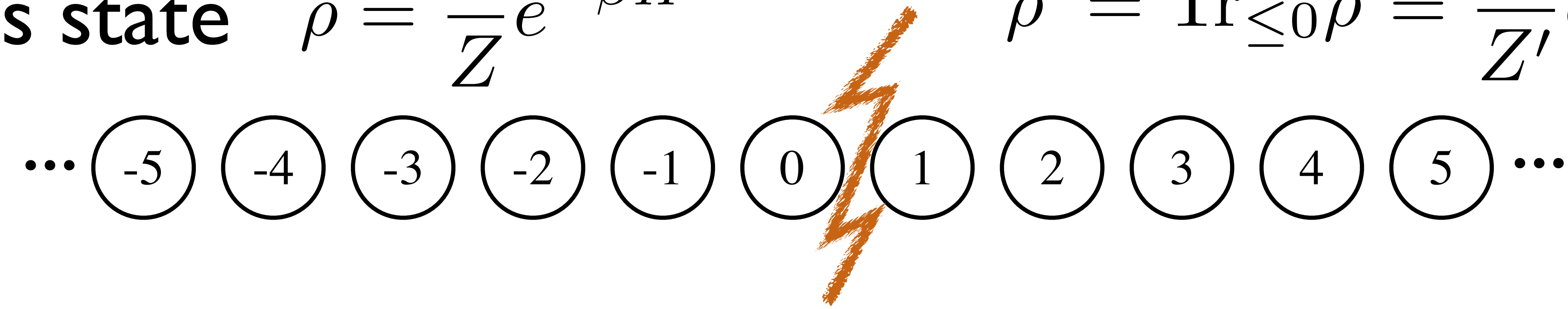

What is H'?

$$H' = \sum_{i>0} h_{i,i+1}$$

Cavity field

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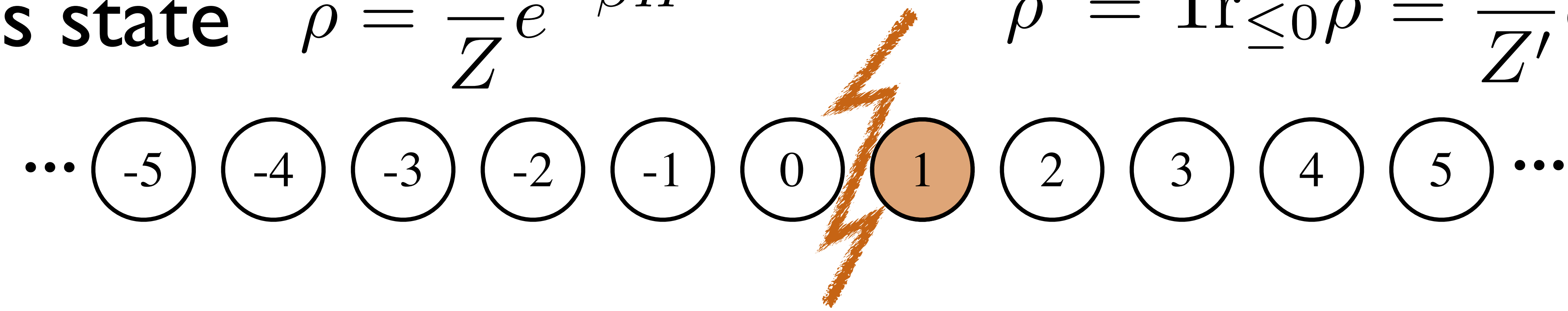

What is H'? $H' = \sum_{i>0} h_{i,i+1} + V_1$

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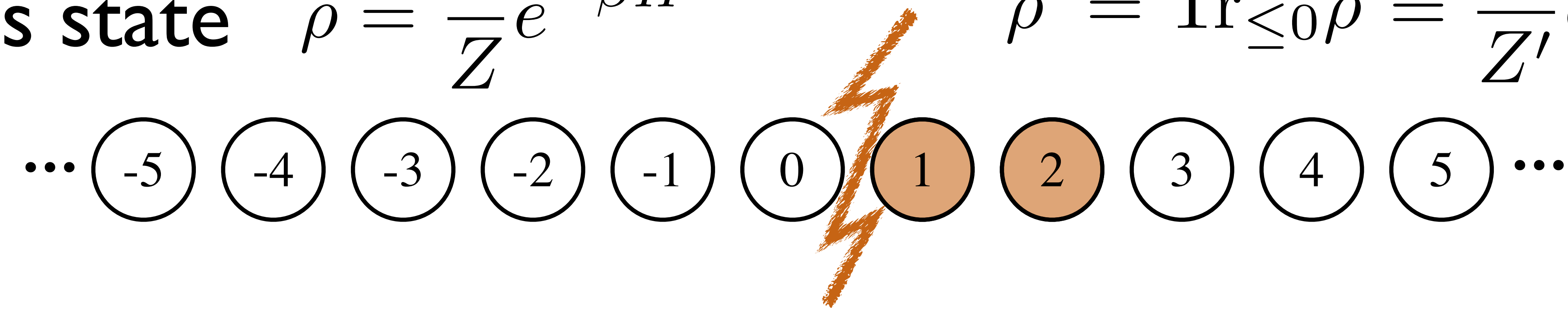
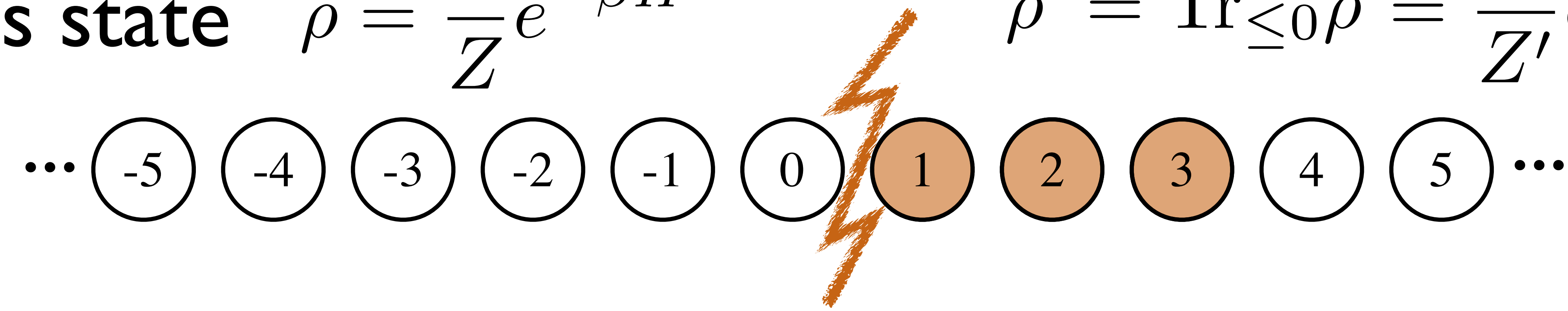


What is H'?

$$H' = \sum_{i>0} h_{i,i+1} + V_1 + V_{12}$$

Cavity field

Gibbs state $\rho = \frac{1}{Z} e^{-\beta H}$ $\rho' = \mathrm{Tr}_{\leq 0}\rho = \frac{1}{Z'} e^{-\beta H'}$



What is H'?

$$H' = \sum_{i>0} h_{i,i+1} + V_1 + V_{12} + V_{123}$$

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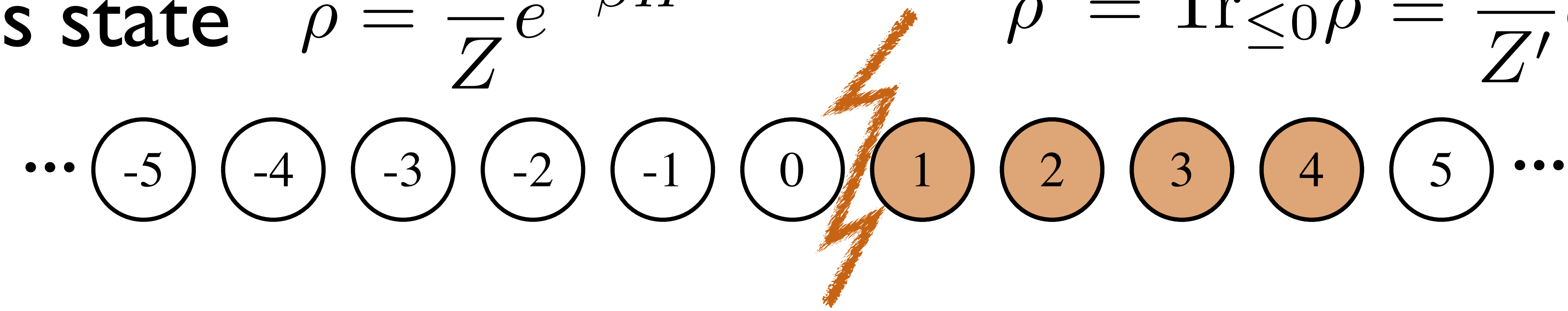
Gibbs state $\rho = \frac{1}{Z} e^{-\beta H}$ $\qquad \rho' = \mathrm{Tr}_{\leq 0}\rho = \frac{1}{Z'} e^{-\beta H'}$



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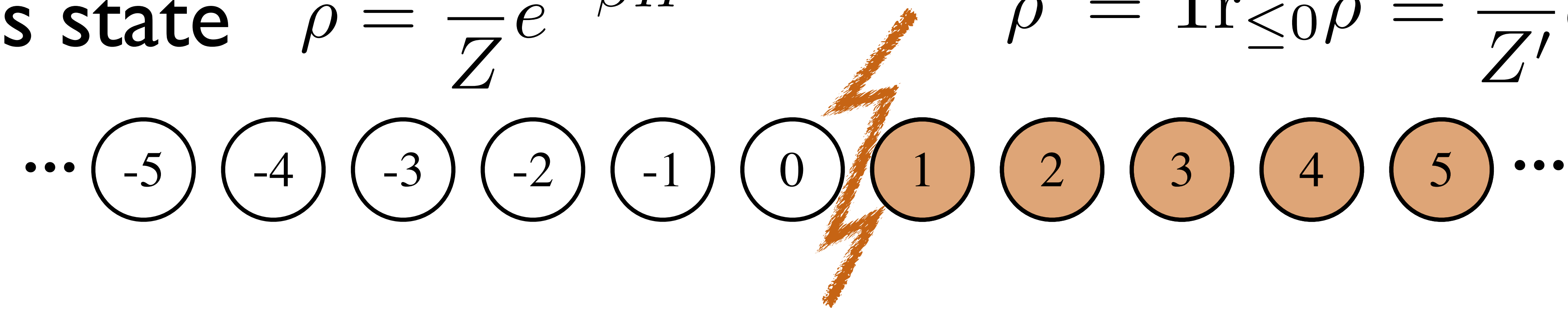

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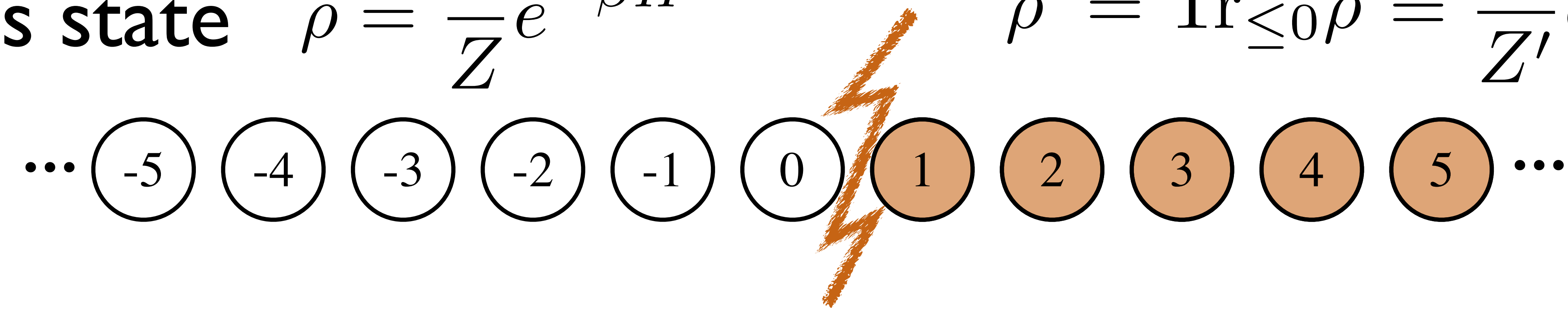

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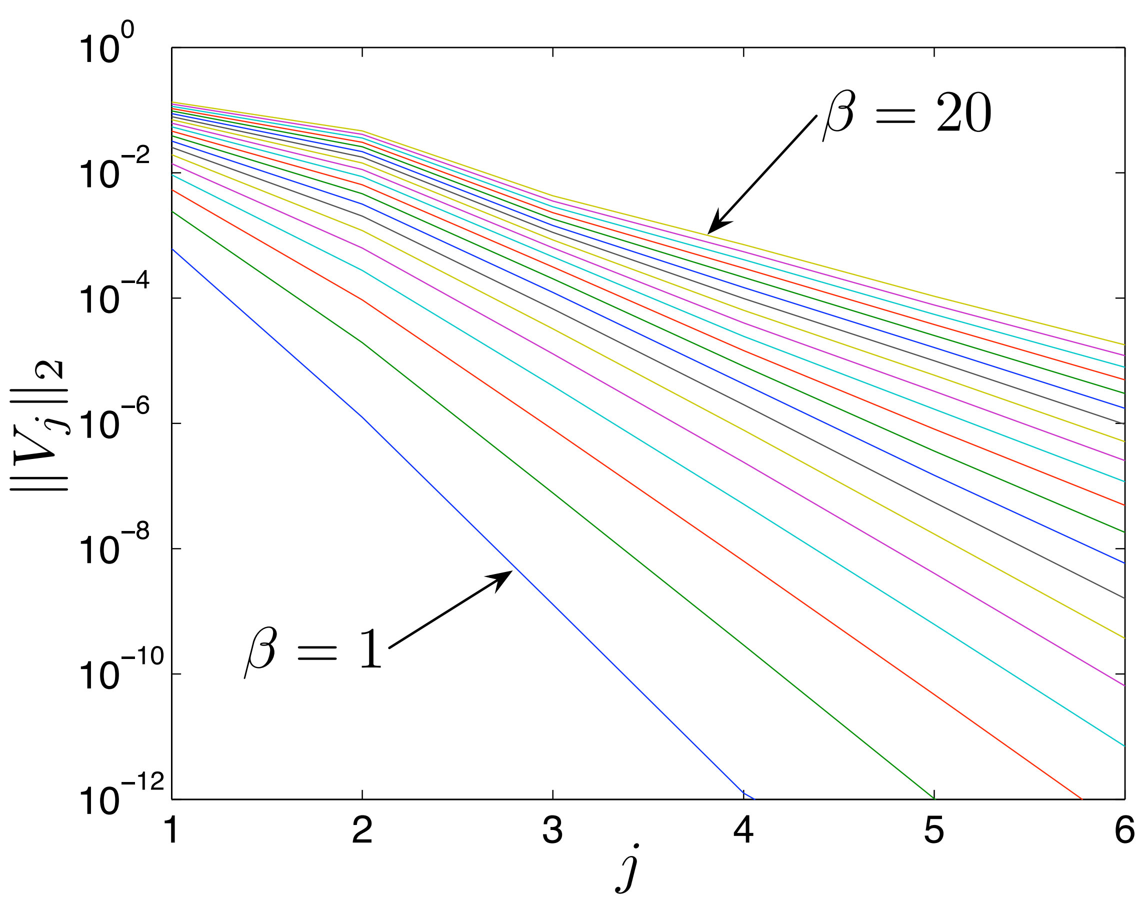

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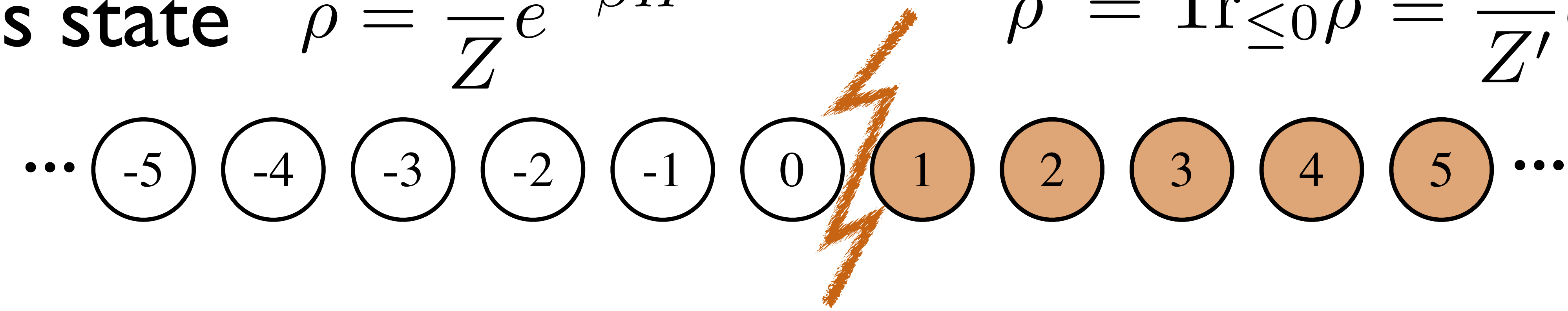

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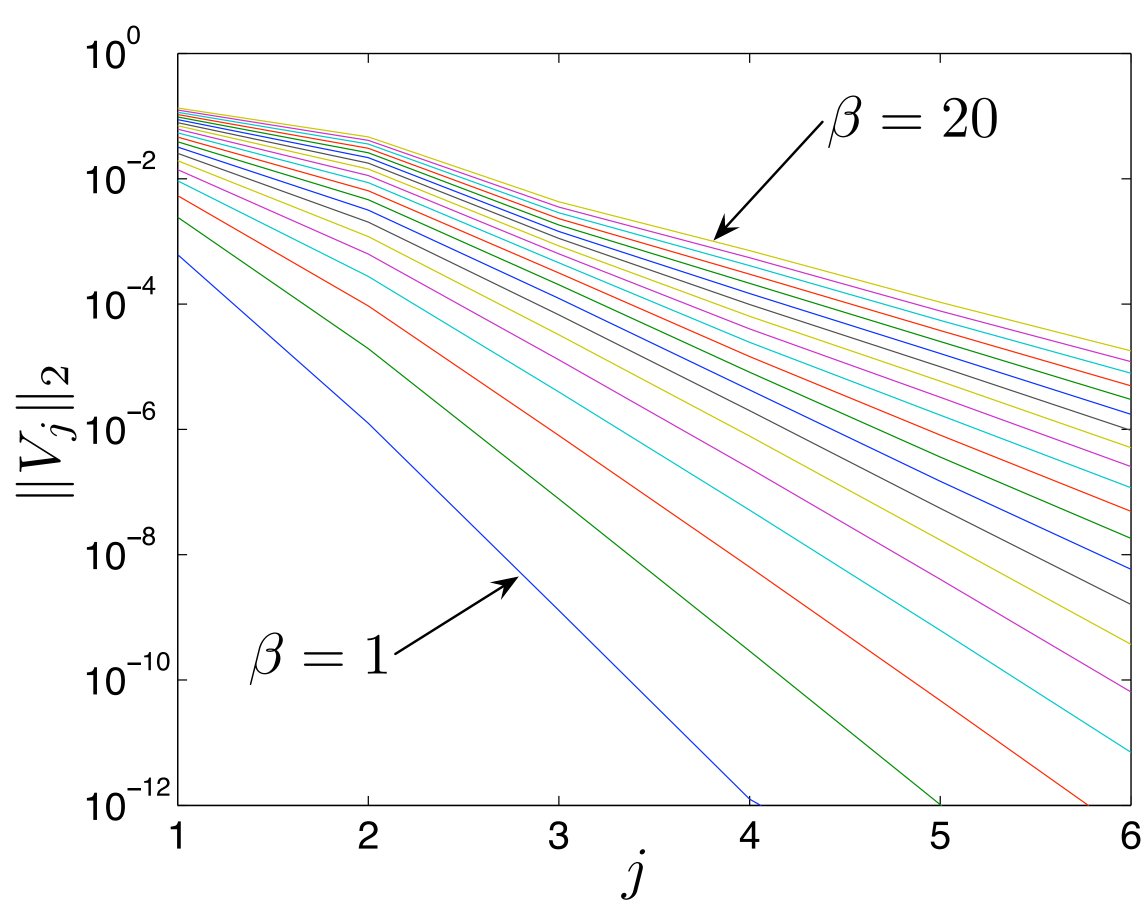

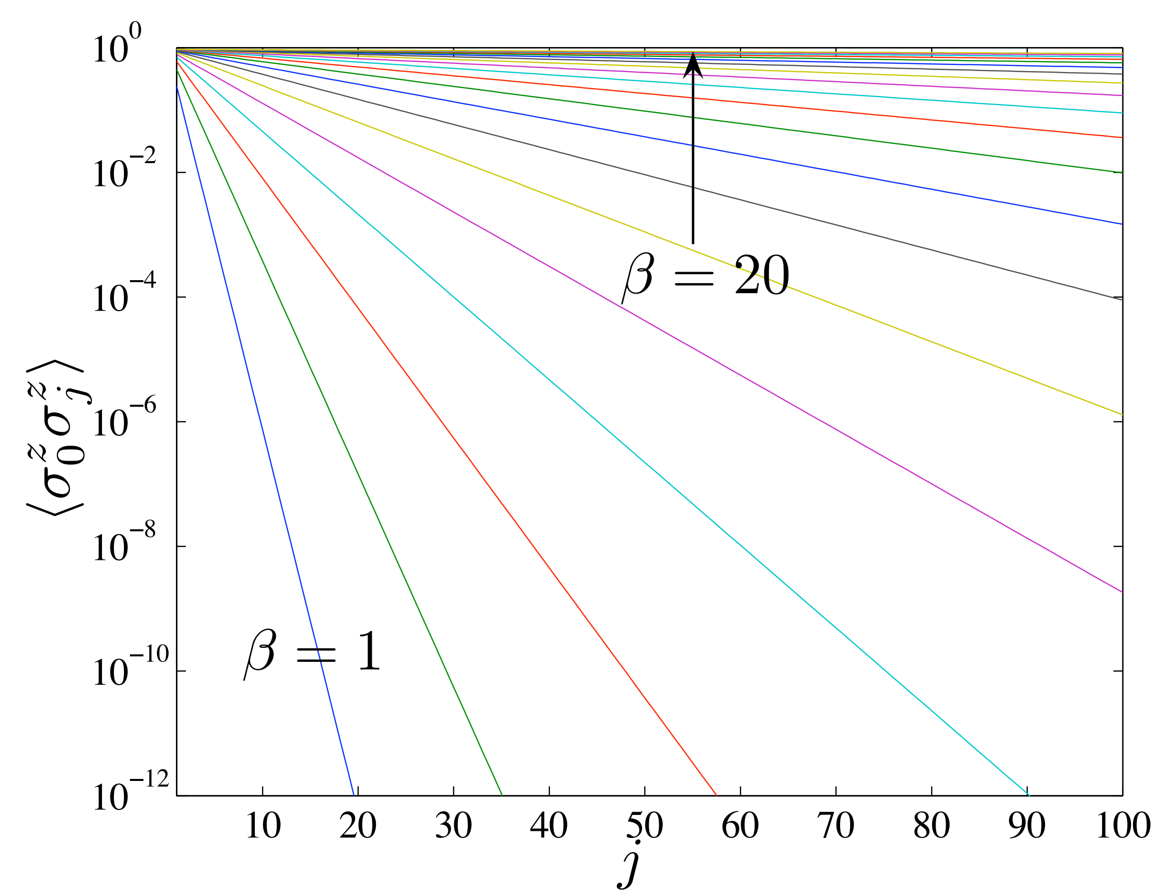

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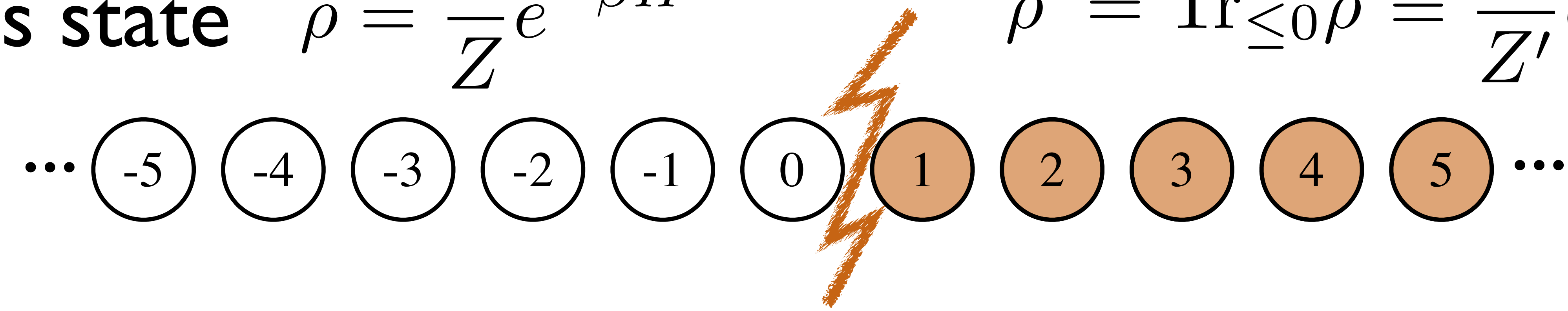

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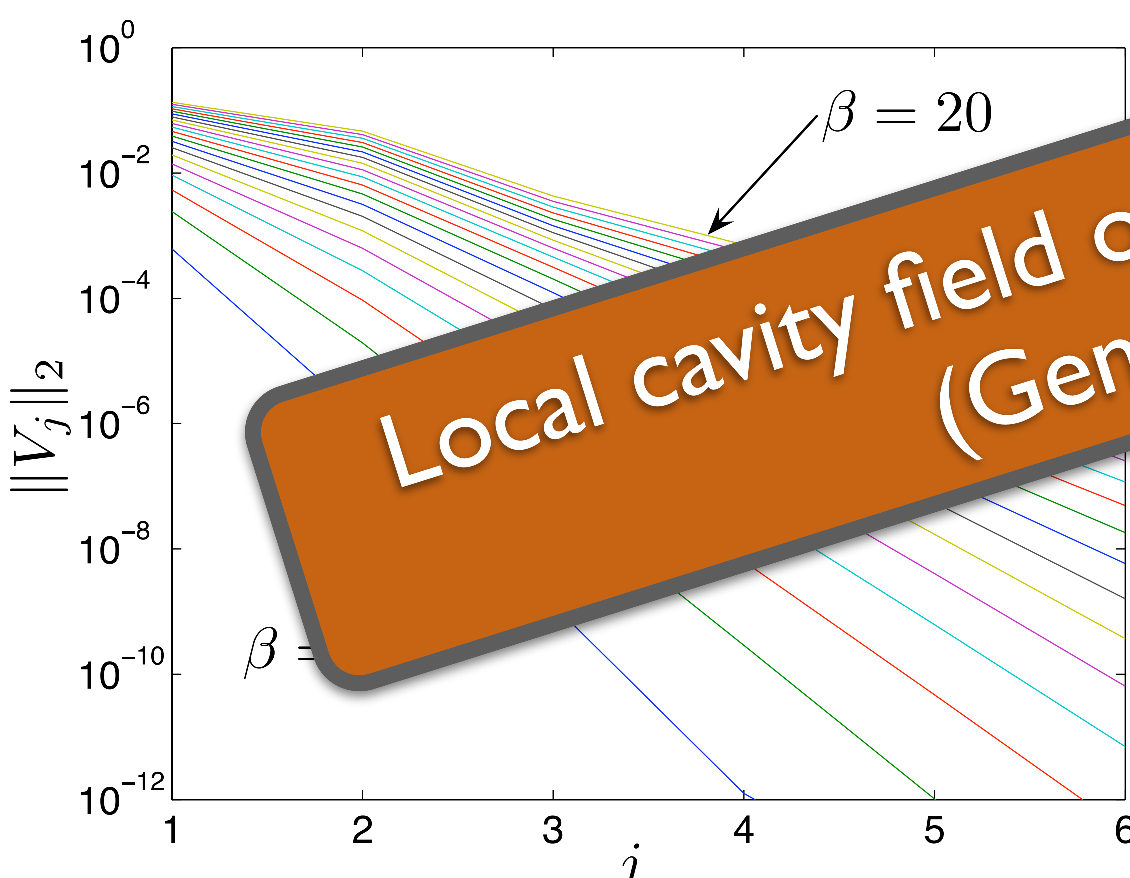
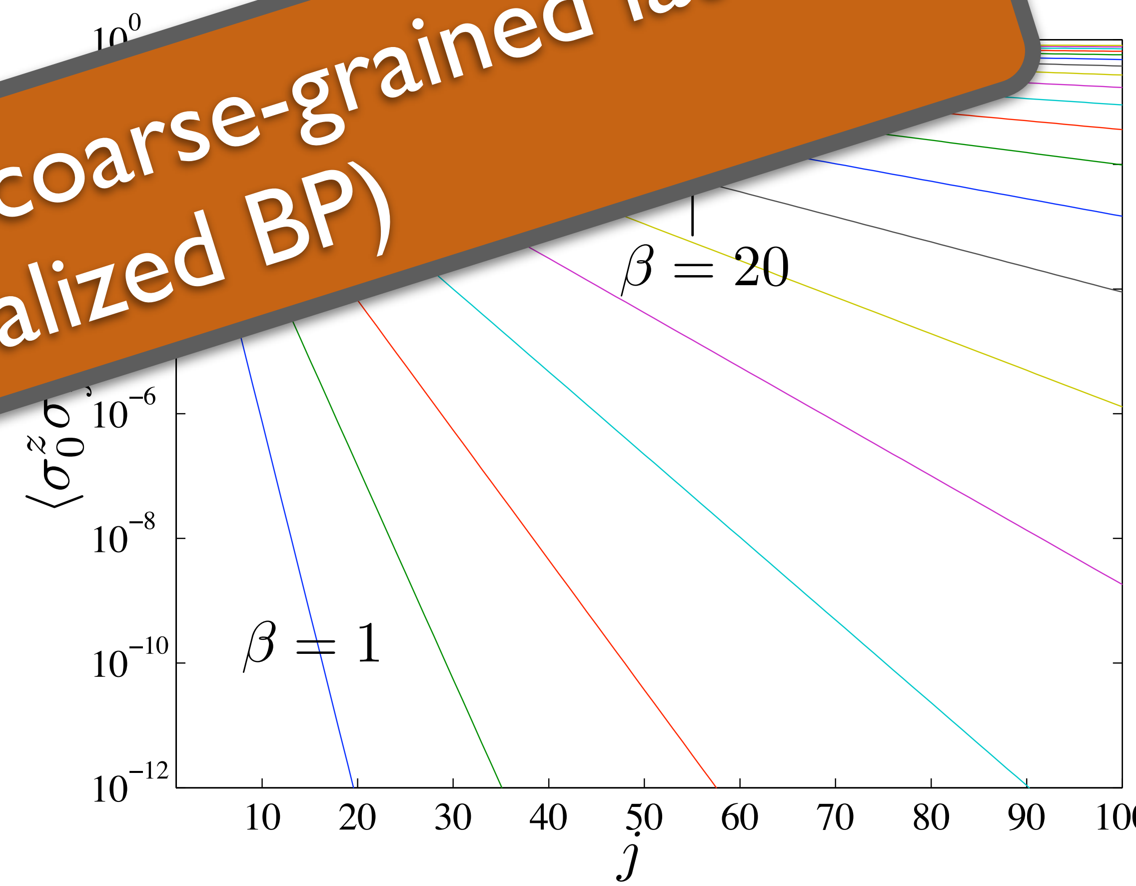


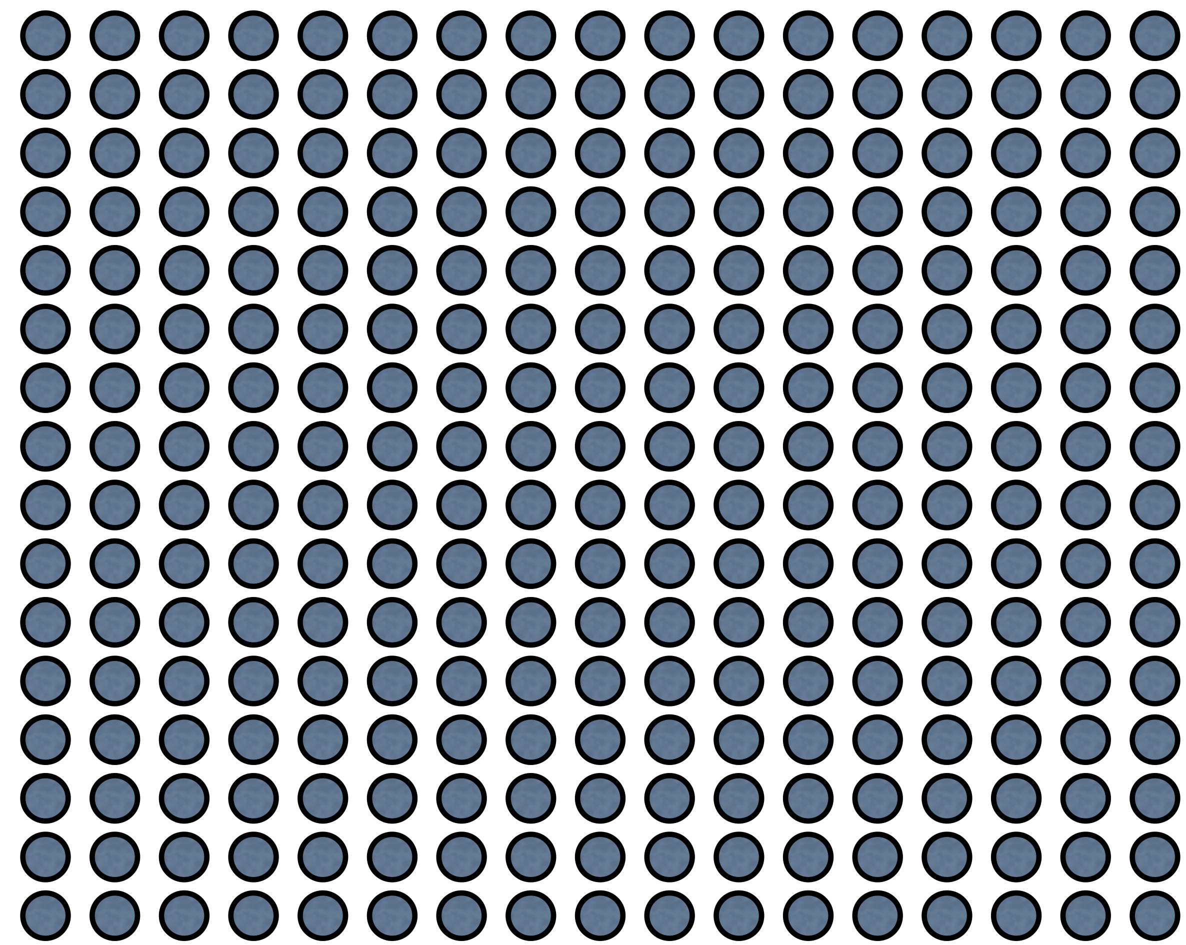


Local cavity field on coarse-grained lattice (Generalized BP)

Markov network?

Gibbs state $\rho = \frac{1}{Z} e^{-\beta H}$

$H =$ Nearest neighbor



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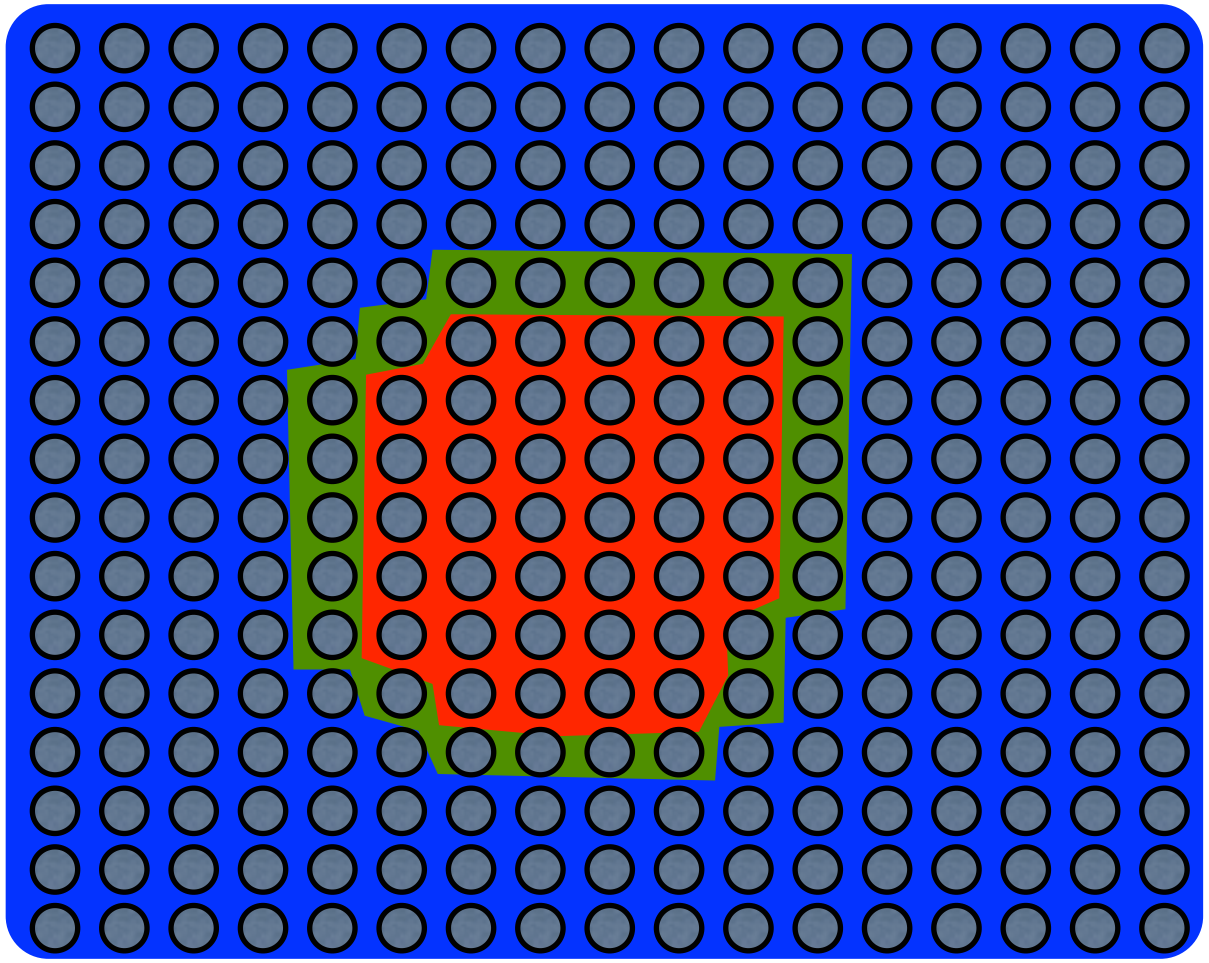
I(A:B|C)≠0



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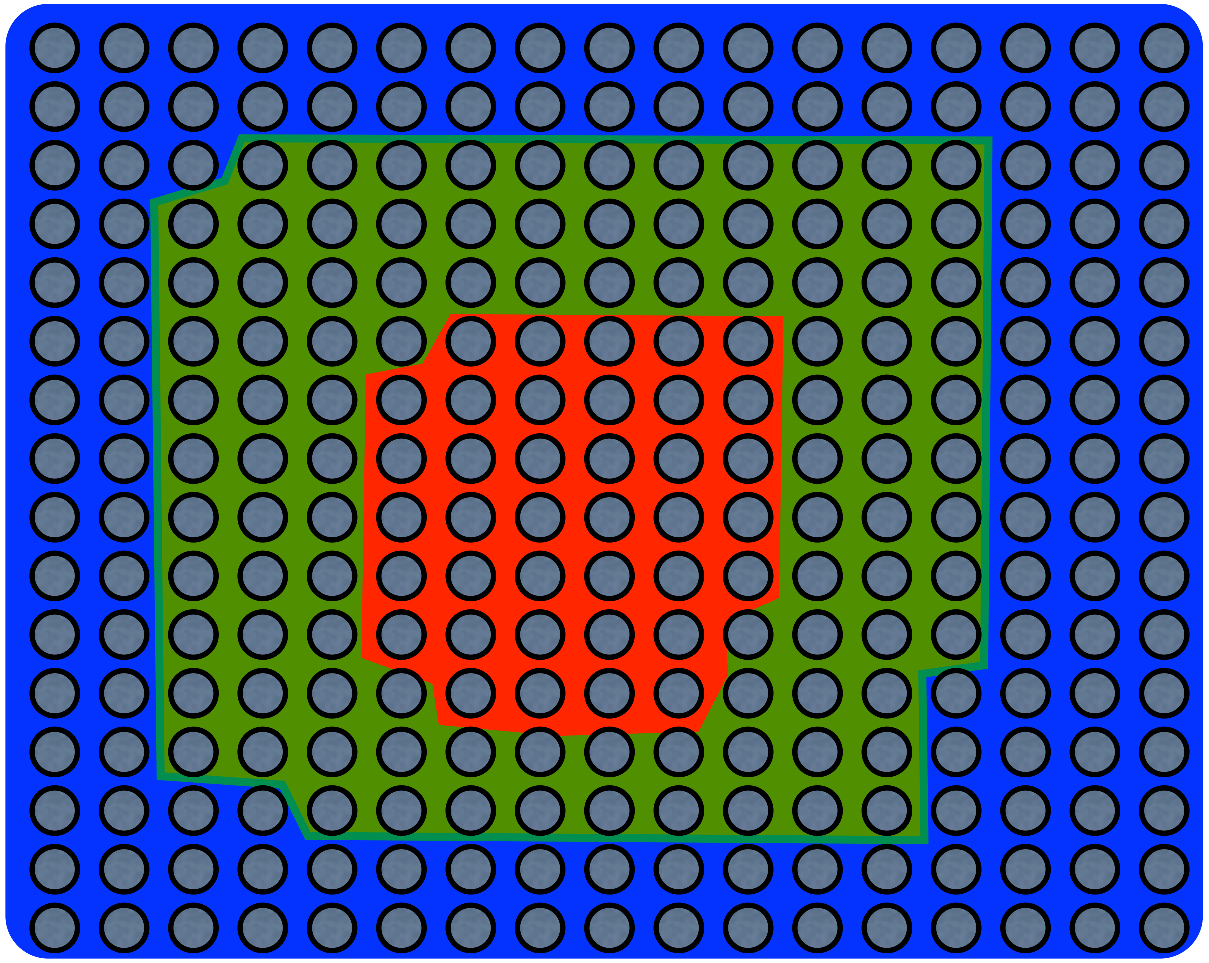

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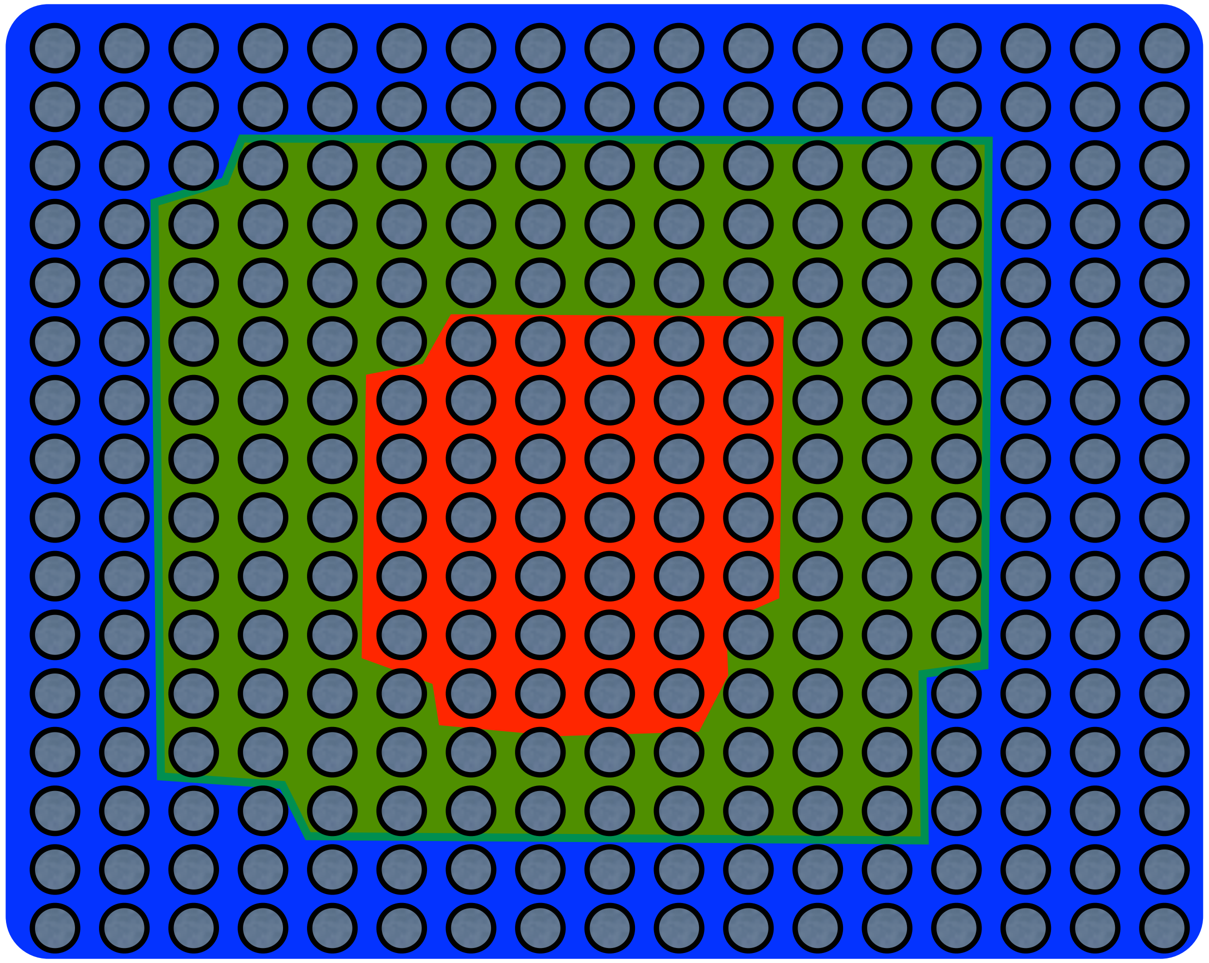
Entanglement area law



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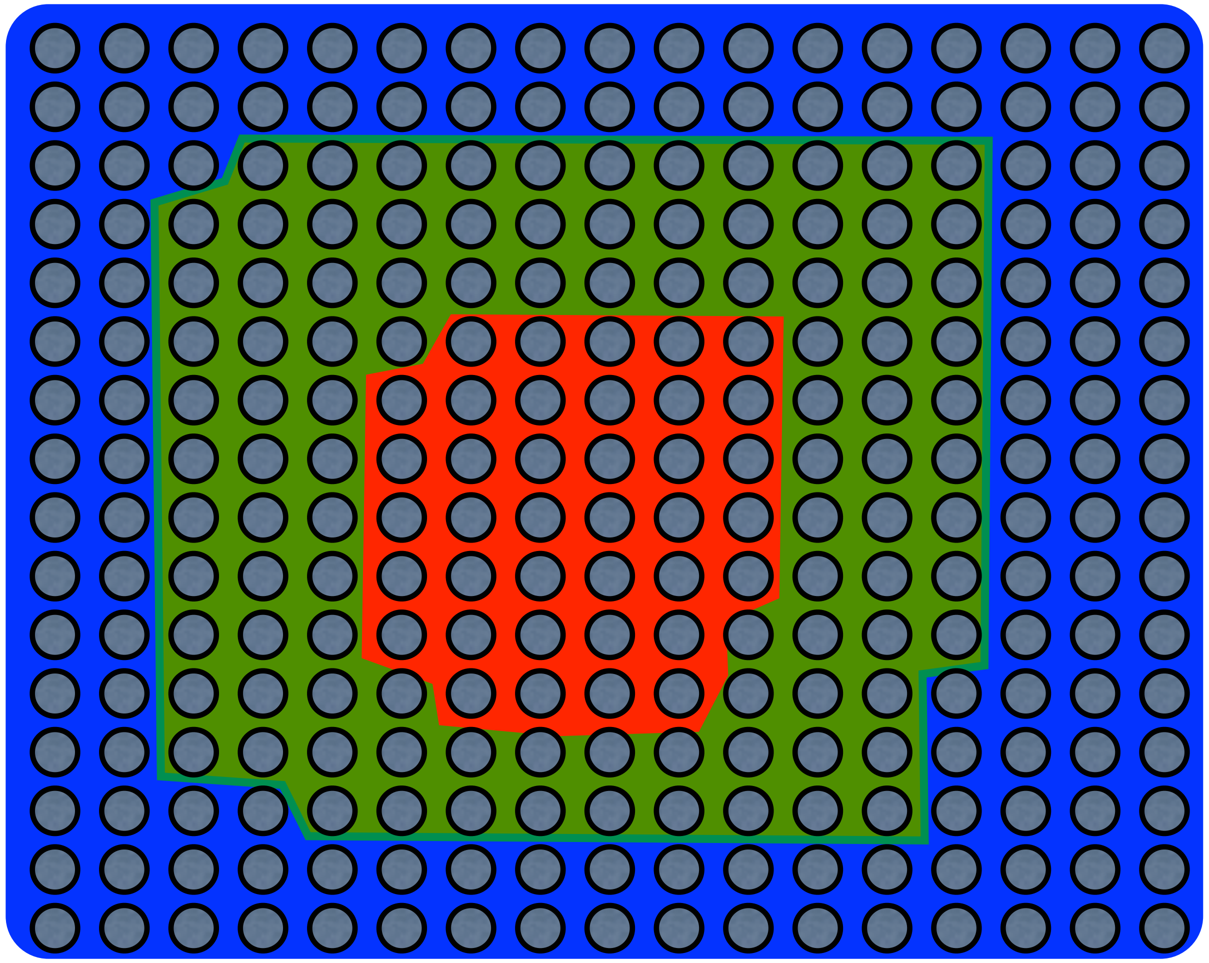

I(A:B|C')=0 ⇒ Entropy = Sum of local terms

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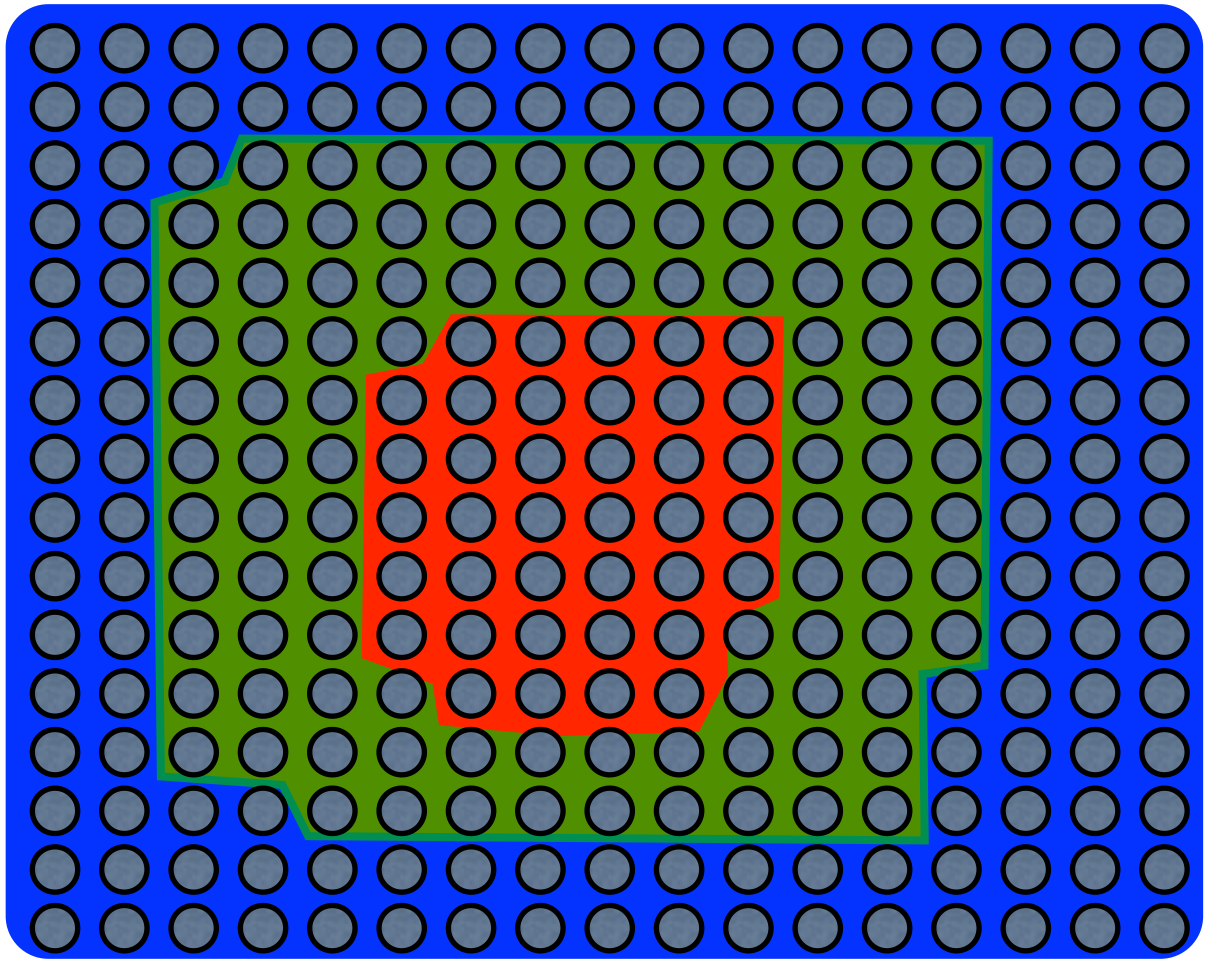

I(A:B|C')=0 ⇒ Entropy = Sum of local terms $S = \sum_i S(i|\mathcal{N}_i)$

Markov network?

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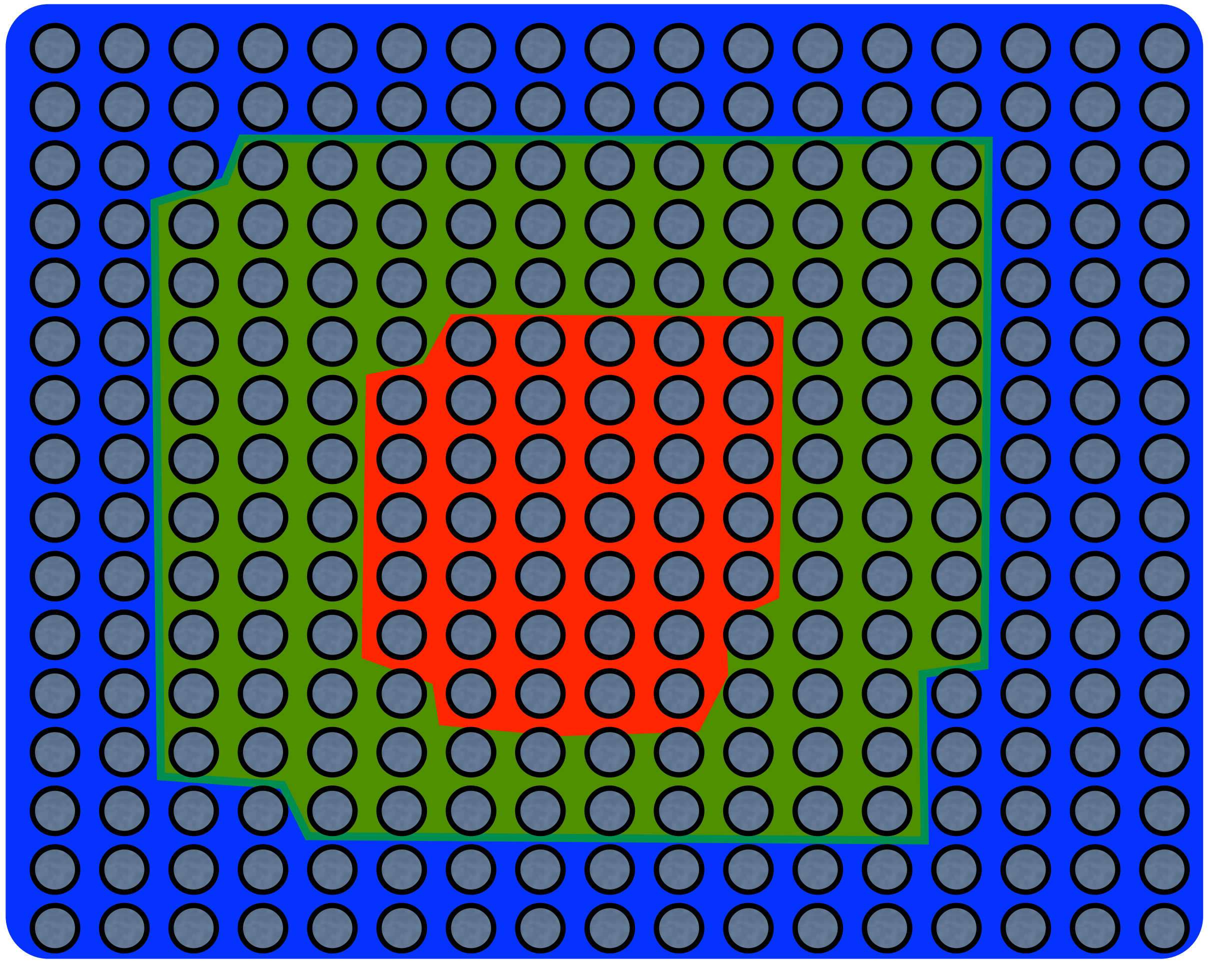

Entanglement area law

I(A:B|C')=0 ⇒ Entropy = Sum of local terms $S = \sum_i S(i|\mathcal{N}_i)$

Variational dual to BP $\min_{\{\rho_{\mathcal{N}_i}\}\text{consistent}} \sum_i \left(\mathrm{Tr}(\rho_{\mathcal{N}_i} h_{\mathcal{N}_i}) - TS(i|\mathcal{N}_i)\right)$

Hammersley-Clifford

When is a Gibbs state associated to a local Hamiltonian a Markov network?

Hammersley-Clifford

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Classically: ALWAYS

Hammersley-Clifford

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(p, G) is a Markov Network

Hammersley-Clifford

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Classically: ALWAYS

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$$\Updownarrow$$

$$p(x_1, x_2, \ldots) = \frac{1}{Z} e^{-H(x_1, x_2, \ldots)}$$

$$H = \sum_{C \in \text{cliques}} h_C(\{x_i \in C\})$$

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(ρ, G) is a Markov Network

$$\Downarrow$$

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Hammersley-Clifford

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(ρ, G) is a Markov Network

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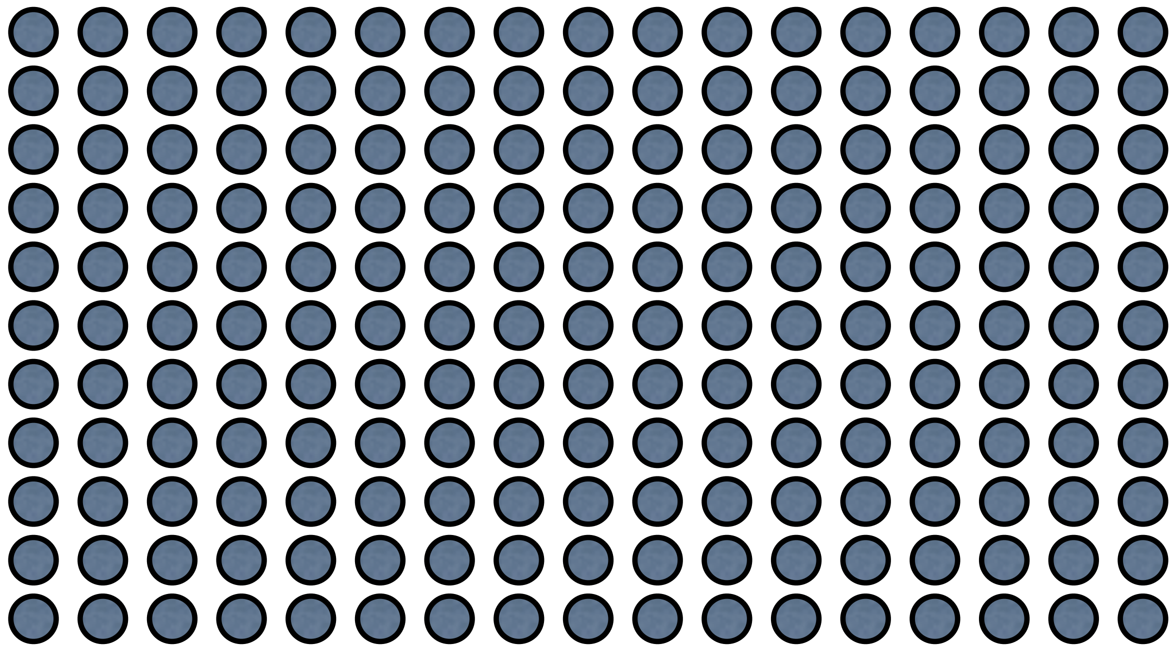
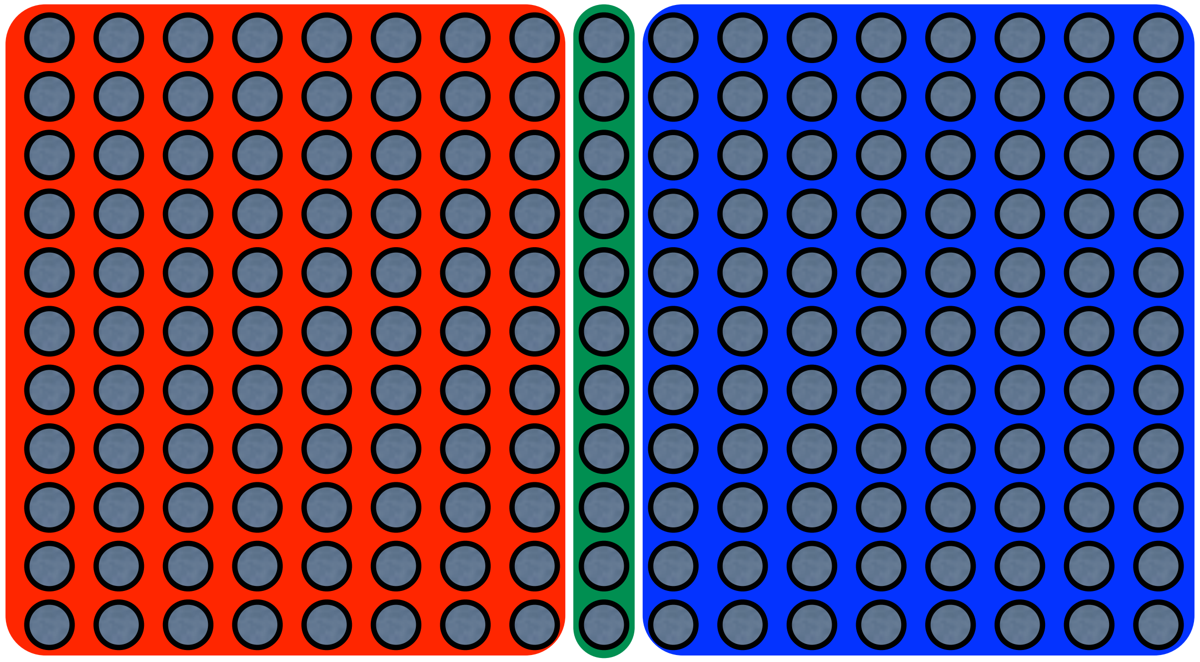
When all h_C commute, the other direction follows

Quantum Hammersley-Clifford



Quantum Hammersley-Clifford

$I(A:B|C)=0 \Rightarrow$



Quantum Hammersley-Clifford

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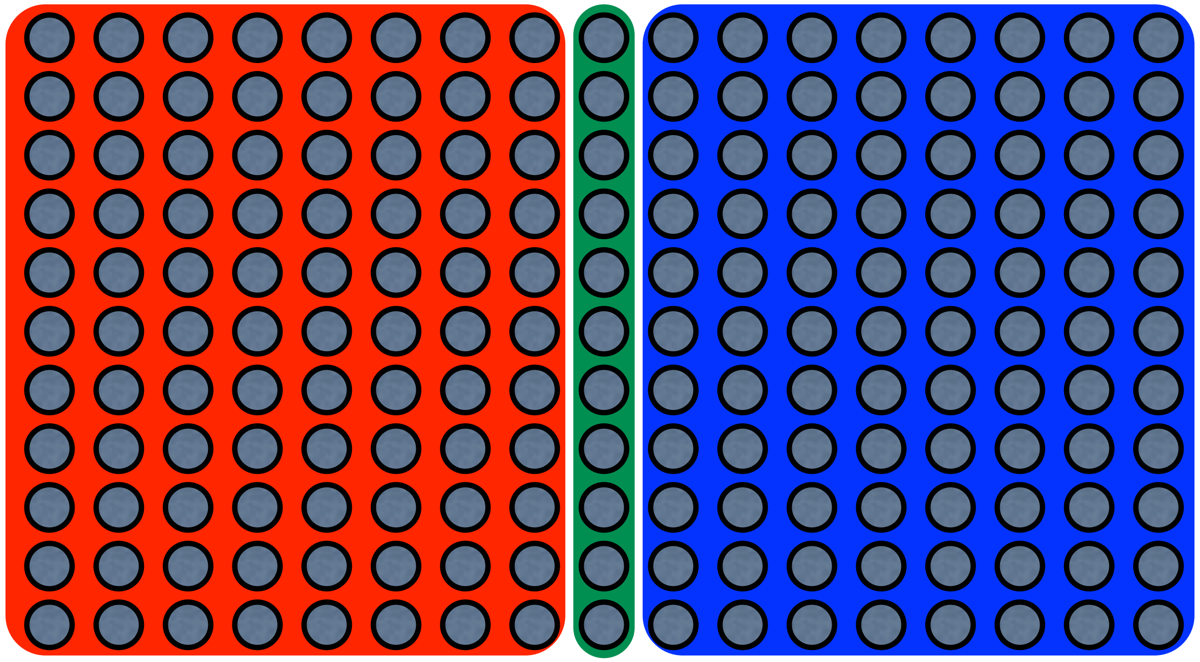
$$\rho = \sigma_{AB}\sigma_{BC} = \sigma_{BA}\sigma_{AB}$$



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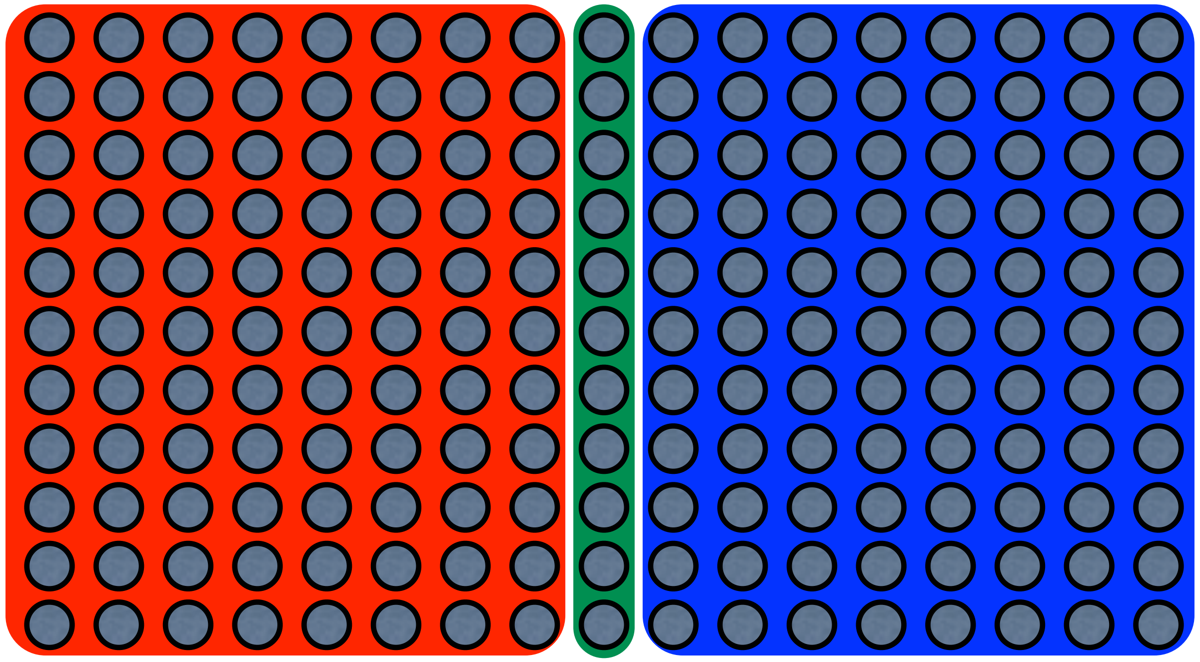

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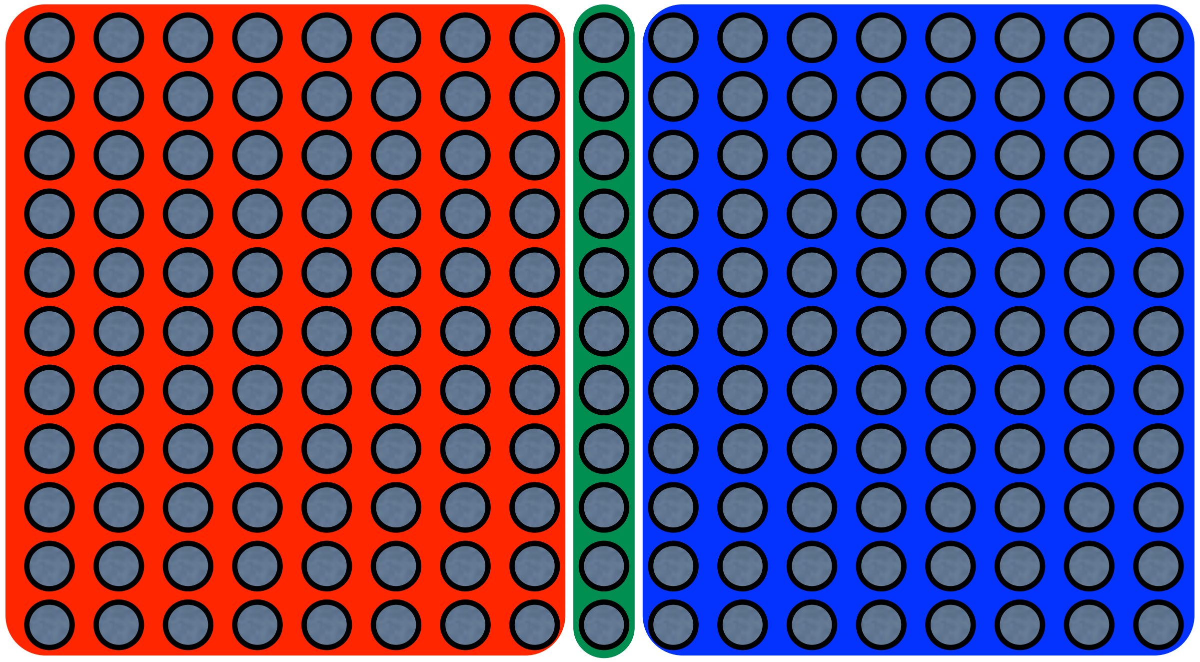
Holds for any cut



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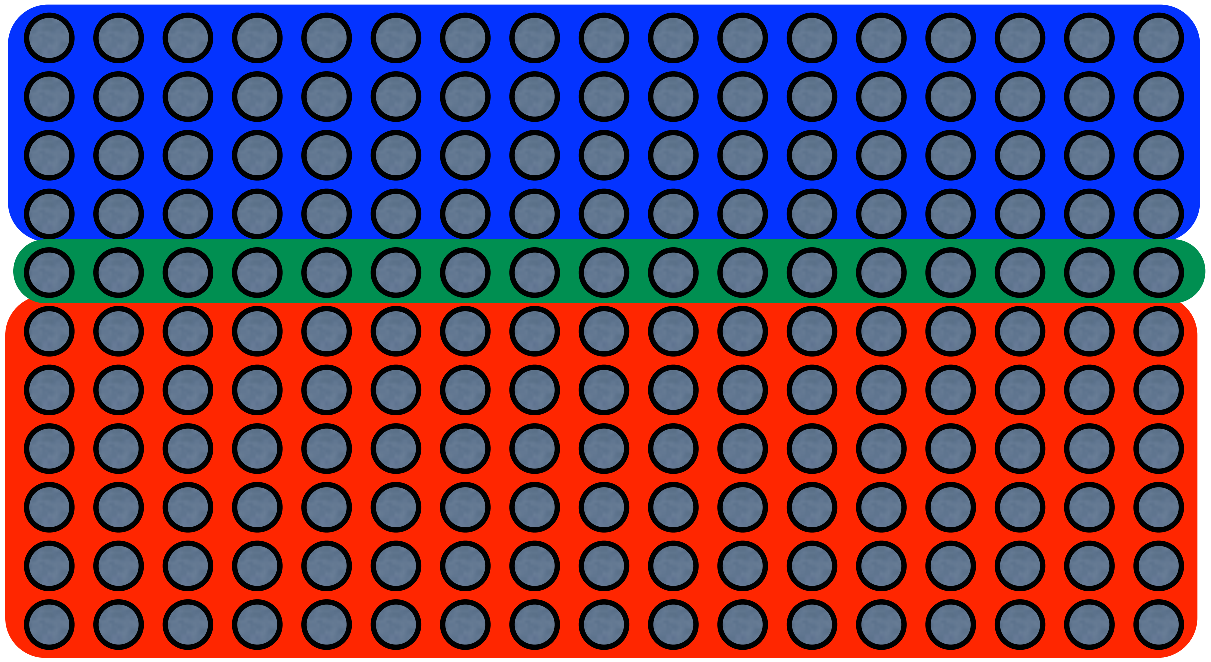

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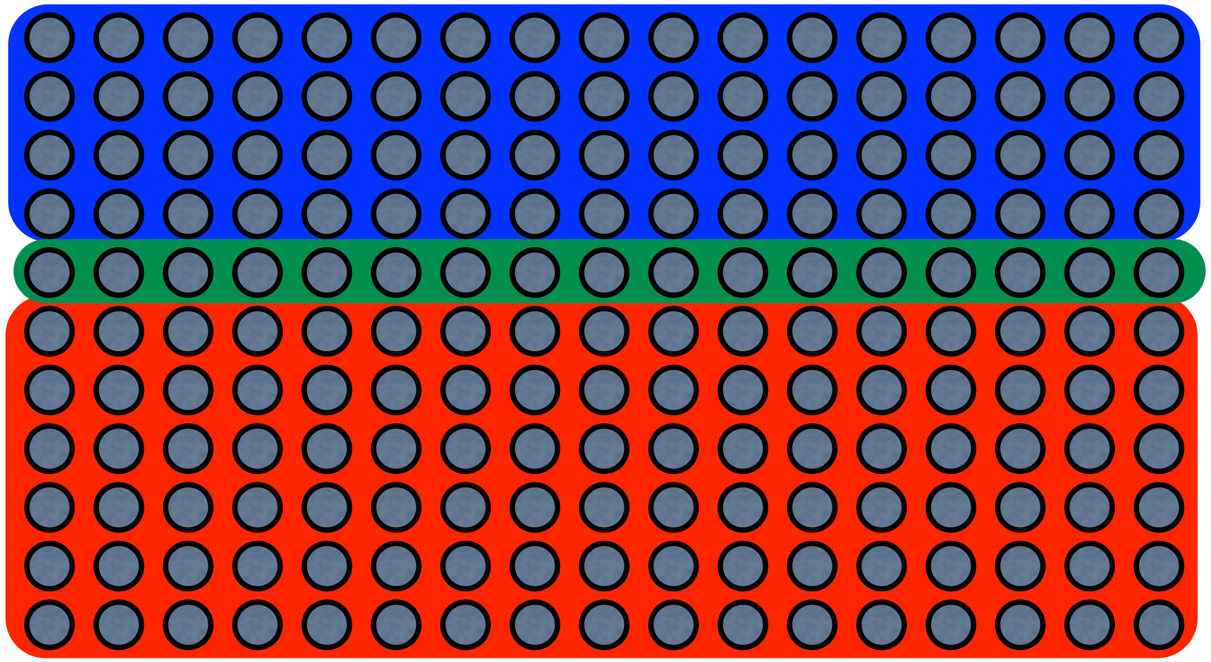
$$H = h_{AB} + h_{BC},\ [h_{AB}, h_{BC}] = 0$$

Holds for any cut



<u>If G is a graph with only two-vertices cliques (square lattice)</u>

(ρ, G) is a Markov chain

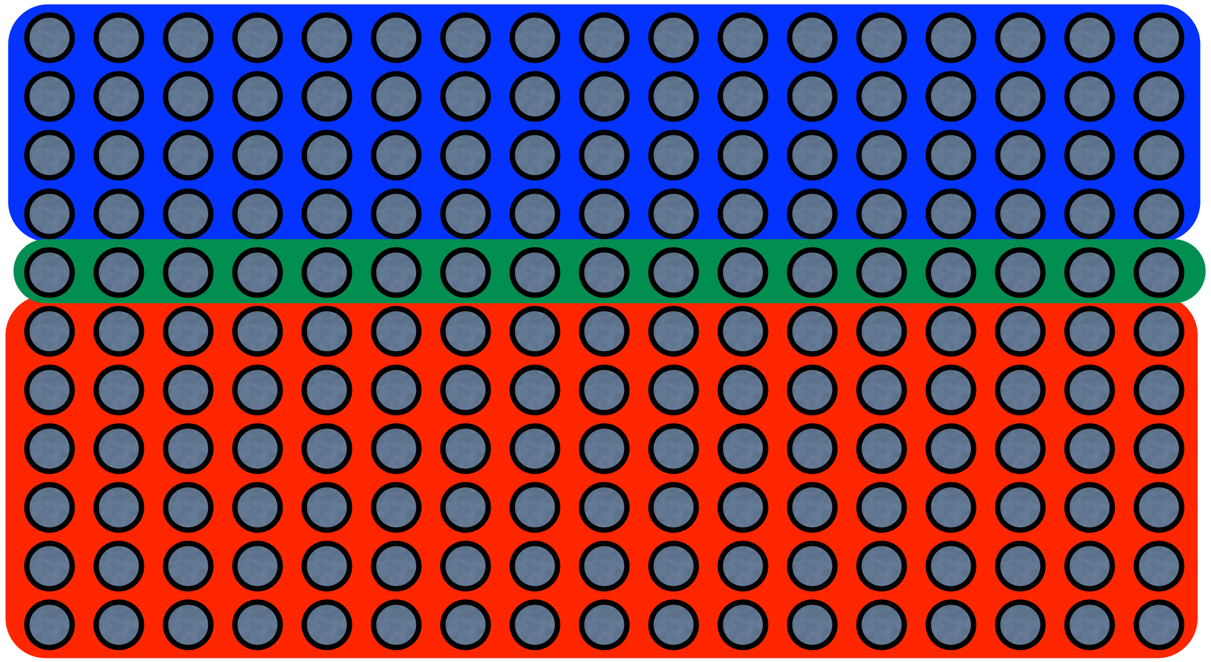
Quantum Hammersley-Clifford

I(A:B|C)=0 $\Rightarrow$

$$\rho = \sigma_{AB}\sigma_{BC} = \sigma_{BA}\sigma_{AB}$$

$$H = h_{AB} + h_{BC},\ [h_{AB}, h_{BC}] = 0$$

Holds for any cut



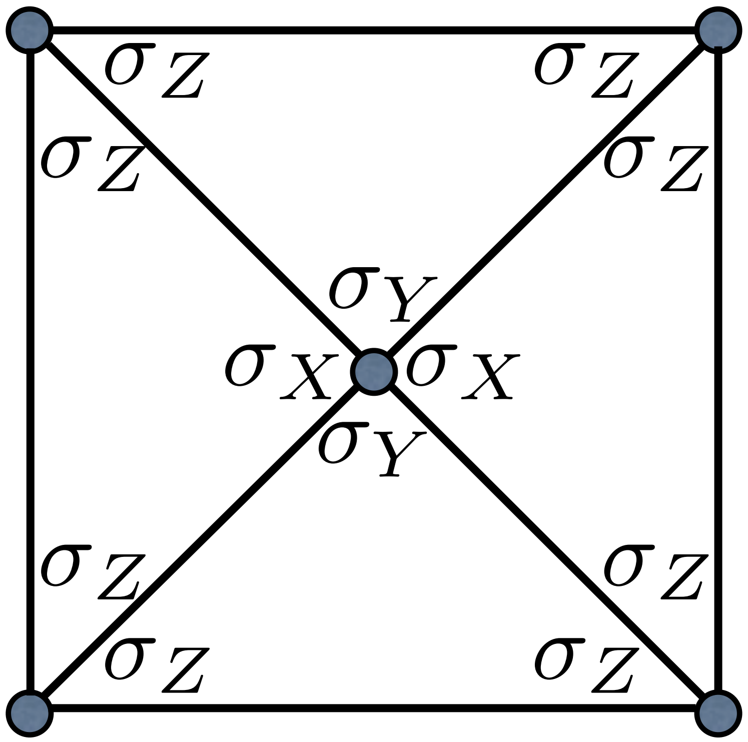
<u>If G is a graph with only two-vertices cliques (square lattice)</u>

(ρ, G) is a Markov chain

$$\Updownarrow$$

$$\rho = \frac{1}{Z}e^{-H} \qquad H = \sum_{C \in \text{cliques}} h_C \qquad h_C h_{C'} = h_{C'} h_C$$

Quantum Hammersley-Clifford



$$\sigma_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

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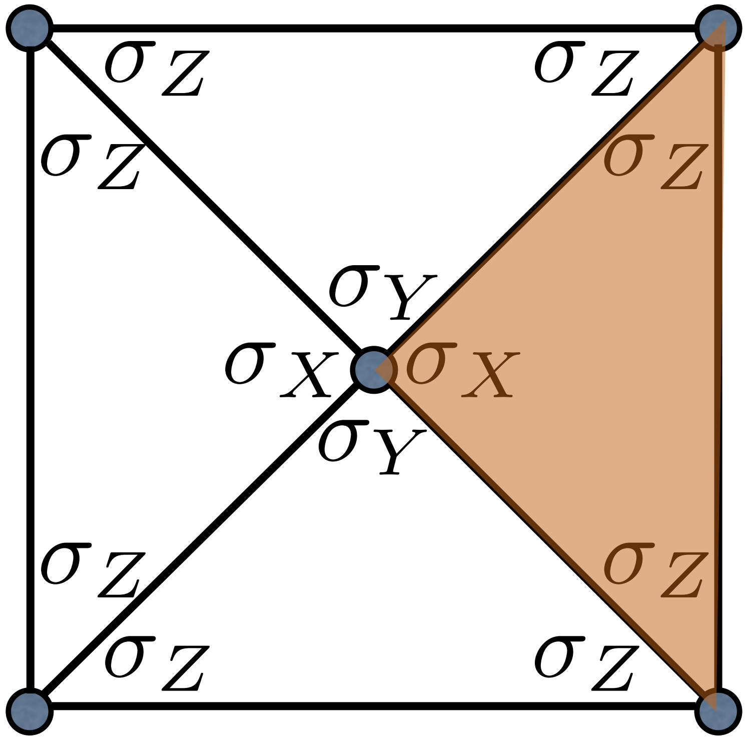

$$h_{\triangleleft} = \sigma_Z \otimes \sigma_X \otimes \sigma_Z$$

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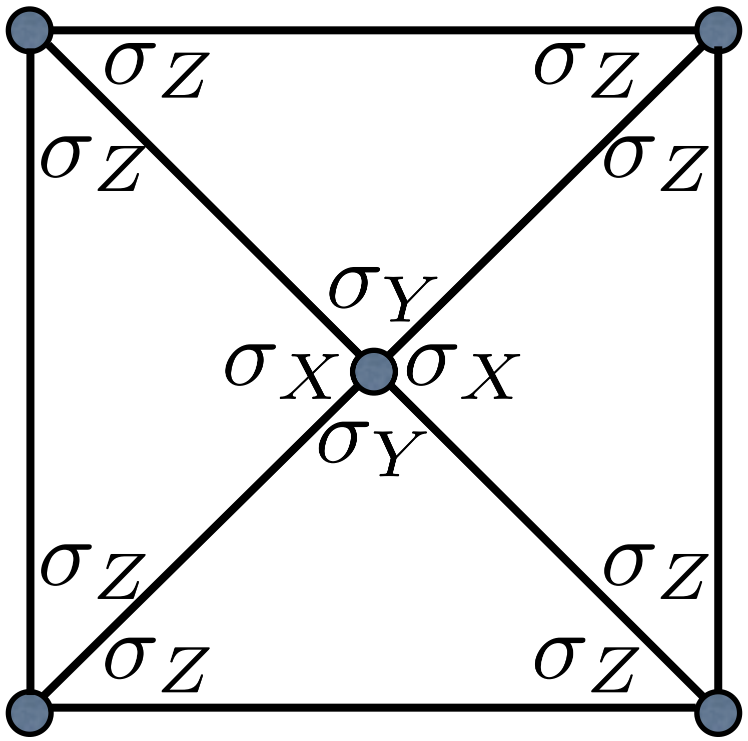

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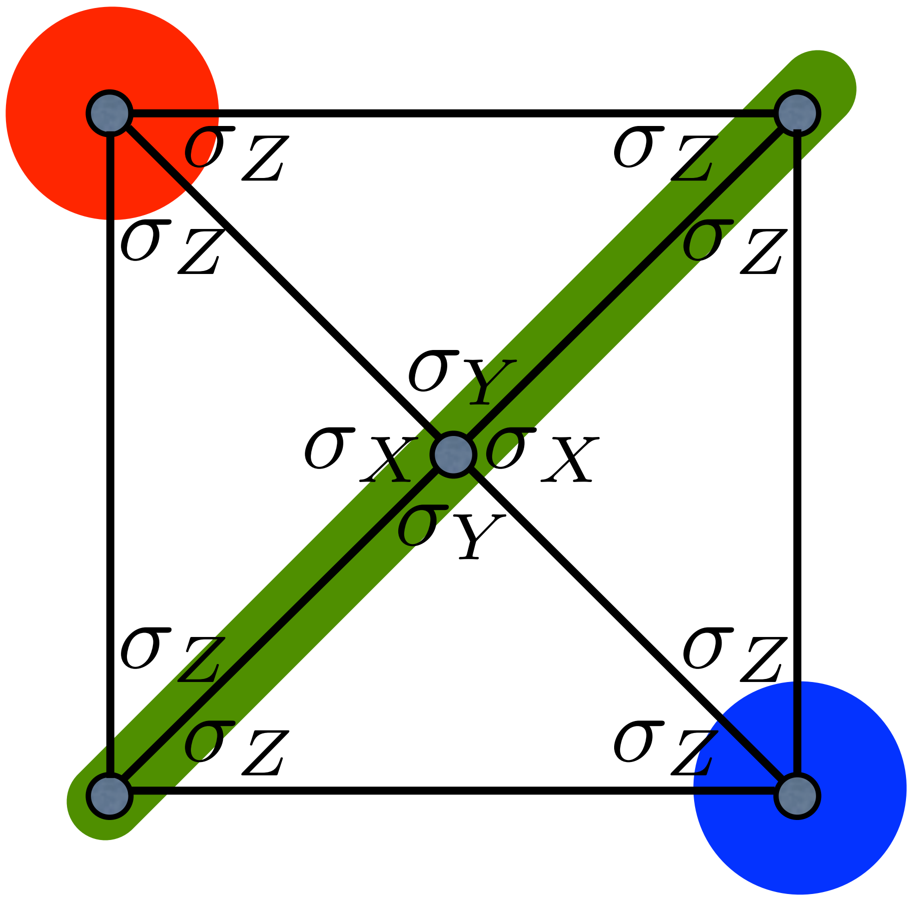

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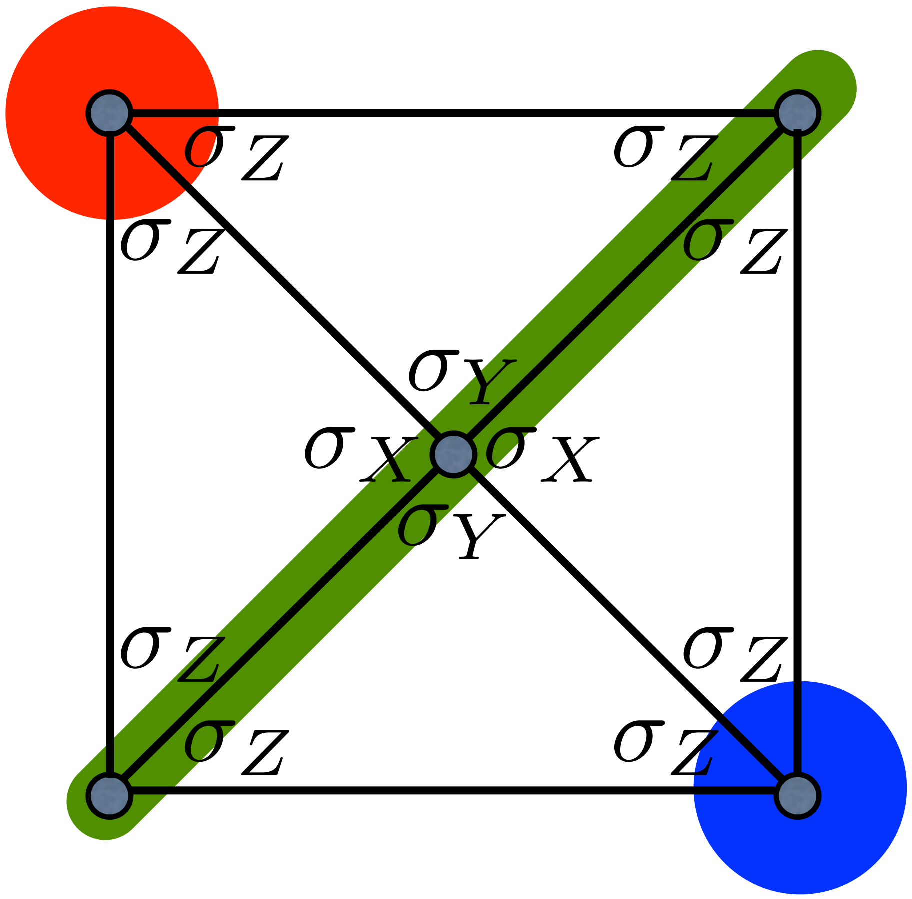

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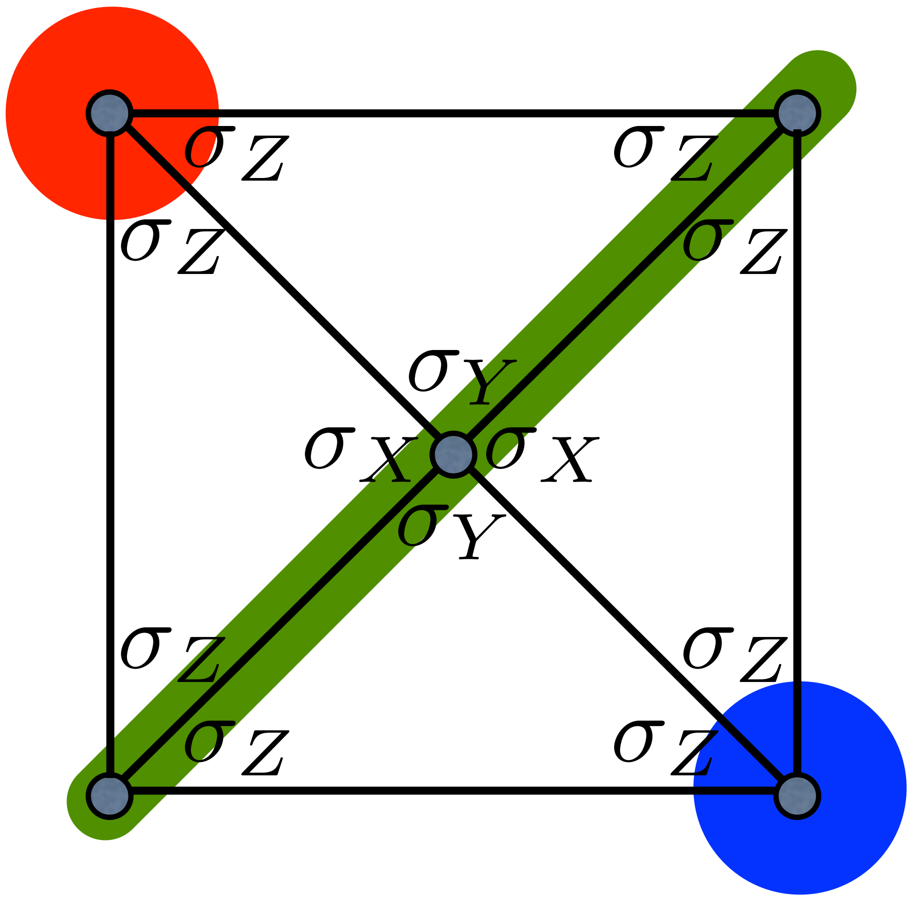

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I(A:B|C)=0 $\quad [h_{AB}, h_{BC}] =$

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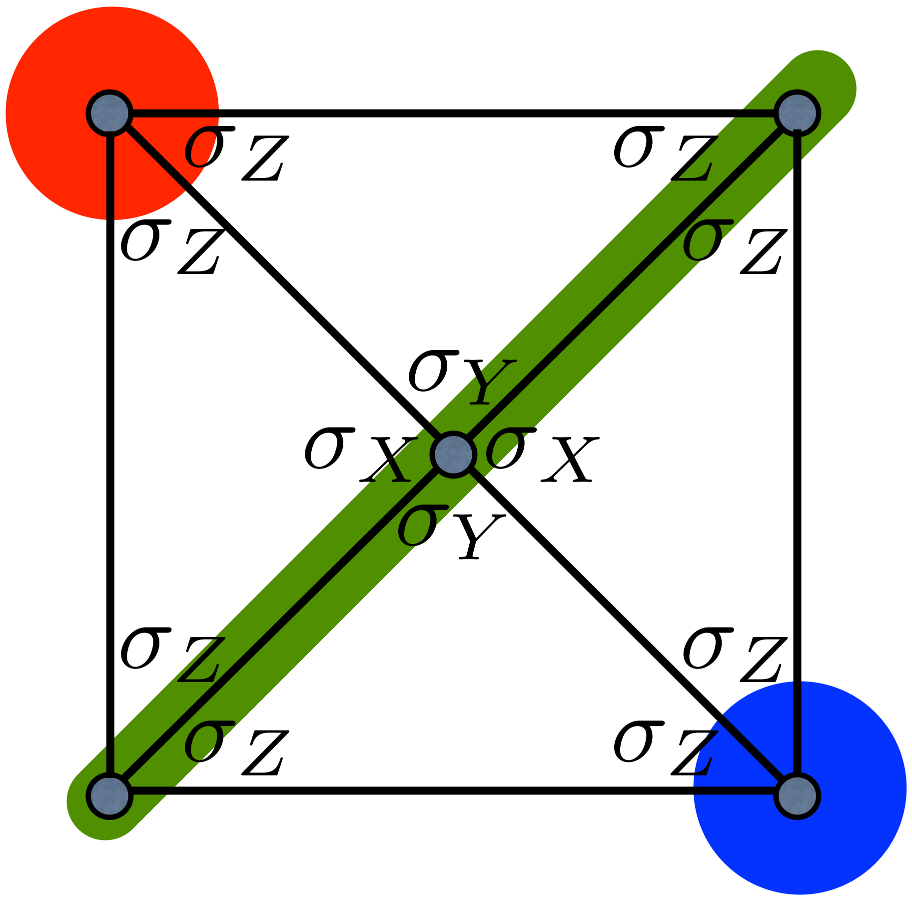

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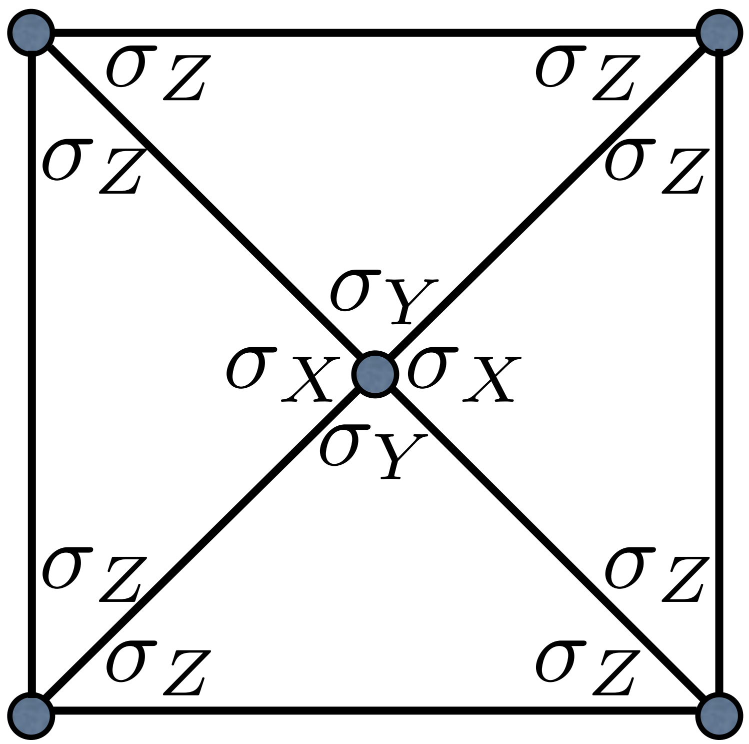

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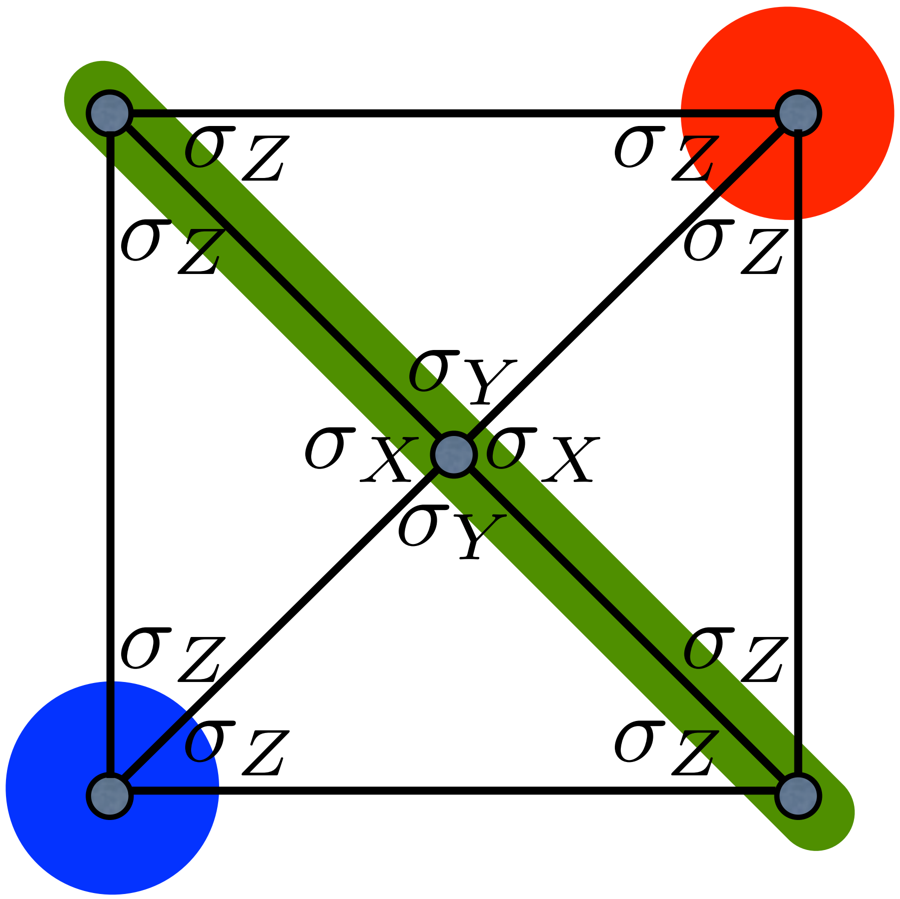

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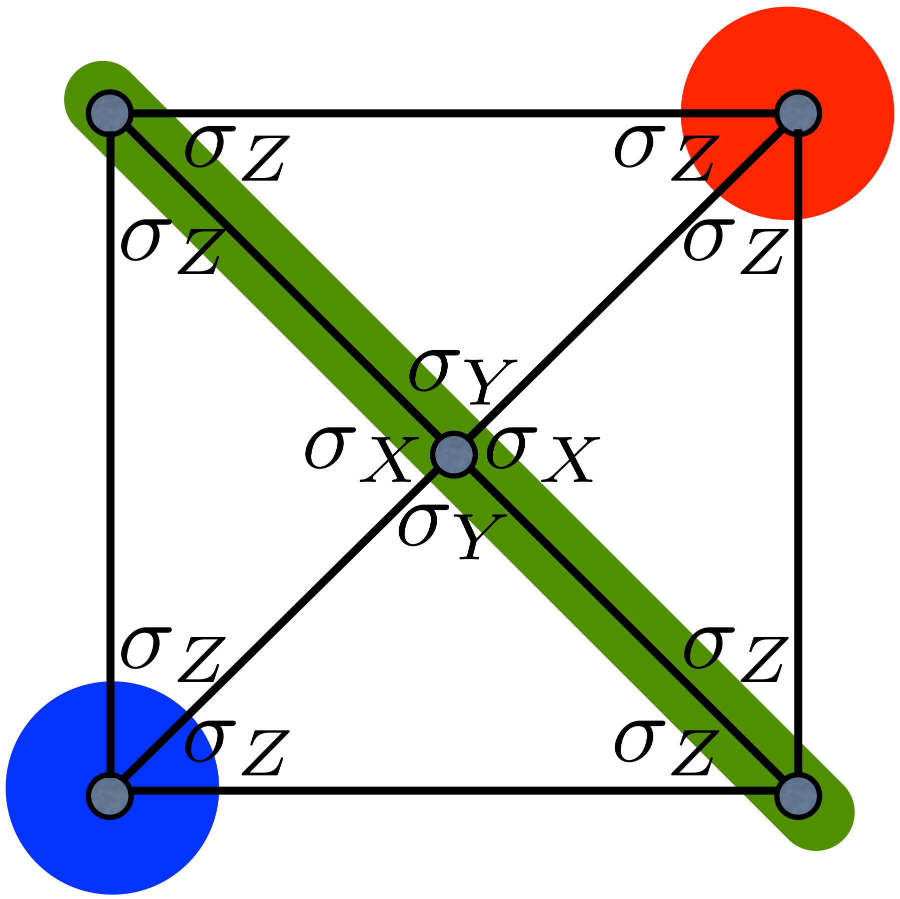

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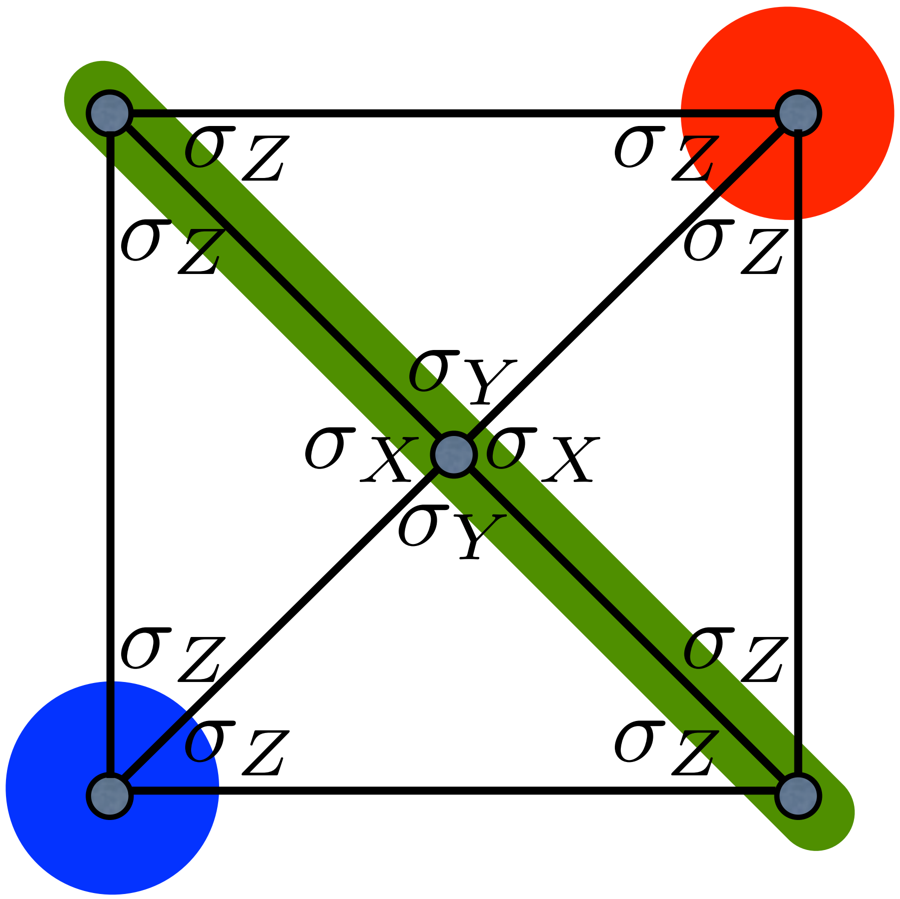

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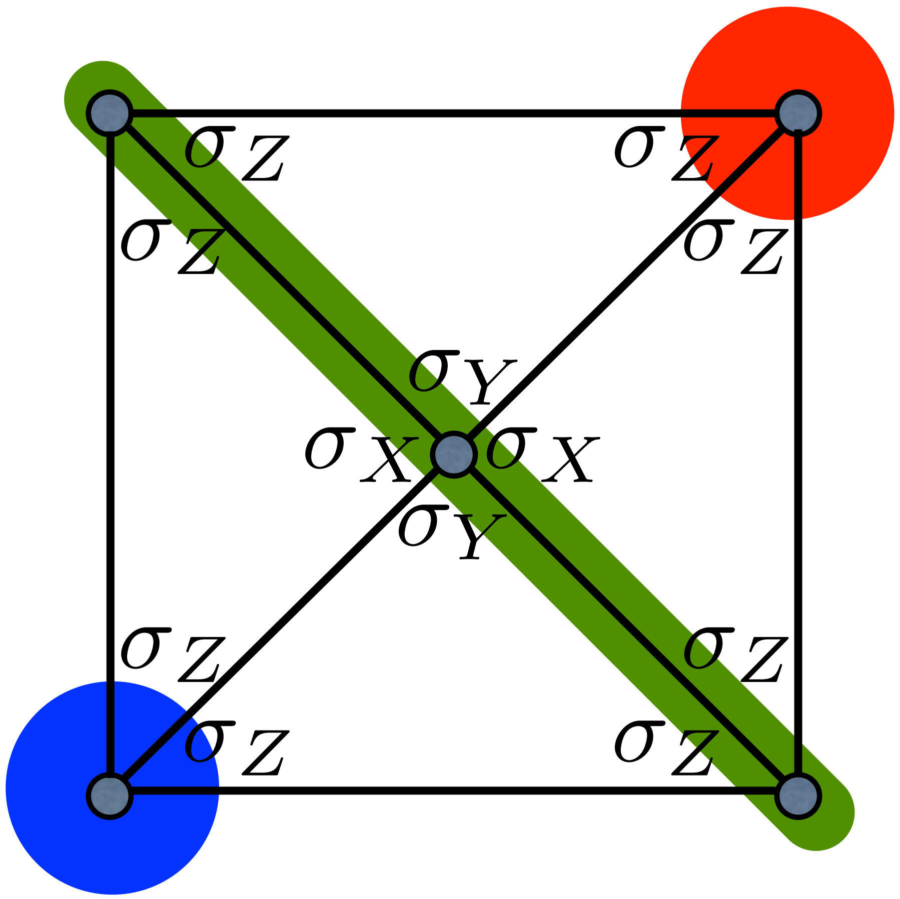

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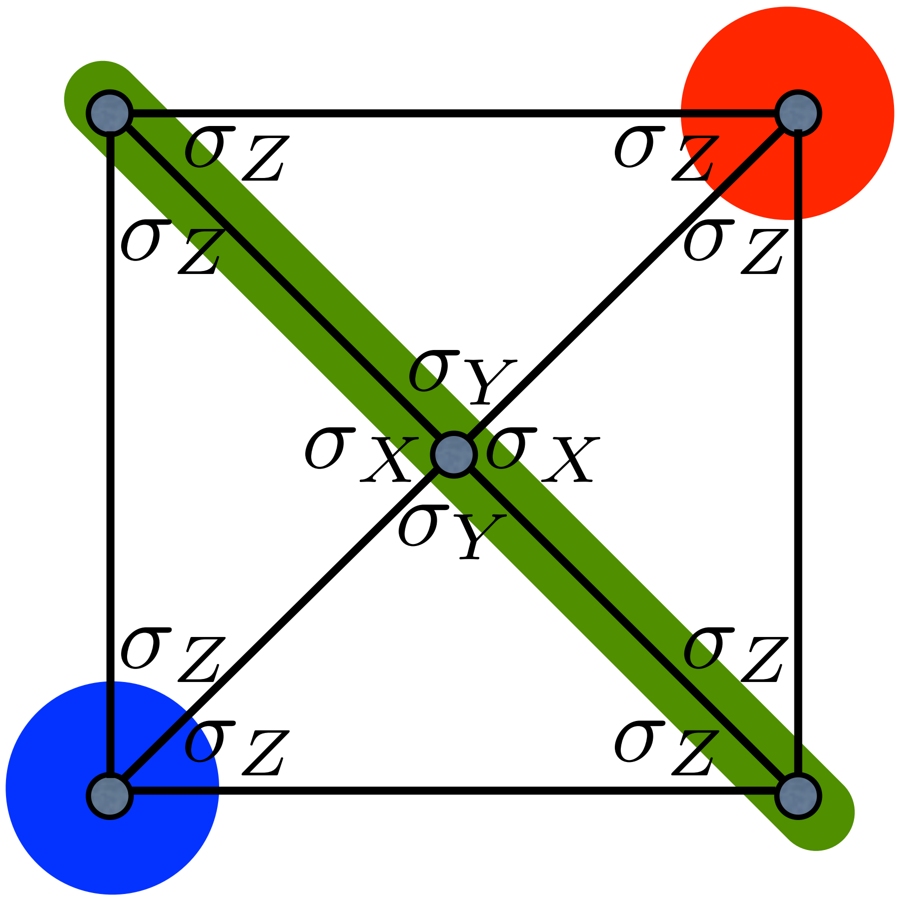

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Despite all this

$[h_{\triangleleft}, h_{\triangledown}] \neq 0 \quad [h_{\triangleright}, h_{\triangledown}] \neq 0$

$[h_{\triangleleft}, h_{\vartriangle}] \neq 0 \quad [h_{\triangleright}, h_{\vartriangle}] \neq 0$

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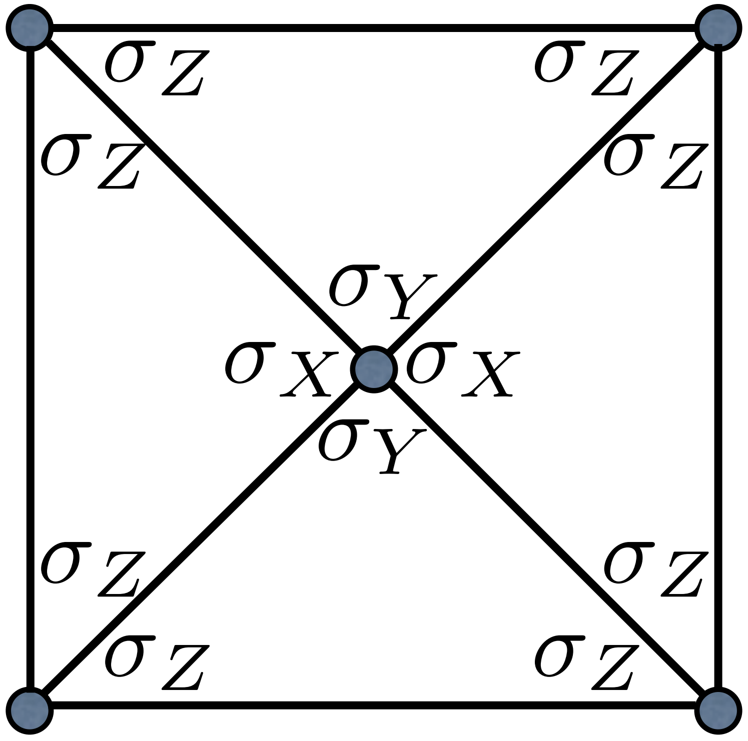

Markov network but not Gibbs state of Hamiltonian with local commuting terms

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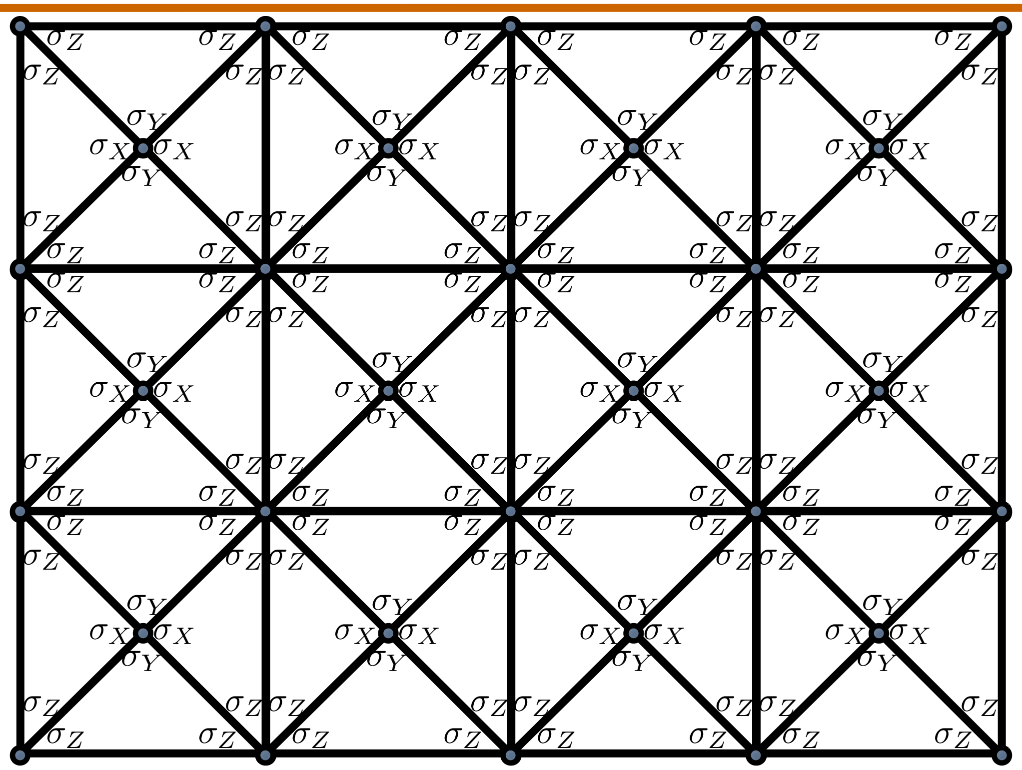

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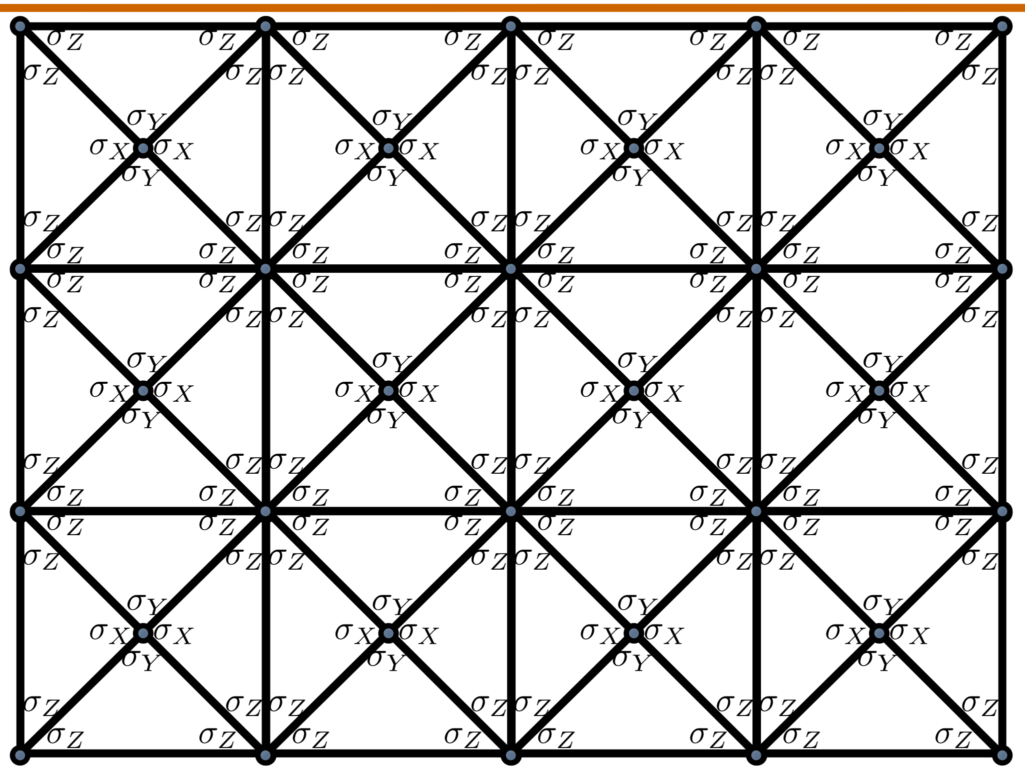

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Coarse graining the lattice eliminates this problem

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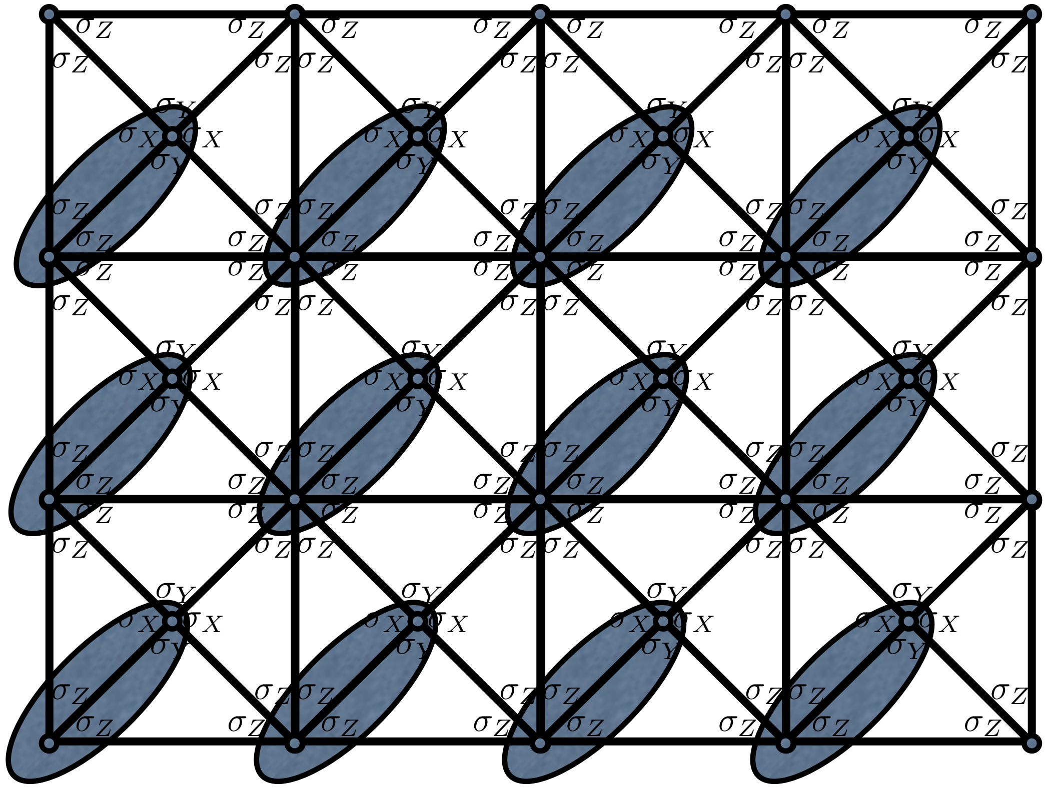

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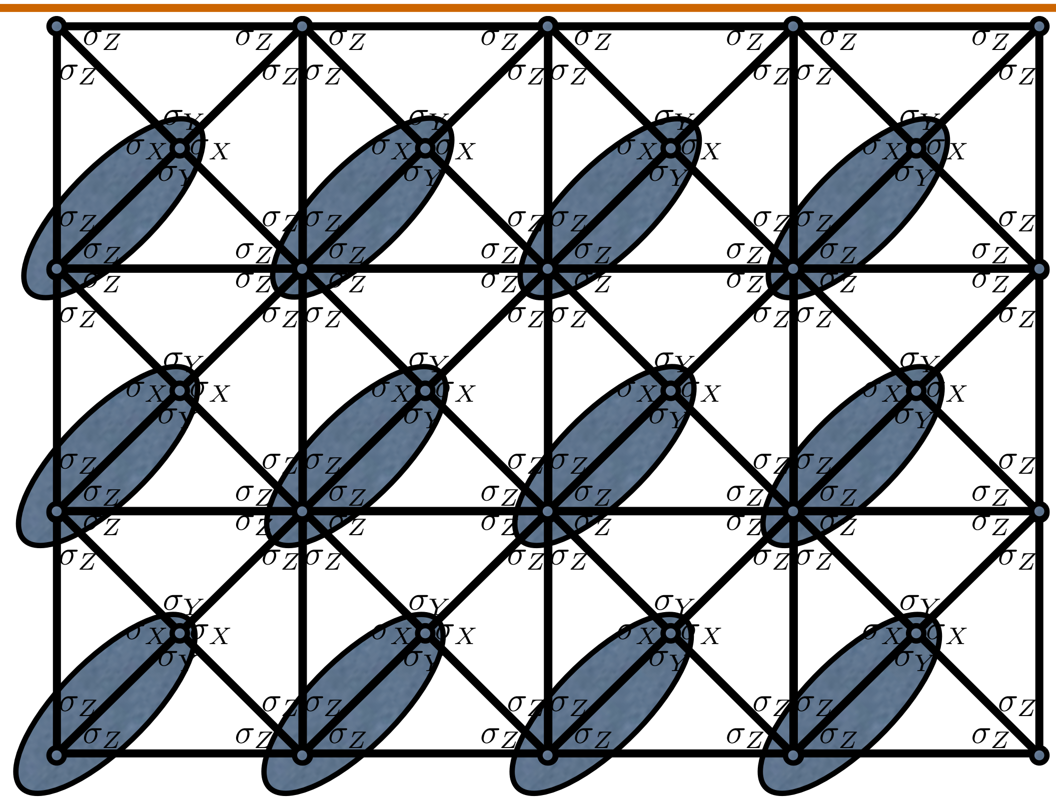

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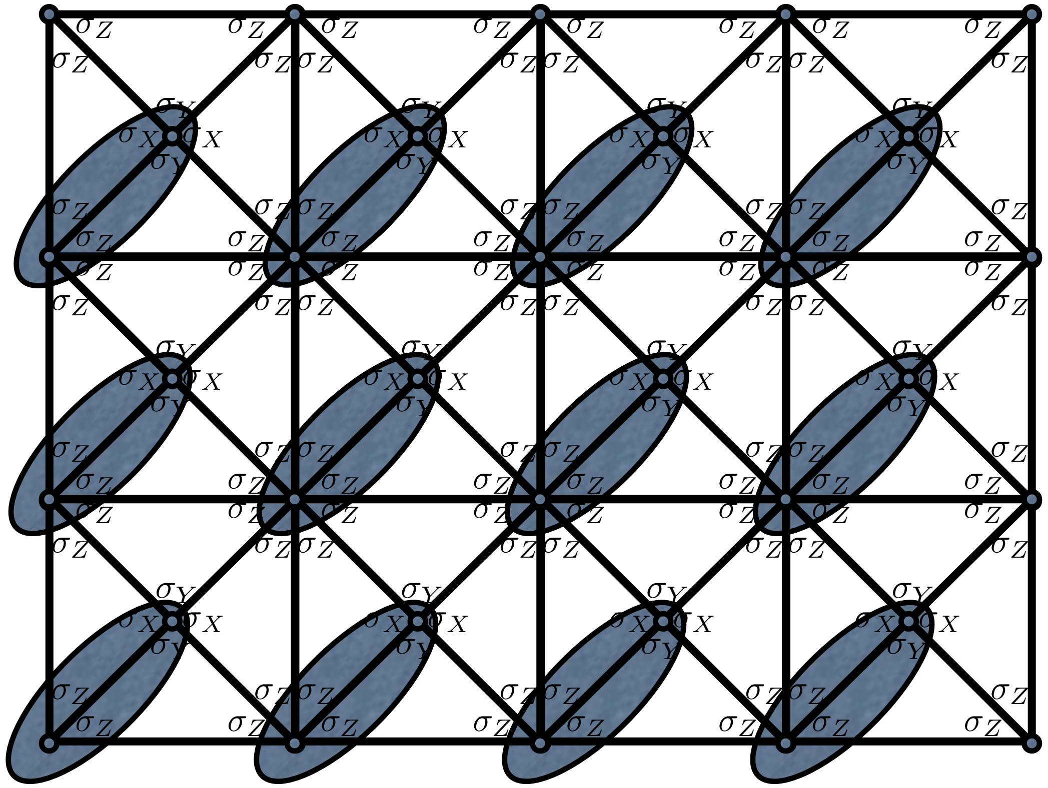

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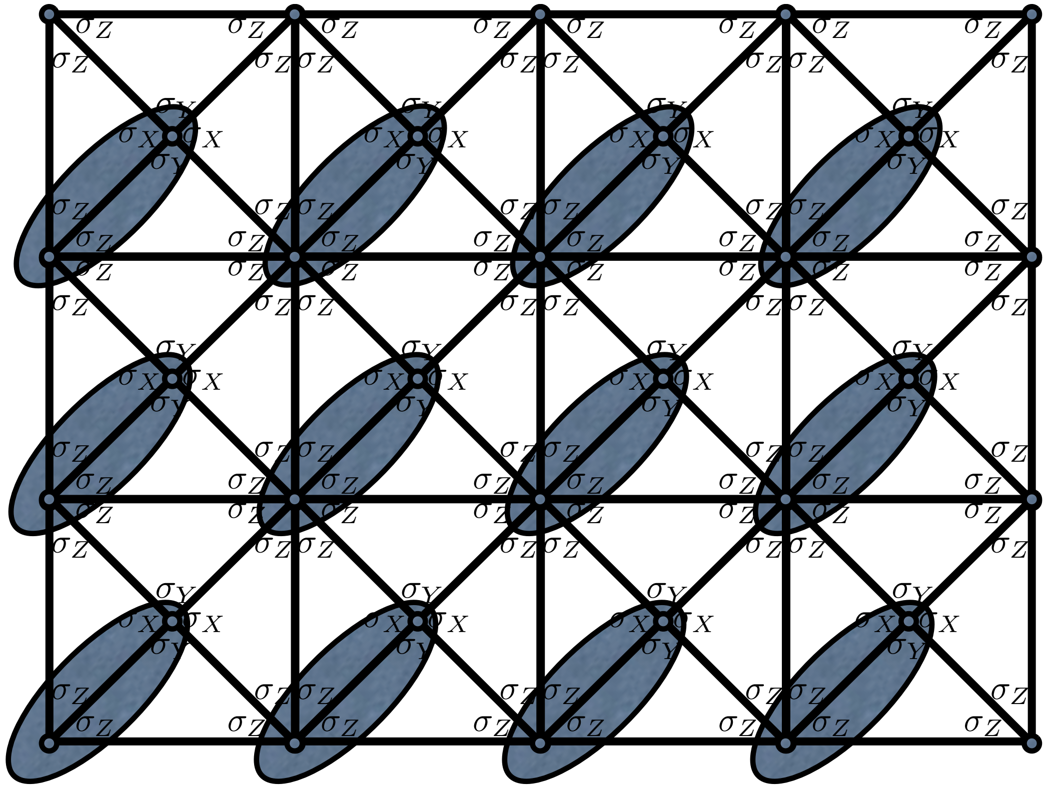

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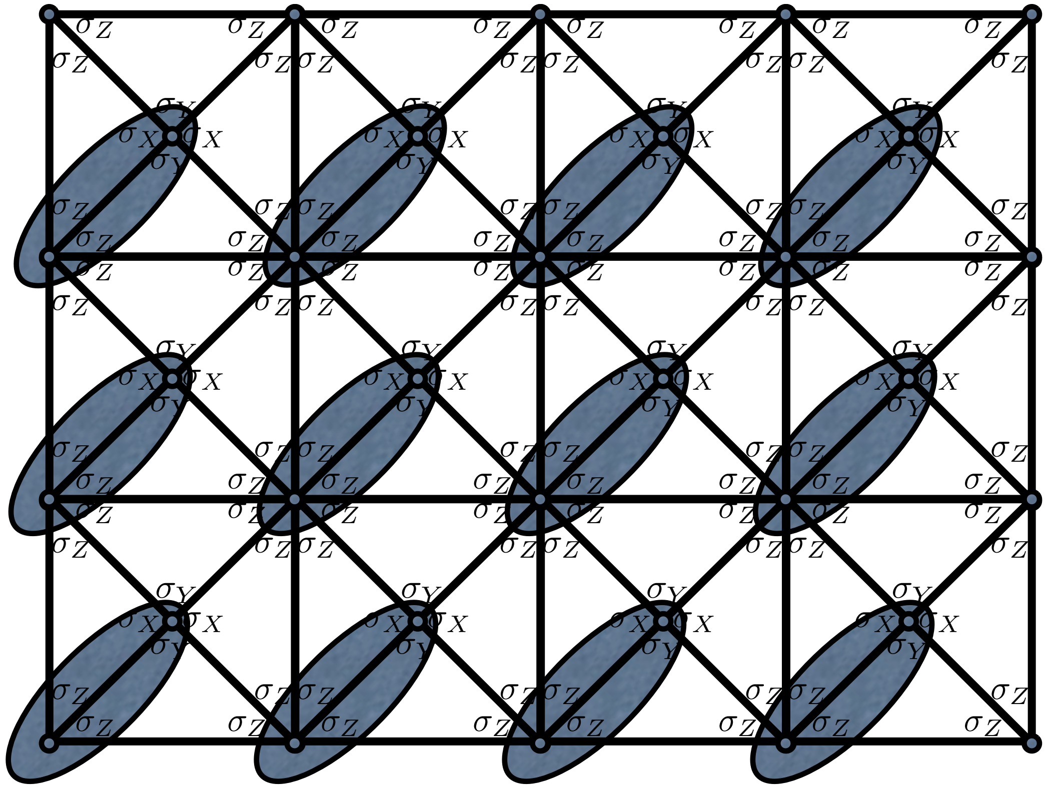

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- H.-C. stable under coarse graining
- Consistent with entanglement area law
- If not, new quantum phase of matter (quantum non-locality without entanglement)

Conclusion

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