6.003 Signals and Systems DUE: 9/12/2003

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Problem Set 1

Problem 1

(a)
$(\sqrt{3}+j)^5 e^{-j\frac{\pi}{3}} = (2e^{j\frac{\pi}{6}})^5 e^{-j\frac{\pi}{3}} = 2e^{j\frac{5\pi}{6}} e^{-j\frac{\pi}{3}} = 2e^{j\frac{\pi}{2}}$.
$|2e^{j\frac{\pi}{2}}| = 2$
$\theta = \frac{\pi}{2}$
See Figure 3

Problem 2

(a)
$x\left(1-\frac{t}{3}\right)$: Shift left 1, flip, expand by 3.
See Figure 4

(b)
$x\,(t-2)\,[\delta(t-\frac{1}{2}) + u(3-t)]$
$x(t-2)$: Shift right 2
$\delta(t-\frac{1}{2})$: Shift right $\frac{1}{2}$
$u(3-t)$: Shift left 3, flip
See Figure 5

Problem 3

(a)
$x[2-n]$: Shift left 2, flip.
See Figure 6
(b)
$x[2n+1]$: Shift left 1, compress by 2.
See Figure 7

Problem 4

Even part:
See Figure 8
Odd part:
See Figure 9

Problem 5

(a)
$x(t) = (\sin(4t-1))^2 = (x_1(t))^2$
We see that $x_1(t)$ oscillates equally above and below the x-axis. Since we sqaure $x_1(t)$, we flip the negative values above the x-axis. So the period will be cut in half. Now, calculate the period of $x_1(t)$.
$x_1(t+T) = \sin(4(t+T)-1) = \sin(4t-1+4T)$
For this to be periodic, we need:
$4T = 2\pi k$
for $k = 0, \pm1, \pm2, \ldots$. Using our above analysis that the period will be cut in half, $T_0 = \frac{1}{2}\frac{\pi}{2} = \frac{\pi}{4}$.
(b)
Let us use the complex exponential:
$x_1[n] = e^{j(4n+\frac{\pi}{4})}$.
So,
$x_1[n+N] = e^{j(4(n+N)+\frac{\pi}{4})} = e^{j4N}e^{j(4n+\frac{\pi}{4})}$.
This is periodic if:
$e^{j4N} = 1 \Leftrightarrow 4N = 2\pi k \Leftrightarrow N = \frac{\pi}{2}k$
for $k = 1, \pm1, \pm2, \ldots$. However, notice that N can never be an integer. So, $x[n]$ cannot be periodic.
(c)
Let us use complex exponentials:
$x_1[n] = (-1)^n e^{j\frac{2\pi n}{7}} = (-1)^n x_2[n]$.
Let us first look at $x_2[n]$:
$x_2[n+N] = e^{j\frac{2\pi(n+N)}{7}} = e^{j\frac{2\pi N}{7}}e^{j\frac{2\pi n}{7}}$.
This is periodic if:
$e^{j\frac{2\pi(n+N)}{7}} = 1 \Leftrightarrow \frac{2\pi N}{7} = 2\pi k \Leftrightarrow N = 7k$
for $k = 1, \pm1, \pm2, \ldots$. We can easily observe that $(-1)^n$ has period of 2. So, $N_0 = 14$.

Problem 6

(a)

1. Memoryless? No—y depends on future and past values of x.

2. TI? No
   $x_2(t) = x_1(t - t_0)$
   $y_2(t) = x_2(t+3) - x_2(1-t) = x_1(t+3-t_0) - x_1(1-t-t_0)$
   $y_1(t-t_0) = x_1(t-t_0+3) - x_1(1-(t-t_0))$

3. Linear? Yes
   $x_3(t) = ax_1(t) + bx_2(t)$
   $y_3(t) = x_3(t+3) - x_3(1-t) = ax_1(t+3) + bx_2(t+3) - ax_1(1-t) - bx_2(1-t) = a(x_1(t+3) - x_1(1-t)) + b(x_2(t+3) - x_2(1-t)) = ay_1(t) + by_2(t)$

4. Causal? No. For $t = -1$, $y(-1)$ depends on $x(2)$.

5. Stable? Yes. Suppose $|x(t)| \leq B \; \forall t$. Then $|y(t)| \leq 2B$.

(b)

1. Memoryless? Yes, since y depends only on current values of x.

2. TI? No. Let:

   $x_1[n] = \delta[n] \rightarrow y_1[n] = \delta[n]$
   $x_2[n] = \delta[n+1] \rightarrow y_2[n] = -\delta[n+1]$

3. Linear? No. Let:

   $x_1[n] = \delta[n]$
   $x_2[n] = (-1)x_1[n] = -\delta[n] \rightarrow y_2[n] = -2\delta[n] \neq (-1)y_1[n] = -\delta[n]$

4. Causal? Yes since it is memoryless.

5. Stable? Yes. If $|x[n]| \leq B$, $|y[n]| \leq 2B$.

(c)

1. Memoryless? No since $y[n]$ depends on future values of $x[n]$.

2. TI? Yes. $x_2[n] = x_1[n-n_0] \rightarrow y_2[n] = \sum_{k=n}^{\infty} x_2[k] = \sum_{k=n}^{\infty} x_1[k-n_0] = \sum_{k=n-n_0}^{\infty} x_1[k]$
   $y_1[n-n_0] = \sum_{k=n-n_0}^{\infty} x_1[k]$.

3. Linear? Yes. $x_3[n] = ax_1[n] + bx_2[n] \rightarrow y_3[n] = \sum_{k=n}^{\infty} x_3[k] = \sum_{k=n}^{\infty} ax_1[k] + bx_2[k] = a\sum_{k=n}^{\infty} x_1[k] + b\sum_{k=n}^{\infty} x_2[k] = ay_1[n] + by_2[n]$.

4. Causal? No, see memoryless answer.

5. Stable? No because we sum all future samples of x, which imples that y is unbounded.

Problem 7

$x_2(t) = -x_1(t-1) - 2x_1(t-2)$
$y_2(t) = -y_1(t-1) - 2y_1(t-2)$
See Figure 10

Problem 8

```
% (a)
nx = -3:7;
x = [0 0 0 2 0 1 -1 3 0 0 0];
figure(1);
stem(nx,x);
title('x');

% (b)
y1 = x;
y2 = x;
y3 = x;
y4 = x;
ny1 = -1:9;
ny2 = -4:6;
ny3 = 3:-1:-7;
ny4 = 4:-1:-6;

% (c)
% y1 - Delayed by 2.
% y2 - Advanced by 1.
% y3 - Flipped.
% y4 - Flipped and then delayed by 1.
figure(2);
subplot(2,2,1);
stem(ny1,y1);
title('y1');
subplot(2,2,2);
stem(ny2,y2);
title('y2');
subplot(2,2,3);
stem(ny3,y3);
title('y3');
subplot(2,2,4);
stem(ny4,y4);
title('y4');
```

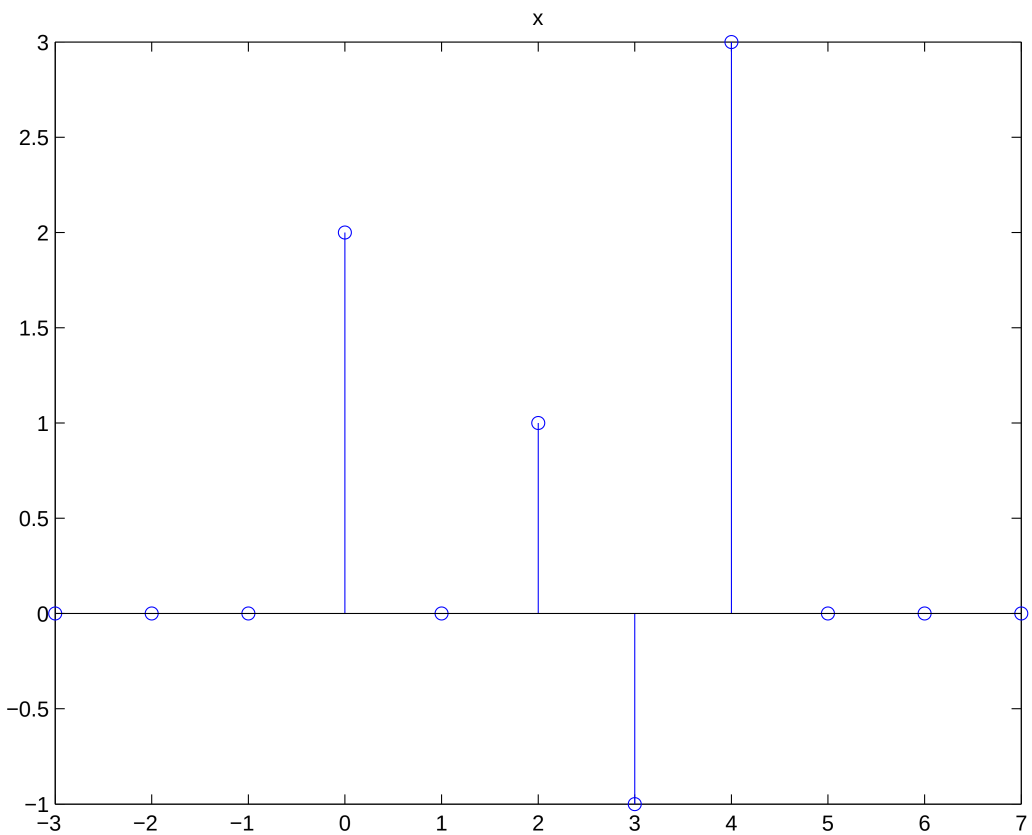

Figure 1: Figure for part 8(a).

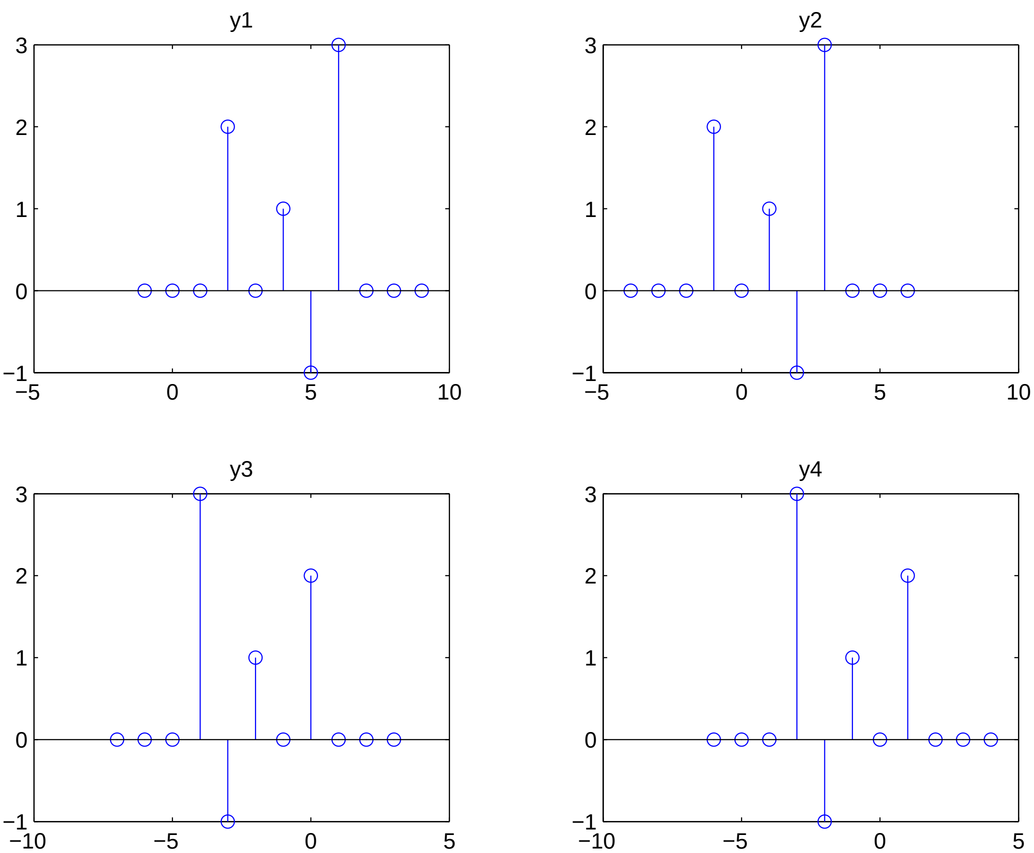

Figure 2: Figure for part 8(c).