REAL TIME PRICING TO ASSIST IN LOAD FREQUENCY CONTROL

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ABSTRACT

We study the use of real time prices to assist in the control of frequency and tie line deviations in electric power systems. The role of such prices, if any, would yield the practical limit to the trend in electric power systems of varying prices on ever faster time scales. Under idealized assumptions, we derive prices for load frequency control that reflect underlying physical relationships of the power system. Applying the theoretical results to a simple example, real time prices, determined by a feedback control law of frequency deviations, are shown to aid in load frequency control. We conclude that real time pricing could potentially serve as an economic load shedding policy to assist the direct control by the electric utility.

1. INTRODUCTION

The application of prices in electric power systems to increase the efficient use of resources is an established technique [1]. Also, the growth of privately owned generation, typically industrial cogeneration, and proposals to deregulate electric utilities pose further opportunities for the use of prices [2]. Pricing schemes can be classified by time scales. Time of day pricing, which varies two or three times a day, has been used for decades in Europe as a means of flattening out the daily demand curve [3]. The power brokerage system of 18 Florida utilities operates on an hourly time scale. The Florida utilities obtain lower costs to the consumers by buying and selling power amongst themselves, taking advantage of the diversity of generation costs [4].

In a spot price based energy marketplace, prices adapt to system operation conditions such as changes in system lambda, the effect of generation shortages, and the effect of line overloads. The fastest spot price that has been implemented is 30 minutes, (most implementations involve 1 hour time steps, which may be prespecified 24 hours in advance). Many papers have been written on spot pricing, starting with [5] and the homeostatic control concepts of [6]. The book by Schweppe, Caramanis, Tabors, and Bohn [7] summarizes the ideas. A key assumption of spot pricing is that the power system is in quasi-steady state; i.e. power system dynamics involving frequency, voltage, etc. are ignored, and only Kirchoff's laws for network flow are considered. The quasi-steady state assumption can be used for time scales as fast as economic dispatch updates, e.g. 5 minutes. The related literature is vast and [1] to [7] are only sample references.

89 WM 132-2 PWRS A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1989 Winter Meeting, New York, New York, January 29 - February 3, 1989. Manuscript submitted March 2, 1988; made available for printing December 21, 1988.

This paper explores pricing at time scales where the quasi-steady state assumption is no longer valid. In particular, we discuss pricing on the time scale of seconds to control frequency deviations that are caused by a temporary imbalance between mechanical power driving generation shafts and electrical power furnished to the loads. (The basic ideas extend naturally to control of voltage swings by excitor control.) Such pricing could be viewed as very fast spot prices, but we choose to reserve the use of the term "spot prices" for quasi-steady state conditions. We use the term "real time pricing" to refer to pricing in the presence of system dynamics. (Spot pricing is sometimes called dynamics pricing, responsive pricing, or real time pricing, depending on the author.)

Real time pricing of system dynamics is contemplated for a regulated utility where independently owned generation and major loads have the option to automatically monitor a time varying price and to respond according to a criterion pre-set by the user. Real time pricing can be conceptually applied also to a deregulated environment. Here, we make no attempt to discuss the pros and cons of deregulation.

Real time pricing is a complex topic, and this paper does not claim to provide final answers on how, or whether, to proceed. The goal of the paper is to stimulate discussion on a concept which has the potential to have a major impact in the future. The paper uses theoretical results and simplified examples; a more detailed presentation can be found in [8].

In the following section, we present a standard model for the frequency and tie line deviations. In Section 3, we examine whether pricing to control frequency and tie line deviations makes any sense, at least theoretically. In Section 4, we drop the idealized viewpoint of Section 3, and, by way of an example, consider a practical scenario.

2. LOAD FREQUENCY CONTROL

Load Frequency Control, or Automatic Generation Control, is concerned with the maintenance of frequency at its set point of 50 or 60 Hz., and tie line power flows at their scheduled levels when the system is subjected to small disturbances that do not, or no longer, threaten the synchronous operation of the power system. The appropriate mathematical model for such phenomena is called the average system frequency model. For simplicity, we consider the case of two areas and ignore dampening.[1]. The equations for the model are:

* This work was done while A. W. Berger was with the Division of Applied Sciences, Harvard University. The work was supported by the M.I.T. Energy Lab. contract # EX-76-A-012295 and the Office of Naval Research contract # N00014-75-C-0648.

1. In [8] we present the case of an arbitrary number of areas, and include dampening.

$$\dot{\delta}_i(t) = f_i(t) \qquad \delta_i(t_o) \text{ given}^2 \qquad i=1,2 \tag{1a}$$

$$H_i \dot{f}_i(t) = \sum_{j=1}^{NG_i} M_{ij} - G_i \qquad f_i(t_o) \text{given} \qquad i=1,2 \tag{1b}$$

$$G_1 = \sum_{j=1}^{NL1} D_{1j} + Loss_1 (\sum_{j=1}^{NL1} D_{1j}) + Tie \tag{1c}$$

$$G_2 = \sum_{j=1}^{NL2} D_{2j} + Loss_2 (\sum_{j=1}^{NL2} D_{2j}) - Tie \tag{1d}$$

$$Tie(\delta_1, \delta_2) = (\delta_1 - \delta_2) / X \tag{1e}$$

$$\dot{x}_{ij} = A_{ij} x_{ij} + B_{ij} u_{ij} \qquad x_{ij}(t_o) \text{ given} \tag{1f}$$

$$M_{ij} = C_{ij} x_{ij} \qquad j=1,...,NG_i \qquad i=1,2 \tag{1g}$$

A glossary of symbols is given at the end of the paper.

The load frequency control problem determines the speed changer controls u_{ij} such that frequency and tie line power remain at their set points in face of exogenous disturbances. Many direct control strategies have been studied; references [9] and [10] are examples. This paper considers indirect control via prices.

3. ECONOMIC THEORY OF PRICING IN LOAD FREQUENCY CONTROL

In this section, our interest is: if generators and loads were to respond to a time varying price, then what should the price be so that frequency and tie line deviations are controlled? Under idealized assumptions, we derive such coordinating prices. The prices have intuitive meaning: they reflect the underlying physical relationships of the power system.

We assume generation plants and loads have automatic control devices that monitor and react to time varying prices. The electric utility company administers the prices via the energy control center. The Public Utility Commission (PUC) sets the rules that determine the prices. The PUC requires that prices be set to maximize social welfare. Social welfare is defined to be the sum of the consumers' satisfaction indices minus the costs to the plants minus penalty functions for the operating constraints of the power system. The plants operate to maximize profits, and the loads operate to maximize a satisfaction index. The energy control center has perfect information, i.e. it knows the models used in the automatic control devices of the plants and the loads. Lastly, all computations take zero time.

The social welfare problem is formulated as:

$$\underset{u_{ij},\, D_{ij}}{maximize} \int_{t_o}^{t_f} \{ \sum_{i=1}^{2} [\sum_{j=1}^{NL_i} S_{ij}(D_{ij}) - \sum_{j=1}^{NG_i} l_{ij}(x_{ij}, u_{ij})] \tag{2}$$

$$- l_f(f) - l_\delta(\delta) \} \; dt$$

$$+ \sum_{i=1}^{2} \sum_{j=1}^{NG_i} \phi_{ij}(x_{ij}(t_f)) - \phi_f(f(t_f)) - \phi_\delta(\delta(t_f))$$

subject to the constraints of the load frequency model, equation 1.

$S_{ij}(D_{ij})$ is the satisfaction index of load j in area i. $l_{ij}(.)$ is the instantaneous cost and $\phi_{ij}(.)$ is the "excess energy" left in the boiler turbine at the terminal time. (The limiting values on u_{ij}, x_{ij} and D_{ij} can be captured by having $l_{ij}(.)$ and $S_{ij}(D_{ij})$ respectively increase or decrease precipitously when the arguments reach their limiting values.) $l_f(f)$ models the concern that at no time are the frequency deviations too large, and the term $\phi_f(f(t_f))$ models the desire that at the terminal time the frequency deviations are close to zero. $l_\delta(\delta)$ models the concern that the stability limit of the inter-area tie line is met, i.e. that the pooled power system remains coupled together. Mathematically this means that the phase angle difference, $\delta_1 - \delta_2$, does not become too large. Thus the function $l_\delta(\delta)$ is in fact a function of $\delta_1 - \delta_2$. Lastly, the term $\phi_\delta(\delta(t_f))$ is also a function of $\delta_1 - \delta_2$ and models the desire that at the terminal time the tie line power flows will be close to their scheduled values.

We are not interested in solving the social welfare problem per se, but rather, given that the optimal controls exists, can they be induced by prices? Thus, we make assumptions that guarantee the existence of the social welfare optimal controls. Assume the terms in the objective function, (2), and the loss function $Loss_i(\cdot)$, (1d), are differentiable with respect to their arguments. Also, assume that S_{ij} and ϕ_{ij} are concave; that l_{ij}, l_f, l_δ, ϕ_f, ϕ_δ are convex; that l_{ij} is separable in x_{ij} and u_{ij}; and that u_{ij} and D_{ij} are elements of the set of piecewise continuous functions on $[t_o, t_f]$. Then the necessary conditions of Pontryagin's maximum principle become sufficient conditions [11]. Lastly, assume the sufficient conditions have a unique solution. The Hamiltonian and necessary conditions for the social welfare problem, (2), are given in the appendix.

3.1 Pricing Mechanisms

We assume the electric utility can solve the social welfare problem and hence knows the social welfare optimal controls, denoted u_{ij}^{sw}, D_{ij}^{sw}. The interest of this section is: can the utility choose prices such that the resulting optimal control for the generators and loads is equal to u_{ij}^{sw}, D_{ij}^{sw} respectively?

Consider first the viewpoint of the generators. The natural entity to price is the electric power generated. This is related to the plant's mechanical power by the individual swing equation:

$$H_{ij} \dot{f}_{ij} = M_{ij} - G_{ij} \tag{3}$$

Suppose that the individual power plant considers the frequency deviation to be exogenous to its influence. (This is the common infinite bus assumption.) Thus the generator's optimization is:

$$\max \int_{t_o}^{t_f} \{ \rho_{G_{ij}} (M_{ij} - H_{ij} \dot{f}_{ij}) - l_{ij}(x_{ij}, u_{ij}) \} dt + \phi_{ij}(x_{ij}(t_f)) \tag{4}$$

subject to equations 1f,g

where $\rho_{G_{ij}}(t), t \in [t_o, t_f]$ is the price stated by the utility and is an exogenous input to the generator. The term: $\int_{t_o}^{t_f} - \rho_{G_{ij}} H_{ij} \dot{f}_{ij} \, dt$ is also exogenous and thus does not influence the individual generator's optimal control, denoted u_{ij}^{ind}. We have the following result.

Theorem 1: The optimal control for the individual generator ij, $u_{ij}^{ind}(t)$, equals the optimal control for social welfare, $u_{ij}^{sw}(t)$,

2. It is necessary to choose one phase angle, say $\delta_1(t_o)$, as a reference point. Then for all $t \geq t_o$ and for i=1,2, $\delta_i(t)$ is measured with respect to $\delta_1(t_o)$.

if the price $\rho_{G_{ij}}(t)$ is chosen to be $\mu_{ij}^{sw}(t)$, the social welfare value for the Lagrange multiplier associated with the generator ij's output function, (1g). Moreover, $\mu_{ij}^{sw}(t)$ equals $\gamma_i^{sw}(t)$, the social welfare value for the Lagrange multiplier of the energy balance equation, (1cd), and equals $\beta_i^{sw}(t)/H_i$, the costate vector of the swing equation, (1b), divided by the area inertial constant.

The proof of Theorem 1 is given in the appendix. The idea of the proof is to choose a price so that the conditions of Pontryagin's Maximum Principle for the generator's optimization match those of the social welfare optimization.

Before discussing Theorem 1, we present the analogous result for the loads. If the entity priced is the power consumed, D_{ij}, and the price is denoted $\rho_{D_{ij}}$ then load ij's optimization is:

$$\underset{D_{ij}}{maximize} \int_{t_o}^{t_f} \{ S_{ij}(D_{ij}) - \rho_{D_{ij}}(t) D_{ij}(t) \} \, dt \tag{5}$$

This optimization trivializes to maximizing the integrand at each point in time. Thus the load's optimal control satisfies the condition: marginal satisfaction equals price.

$$\frac{\partial S_{ij}}{\partial D_{ij}} - \rho_{D_{ij}} = 0 \tag{6}$$

Solving equation (6) for $D_{ij}(t)$ yields the optimal control for the individual load, denoted D_{ij}^{ind}. Furthermore, from equation A.1a of the appendix and Theorem 1, we have:

Corollary: The optimal control for the individual load ij, D_{ij}^{ind}, equals the social welfare value, D_{ij}^{sw}, if:

$$\rho_{D_{ij}} = \rho_{G_{ij}} \left(1 + \frac{\partial Loss_i}{\partial D_{ij}} \right) \Bigg|_{sw}$$

3.2 Discussion of Prices

3.2.1 Common Price for Generation Within an Area: Since the price for generation $\rho_{G_{ij}}$ equals $\gamma_i^{sw}(t)$ and $\beta_i^{sw}(t)/H_i$ and since these multipliers are the same for all plants in a given area (i.e., they do not depend on j), then the price for generation is the same for all plants in a given area. Notationally, we can write $\rho_{G_{ij}}(t)$ as $\rho_{G_i}(t)$. Likewise, since

$$\frac{\partial}{\partial D_{ij}} Loss_i\left(\sum_{j=1}^{NL_i} D_{ij} \right) = \frac{\partial}{\partial D_i} Loss_i(D_i) \cdot \frac{\partial D_i}{\partial D_{ij}}$$

$$= \frac{\partial}{\partial D_i} Loss_i(D_i) \cdot 1$$

then, the price for demand is:

$$\rho_{D_{ij}} = \rho_{G_i} \left(1 + \frac{\partial}{\partial D_i} Loss_i(D_i) \right) \Bigg|_{sw} \tag{7}$$

and hence is the same for all loads in a given area. Again, notationally we can write $\rho_{D_{ij}}(t)$ as $\rho_{D_i}(t)$.

The above result is due to a feature of the average system frequency model. The model collapses the Transmission and Distribution system within a given area to a single bus, and thus the spatial losses within an area are suppressed. If they had been included then the price would have differed between generators and would have differed between loads. Note that if there were no losses in the transmission and distribution system, $Loss_i \equiv 0$, then equation (7) implies that the price would be the same for both the generators and the loads, in a given area.

3.2.2 Constraints on Frequency Deviations and Tie Line Power Determine Price: The necessary conditions of social welfare, equations A.1defg of the appendix, yield differential equations for the price.

Lemma 1: The price for generation in area 1, ρ_{G_1}, satisfies:

$$\ddot{\rho}_{G_1} + \frac{1}{XH_1}(\rho_{G_1} - \rho_{G_2}) = \frac{1}{H_1}\left(\frac{d}{dt}\frac{\partial l_f}{\partial f_1} - \frac{\partial l_\delta}{\partial \delta_1} \right) \Bigg|_{sw} \tag{8a}$$

with boundary conditions:

$$\rho_{G_1}(t_f) = - \frac{1}{H_1}\frac{\partial \phi_f}{\partial f_1} \Bigg|_{sw\; t_f} \tag{8b}$$

$$\dot{\rho}_{G_1}(t_f) = \frac{1}{H_1}\left(\frac{\partial \phi_\delta}{\partial \delta_1} + \frac{\partial l_f}{\partial f_1} \right) \Bigg|_{sw\; t_f} \tag{8c}$$

The price for generation in area 2 satisfies the same equation except with the subscripts "1" and "2" reversed.

Note that what drives the prices from being zero is the penalty function for deviation in tie line power and frequency. Note also that the equations for the two prices are coupled by the term: $\rho_{G_1} - \rho_{G_2}$; this difference is the subject of the next section.

3.2.3 Price Difference Between Areas: The prices also differ from area to area. Manipulating equations A.1defg of the appendix, we obtain:

Lemma 2: The difference in the price for generated power between the two areas, $\Delta\rho = \rho_{G_1} - \rho_{G_2}$, satisfies:

$$\Delta\ddot{\rho} + \frac{1}{X}\left(\frac{1}{H_1} + \frac{1}{H_2} \right)\Delta\rho = \tag{9a}$$

$$\frac{d}{dt}\left(\frac{1}{H_1}\frac{\partial l_f}{\partial f_1} - \frac{1}{H_2}\frac{\partial l_f}{\partial f_2} \right)\Bigg|_{sw} - \left(\frac{1}{H_1} + \frac{1}{H_2} \right)\frac{\partial l_\delta}{\partial \delta_1}\Bigg|_{sw}$$

with boundary conditions:

$$\Delta\rho(t_f) = - \left(\frac{1}{H_1}\frac{\partial \phi_f}{\partial f_1} - \frac{1}{H_2}\frac{\partial \phi_f}{\partial f_2} \right)\Bigg|_{sw\; t_f} \tag{9b}$$

$$\Delta\dot{\rho}(t_f) = \left\{ \frac{1}{H_1}\frac{\partial l_f}{\partial f_1} - \frac{1}{H_2}\frac{\partial l_f}{\partial f_2} + \left(\frac{1}{H_1} + \frac{1}{H_2} \right)\frac{\partial \phi_\delta}{\partial \delta_1} \right\}\Bigg|_{sw\; t_f} \tag{9c}$$

Equation 9 has an intuitive interpretation. The forcing function shows that if the areas care differently about deviations in frequency then this difference, appropriately weighted, causes a difference in price between the two areas. Also, the forcing function shows that concern about the stability limits of the tie line, i.e. the finite capacity of the tie line, causes a difference in price between the two areas.

3.2.4 Average System Price: Define the average system price for generation for the two areas as:

$$\bar{\rho}_G \triangleq \frac{H_1\rho_{G1} + H_2\rho_{G2}}{H_1 + H_2}$$

Using equations A.1efg of the appendix, we obtain:

Lemma 3: The average system price satisfies:

$$\frac{d}{dt}\bar{\rho}_G = \frac{1}{H_1 + H_2}\sum_{i=1}^{2} \left.\frac{\partial l_f}{\partial f_i}\right|_{sw} \tag{10a}$$

with boundary condition

$$\bar{\rho}_G(t_f) = -\frac{1}{H_1 + H_2}\sum_{i=1}^{2} \left.\frac{\partial \phi_f}{\partial f_i}\right|_{\substack{sw\\t_f}} \tag{10b}$$

The above equation says that the overall price is determined by concern about frequency deviations, the inter-area concern of tie line power $l_\delta(.)$, has dropped out. If frequency deviations did not matter ($l_f(\cdot)$ and $\phi_f(\cdot)$ equal to zero) then the overall price would be zero. This makes sense, because if frequency deviations did not matter, then there would be a free energy source: infinite stored kinetic energy of the turbine-generator shaft.

4. APPLICATION OF ECONOMIC THEORY

To apply the results of the previous section to the real world, a naive approach would be to try to estimate the social welfare optimization problem, and then use the price obtained as an open loop control law. Unfortunately, such an approach is not robust to modeling errors. One can construct simple examples where a small error in the estimated optimization yields a price trajectory that causes instability in the frequency deviation.

A more practical approach would be to consider a closed loop control law that exploits the results of the idealized formulation. We apply this idea to a simple example where frequency is controlled both directly by the utility and indirectly via time varying prices to independently owned generation and loads. In the example, we drop the assumptions of perfect information, and of infinite computational power that were made in Section 3.

4.1 Base Case: No Real Time Pricing

For simplicity, consider a single area, with aggregate load $D(t)$, no losses, and with two aggregate power plants: plant #1 owned by the electric utility and plant #2 owned by independent firms. Suppose there is no real time pricing to control frequency deviations. The utility plant has a governor and automatic generation control yielding proportional plus integral feedback, while the independent plant has not installed controls. Suppose there is step increase in demand of .05 pu. Considering a first order equation for the generator, the simple average system frequency model becomes:

$$H\dot{f}(t) = M_1(t) + M_2(t) - D(t) \qquad f(0) = 0 \tag{11a}$$

$$T_1\dot{M}_1(t) = -M_1(t) + u(t) \qquad M_1(0) = 0 \tag{11b}$$

$$u(t) = \frac{1}{R_1}f(t) + c_1\int_0^t f(\tau)d\tau \tag{11c}$$

$$M_2(t) = 0 \tag{11d}$$

$$D(t) = .05 \tag{11e}$$

Let the plant parameters be: $H = 10$ secs, and $T_1 = .1$ secs. We choose the control parameters, R_1 and c_1 to be 0.1 and 5.0 respectively so that the frequency is tightly controlled. Additional decrease in R_1, or increase in c_1 yields increased oscillatory response.

4.2 Real Time Pricing of Generation

As an alternative to the base case, consider a real time price for generation from the independent plants. The utility company would need to know how changes in the price would approximately affect the independent generation. This would largely be obtained by experience, analogous to how changes in bus voltages affect demand. For the sake of discussion, we will make up some numbers. Suppose that the independent plants have 10% of the system capacity, are currently operating at 50% of their capacity, and half of this capacity is under automatic control that is responsive to the varying price. Suppose also that a \$0.005/kWh increase in price would increase the price sensitive generation by 1%. Thus:

$$\frac{\Delta generation}{\Delta price} = \frac{.10pu \cdot 50\% \cdot 50\% \cdot 1\%}{\$0.005/kWh} = .05\frac{pu}{\$/kWh}$$

Assume a delay between a change in price and a resulting change in mechanical power. The delay would be partially due to computation, but mostly due to the plant dynamics. Analogous to the $T_1 = .1$ secs assumed for the utility plant, assume $T_2 = .2$ secs for the independent generators. Thus, equation 11d would change to:

$$T_2\dot{M}_2(t) = -M_2(t) + .05\rho(t) \qquad M_2(0) = 0 \tag{11d'}$$

where $\rho(t)$ is the price deviation in \$/kWh.

To choose the price, recall that the social welfare maximizing price is determined by penalty functions for frequency and tie line deviations, equation 8. Thus, a natural, heuristic choice for the price is a feedback control law of frequency deviation (there being no tie lines in this example of a single area). Furthermore, as a straw man choice, consider a proportional plus integral control law:

$$\rho(t) = -c_2 f(t) - c_3\int_0^t f(\tau)d\tau \tag{13}$$

A key feature of this pricing scheme is that the independent power plants can themselves monitor the frequency deviations and thus no real time signal needs to be sent by the electric utility. This eliminates the problem of how the utility could compute and transmit the price faster than the time scale to be controlled. The proportionality constants, c_2 & c_3, would be set off line as part of the utility rate schedule, and they could be prescribed to vary with the time of day.

With $c_2 = 100$ and $c_3 = 2$, the pricing scheme yields reasonable closed loop response, as plotted in Figures 1, 2, & 3. The major difference from the base case is the elimination of the overshoot of mechanical power of the utility plants. Without pricing, this overshoot would necessarily be present to bring frequency back to its set point. With pricing, this action has shifted to the independent plants. In addition, the frequency deviation is lessened. The price deviation is substantial, to handle the shock of the step change in demand, but then it settles down to a new steady state value reflecting the new steady state operating point. Although the real time pricing has yielded improved response, the improvement is not remarkable. However, the utility power plant has the potential option not to control as tightly (relax R_1, c_1), and to let the independent plant and loads (see next section) take up the slack, yielding additional assistance to the utility. We conclude that a price control law like (13) could play a supportive role. This may be the practical limit to the trend of using prices on ever faster time scales in electric power systems.

4.3 Real Time Pricing of Demand

As a second alternative to the base case, suppose the demand is offered a time varying price. This would constitute economic load shedding. Consumers could elect to receive a time varying price, whose average value would be lower than the regular static price. During moments of excess demand the time varying price would increase. A control device at the loads would automatically monitor the price and, at thresholds pre-set by the customer, would turn on or off the load. Prime candidates for such control would be air conditioning and space heating in office buildings where small deviations in the natural on off cycle would not be noticeable.

For the sake of discussion, consider some hypothetical numbers. Suppose the demand is at 70% system capacity, .7 pu, and 10% of the demand is by customers who have elected real time pricing. Suppose the current price is \$0.10/kWh and a 10% increase in price would cause a 1% decrease in the price sensitive demand. Thus:

$$\frac{\Delta demand}{\Delta price} = \frac{.70pu \cdot 10\% \cdot (-1\%)}{\$\,0.10/kWh \cdot 10\%} = -.07\frac{pu}{\$/kWh}$$

Ignoring the delay of turning the load on and off, equation 11e would change to:

$$D(t) = .05 - .07\rho(t) \qquad (11e')$$

where $\rho(t)$ is the deviation in price.

Consider the same proportional plus integral control law as before, equation 13 with the same gains. The closed loop response is plotted in Figures 1, 2 & 3. Once again the overshoot in mechanical power of the utility plant is eliminated and, in addition, the frequency deviation is lessened. We reach the same conclusion as in Section 4.2.

5. SUMMARY

We have studied the use of prices in the control of frequency and tie line deviations. Under an idealized social welfare formulation, we have shown that the price for generation in each area, as well as the difference in price between areas and a system wide price satisfy differential equations that are driven by penalty functions for frequency and tie line deviations. Dropping the perfect information assumption of the social welfare formulation, we have shown in a simple example how prices, determined by a proportional plus integral feedback control law of frequency deviations, could assist in load-frequency control. This control law solves the problem of computing and transmitting the price on a time scale faster than the dynamics to be controlled. It allows independently owned plants to assist in the control of frequency deviations. Likewise, large commercial loads, such as space conditioning, could shut off, or turn on, in response to sharp deviations in the price, yielding an economic load shedding policy.

We make no conclusions whether real time pricing of system dynamics will eventually prove to be worthwhile in the real world. We do conclude, however, that it has the potential to assist the direct control of the electric utility and should be studied further.

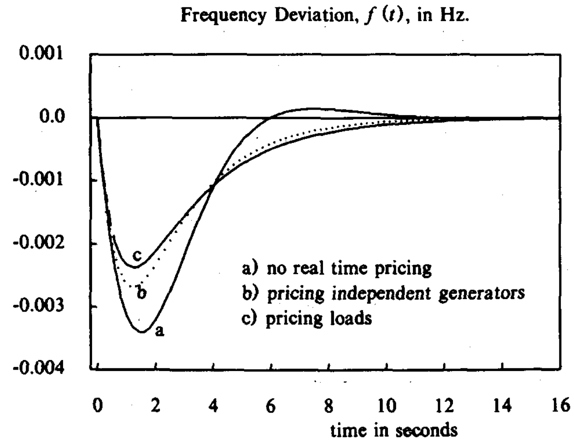

Figure 1.

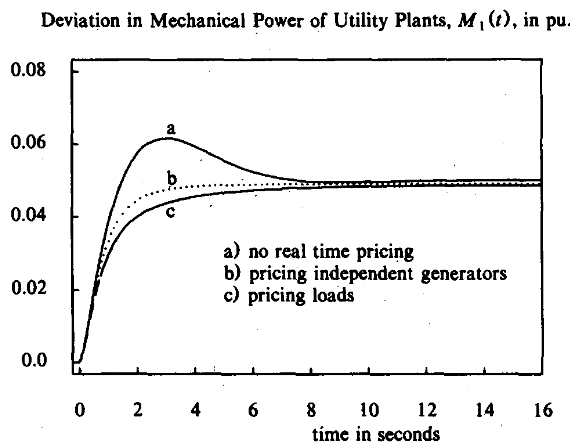

Figure 2.

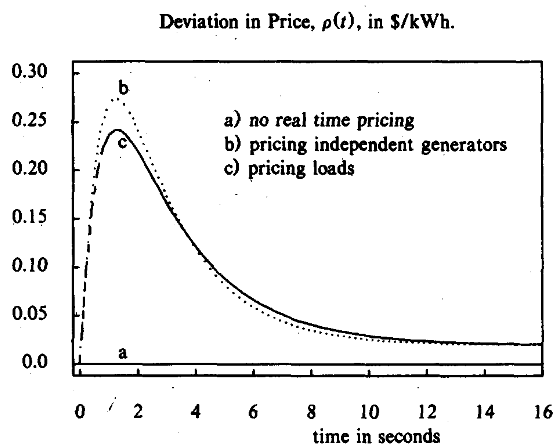

Figure 3.

APPENDIX: PROOF OF THEOREM 1

The idea of the proof is to compare the necessary conditions of the generator with those of the coordinator.

The social welfare Hamiltonian is:

$$H = \sum_{i=1}^{2} \{ \sum_{j=1}^{NL_i} S_{ij}(D_{ij}) - \sum_{j=1}^{NG_i} l_{ij}(x_{ij},u_{ij}) \} - l_f(f) - l_\delta(\delta_1 - \delta_2)$$

$$+ \sum_{i=1}^{2} \{\alpha_i f_i + \beta_i \frac{1}{H_i} (\sum_{j=1}^{NG_i} M_{ij} - G_i) \}$$

$$+ \gamma_1 (G_1 - \sum_{j=1}^{NL_1} D_{1j} - Loss_1(\sum_{j=1}^{NL_1} D_{1j}) - \frac{1}{X}(\delta_1 - \delta_2))$$

$$+ \gamma_2 (G_2 - \sum_{j=1}^{NL_2} D_{2j} - Loss_2(\sum_{j=1}^{NL_2} D_{2j}) + \frac{1}{X}(\delta_1 - \delta_2))$$

$$+ \sum_{i=1}^{2} \sum_{j=1}^{NG_i} \{\lambda_{ij}'(A_{ij}x_{ij} + B_{ij}u_{ij}) + \mu_{ij}(C_{ij}x_{ij} - M_{ij}) \}$$

The social welfare necessary conditions are:

$$\frac{\partial H}{\partial D_{ij}} = 0 = \frac{\partial S_{ij}}{\partial D_{ij}} - \gamma_i \frac{\partial}{\partial D_{ij}} (\sum_{j=1}^{NL_i} D_{ij} + Loss_i(\sum_{j=1}^{NL_i} D_{ij}) \qquad j=1,\ldots,NL_i \quad i=1,2 \tag{A.1a}$$

$$\frac{\partial H}{\partial u_{ij}} = 0 = -\frac{\partial l_{ij}}{\partial u_{ij}} + \lambda_{ij}' B_{ij} \qquad j=1,\ldots,NG_i \quad i=1,2 \tag{A.1b}$$

$$\frac{\partial H}{\partial M_{ij}} = 0 = \frac{\beta_i}{H_i} - \mu_{ij} \tag{A.1c}$$

$$\frac{\partial H}{\partial G_i} = 0 = -\frac{\beta_i}{H_i} + \gamma_i \tag{A.1d}$$

$$\frac{\partial H}{\partial \delta_1} = -\dot{\alpha}_1 = -\frac{\partial}{\partial \delta_1} l_\delta(\delta_1 - \delta_2) - \frac{1}{X}(\gamma_1 - \gamma_2) \tag{A.1e}$$

$$\alpha_1(t_f) = -\frac{\partial}{\partial \delta_1} \phi_\delta(\delta_1 - \delta_2)$$

$$\frac{\partial H}{\partial \delta_2} = -\dot{\alpha}_2 = -\frac{\partial}{\partial \delta_2} l_\delta(\delta_1 - \delta_2) + \frac{1}{X}(\gamma_1 - \gamma_2) \tag{A.1f}$$

$$\alpha_2(t_f) = -\frac{\partial}{\partial \delta_2} \phi_\delta(\delta_1 - \delta_2)$$

$$\frac{\partial H}{\partial f_i} = -\dot{\beta}_i = -\frac{\partial l_f}{\partial f_i} + \alpha_i \tag{A.1g}$$

$$\beta_i(t_f) = -\left.\frac{\partial \phi_f}{\partial f_j}\right|_{tf}$$

$$\frac{\partial H}{\partial x_{ij}} = -\dot{\lambda}_{ij}' = -\frac{\partial l_{ij}}{\partial x_{ij}} + \lambda_{ij}' A_{ij} + \mu_{ij} C_{ij} \tag{A.1h}$$

$$\lambda_{ij}(t_f) = \left.\frac{\partial \phi_{ij}}{\partial x_{ij}}\right|_{tf}$$

and the system constraints, equation 1.

Likewise, generator ij's Hamiltonian is:

$$H = \rho_{G_{ij}} \cdot G_{ij} - l_{ij}(x_{ij},u_{ij}) + \lambda_{ij}'(A_{ij}x_{ij} + B_{ij}u_{ij}) + \mu_{ij}(C_{ij}x_{ij} - M_{ij})$$

and using the individual swing equation: $H_{ij}\dot{f}_i = G_{ij} - M_{ij}$, we can substitute out G_{ij}.

$$H = \rho_{G_{ij}} \cdot (M_{ij} + H_{ij}\dot{f}_i) - l_{ij}(x_{ij},u_{ij}) + \lambda_{ij}'(A_{ij}x_{ij} + B_{ij}u_{ij}) + \mu_{ij}(C_{ij}x_{ij} - M_{ij})$$

where generator ij treats $\rho_{G_{ij}}$ and $H_{ij}\dot{f}_i$ as exogenous, the latter stemming from generator ij's viewpoint that frequency deviations are exogenous.

Generator ij's necessary conditions are:

$$\frac{\partial H}{\partial u_{ij}} = 0 = -\frac{\partial l_{ij}}{\partial u_{ij}} + \lambda_{ij}' B_{ij} \tag{A.2a}$$

$$\frac{\partial H}{\partial M_{ij}} = 0 = \rho_{G_{ij}} - \mu_{ij} \tag{A.2b}$$

$$\frac{\partial H}{\partial x_{ij}} = -\dot{\lambda}_{ij}' = -\frac{\partial l_{ij}}{\partial x_{ij}} + \lambda_{ij}' A_{ij} + \mu_{ij} C_{ij} \tag{A.2c}$$

$$\lambda_{ij}(t_f) = \left.\frac{\partial \phi_{ij}}{\partial x_{ij}}\right|_{tf}$$

$$\frac{\partial H}{\partial \lambda_{ij}} = \dot{x}_{ij} = A_{ij}x_{ij} + B_{ij}u_{ij} \qquad x_{ij}(t_o) \text{ given} \tag{A.2d}$$

$$\frac{\partial H}{\partial \mu_{ij}} = 0 = C_{ij}x_{ij} - M_{ij} \tag{A.2e}$$

If the price, $\rho_{G_{ij}}$, is chosen to be: $\rho_{G_{ij}}(t) \triangleq \beta_i(t)/H_i$ then the generator's necessary conditions, A.2, match social welfare's conditions A.1b,c,h. Thus the solution to social welfare's necessary conditions, u_{ij}^{sw}, x_{ij}^{sw}, λ_{ij}^{sw}, μ_{ij}^{sw}, β_{ij}^{sw}, satisfies the necessary conditions of the generator. Thus, by the assumed uniqueness of the solution to the necessary conditions, the optimal control of the generator equals social welfare's value, u_{ij}^{sw}. Thus, if

$$\rho_{G_{ij}}(t) \triangleq \left.\frac{\beta_i(t)}{H_i}\right|_{sw}$$

then the optimal control of generator ij equals $u_{ij}^{sw}(t)$.

Furthermore, from the social welfare necessary conditions A.1cd, we have:

$$\left.\mu_{ij}(t)\right|_{sw} = \left.\gamma_i(t)\right|_{sw} = \left.\frac{\beta_i(t)}{H_i}\right|_{sw} \qquad \text{(Q.E.D.)}$$

GLOSSARY OF SYMBOLS

β_i is the costate variable associated with the swing equations, (1b).

D_{ij} is the real power demand of the j^{th} load in area i.

D_i is the real power generated in area i; $D_i = \sum_{j=1}^{NL_i} D_{ij}$

δ_i is the average mechanical angle of the rotors of the generators in area i, which is assumed equal to the average electrical phase angle of the voltage of the aggregate bus of area i. $\delta =$ the vector (δ_1, δ_2).

f_i is the average system frequency deviation from the set point in area i. $f =$ the vector (f_1, f_2).

G_{ij} is the real power generated by the j^{th} generator in area i.

G_i is the real power generated in area i; $G_i = \sum_{j=1}^{NG_i} G_{ij}$

γ_i is the Lagrange multiplier associated with the energy balance equations, (1cd).

H_i is the sum of the inertial constants of the generators in area i.

$(\cdot)^{ind}$ means evaluated at the individual generator or load's optimal value.

$l_\delta(\delta)$ is the penalty function for tie line deviations.

$l_f(f)$ is the penalty function for frequency deviations.

$l_{ij}(\cdot)$ is the instantaneous cost to generator j in area i.

$Loss_i$ is the real power losses in area i.

M_{ij} is the mechanical power to the turbine-generator shaft of generator j in area i.

μ_{ij} is the Lagrange multiplier associated with generator ij's output function, (1g).

NG_i = the number of generators in area i, i=1,2.

NL_i = the number of loads in area i, i=1,2.

$\phi_{ij}(\cdot)$ is the "excess energy" left in the boiler turbine at the terminal time.

$\phi_\delta(\cdot)$ is the penalty function for tie line deviations at the terminal time.

$\phi_f(\cdot)$ is the penalty function for frequency deviations at the terminal time.

$\rho_{G_{ij}}$ is the price to generator j in area i.

$\rho_{D_{ij}}$ is the price to load j in area i.

$S_{ij}(D_{ij})$ is the satisfaction index of load j in area i.

$(\cdot)^{sw}$ means evaluated at the social welfare optimal value.

Tie is the tie line power between areas 1 and 2. Without loss of generality, *Tie* is positive for power flowing out of area 1.

X is the effective line inductance in the tie lines between the two areas.

x_{ij} is the state of the boiler-turbine of generator j in area i.

u_{ij} is governor control valve position of generator j in area i.

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