Computability and Complexity
Homework 2 (due 2PM, 09/25/01)

Problem 1. *Let G be a context-free grammar and let $k > 0$. Let $L_k(G)$ denote the set of all strings over Σ, that have a derivation in G with at most k steps. What is $L_5(G)$ for the following grammar?*

$$\begin{array}{rcl} V & = & \{S\} \\ \Sigma & = & \{(,\ )\ \} \\ R & = & \{S \to e \\ & & \ \ S \to SS \\ & & \ \ S \to (S)\} \end{array}$$

Problem 2. *Using the method described in class, construct a pushdown automaton which accepts the language generated by the grammar from problem #1.*

Problem 3. *A class $\mathcal{N}$ of languages is said to be closed under an operation $\mathcal{O}$, if for every two languages $L_1, L_2 \in \mathcal{N}$, the language obtained by applying $\mathcal{O}$ to L_1 and L_2 is also in class $\mathcal{N}$. (Similarly, for the case of an operation which acts on individual languages.)*

Show that the class of languages accepted by pushdown automata is closed under the operations union, concatenation, and star.

Problem 4. *Given a pushdown automaton G, define* **the language $L^e(G)$ accepted by G by empty store** *as follows:*

$$L^e(G) = \{w \in \Sigma^* : \textit{after } w \textit{ is read off the input tape, the stack is empty.}\}$$

Show that there is a pushdown automaton M' such that $L^e(M) = L(M)$.

Problem 5. *A context-free grammar is called* **linear**, *if no rule has its right-hand side a string with more than one variable. A pushdown automaton $(Q, \Sigma, \Gamma, \Delta, q_0, F)$ is called* **single-turn**, *if for any sequence of transitions leading from the starting state with the empty stack, the height of the stack increases during a number of steps, achieves the maximum, and then decreases with every next step (the decreasing stage can be empty). Show that a language is generated by a linear context-free grammar if and only if it is accepted by a single-turn pushdown automaton.*

Problem 6. *Use the pumping lemma to show that the following languages are not context-free:*

1. $L = \{w_1 \# w_2 : w_1 \textit{ is a substring of } w_2, \textit{ where } w_1, w_2 \in \{a, b\}^*\}$;
2. $L = \{w_1 \# w_2 \# w_3 : w_1, w_2, w_3 \in \{a, b\}^* \textit{ and either } w_1 = w_2, \textit{ or } w_1 = w_3, \textit{ or } w_2 = w_3\}$;

Three out of the six problems will be graded. For a good grade, the solutions must be presented neatly, with details and proofs when appropriate. You can discuss the problems with other students, but write the solutions by yourself. A paper given to me after the beginning of class will lose some points. GOOD LUCK