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Project 1

Part 1 & Part 2

Project 1 : Part 1

Task 1: For Layout 1, U = { T1, T2, R1, R2, R3, R4, S1, S2 },
write the formulas 1 through 6 in a file using the homework language.

Formula 1 (Task 1): expressed in the homework language.
This axiom represents the positive connections in the rail layout. In other words, these are the connections that do exist.

```
connects(R1, R2) ^ connects(R2, R3) ^
connects(R3, R4) ^ connects(R4, R1) ^
```

Formula 2 (Task 1) : expressed in the homework language.
These axioms represent the negative connections in the rail layout.
In other words, these are the connections that do not exist.
Basically, the above connections are the only connections that exist, and a rail is not connected to itself.

```
~connects(R1, R3) ^ ~connects(R1, R4) ^
~connects(R2, R4) ^ ~connects(R2, R1) ^
~connects(R3, R1) ^ ~connects(R3, R2) ^
~connects(R4, R2) ^ ~connects(R4, R3) ^
(all r ~connects(r, r)) ^
```

Formula 3 (Task 1): expressed in the homework language.
A situation is safe if and only if no 2 trains occupy the same rail.

```
(
      all s safe(s) <-> (
            all r all t1 all t2 (
                  on(t1,r,s) ^ on(t2,r,s) -> Equals(t1,t2)
            )
      )
)^
```

Formula 4 (Task 1) : expressed in the homework language.
This asserts that situation 1 is safe.

```
(safe(S1)) ^
```

Formula 5 (Task 1) : expressed in the homework language.
Dynamics: Only move to connected rails.

```
(
      all t all r2 (
            on(t,r2,S2) -> exists r1 (
                  on(t,r1,S1) ^ connects(r1,r2) ^ legal(t,r1,r2)
            )
      )
)^
```

Formula 6 (Task 1) : expressed in the homework language.

Definition of legal: A move is legal if there is no train on r2 in situation S1.

```
(
      all t1 all r1 all r2 (
            legal(t1,r1,r2) <-> (
                  ~(
                        exists t2 (on(t2,r2,S1))
                  )
            )
      )
)^
```

Task 2: You'll now need to add two more axioms to your file, specifying that:
2. The two new axioms, in the homework language.

The following axioms are generic because they do not mention the trains by name. They would therefore work for any universe of s, r, and t.

Axiom 1 (Task 2) : Every train is always on some rail.

```
(all t all s exists r on(t,r,s))^
```

The axiom essentially says: for any give t and s combination that train is on some rail.

Axiom 2 (Task 2) : No train is on two rails at the same time.

```
(
      all t all s all r1 (
            on(t,r1,s) -> (
                  all r2 (
                        on(t,r2,s) -> Equals(r1,r2)
                  )
            )
      )
)^
```

This axiom works because, we are specifying that if a train is on a given rail (in a given situation) then, if the train is on another rail, that rail is (equal to) the first rail.

Task 3: Find, print out and describe a satisfying assignment for the whole specification, using the provided code.

```
task3.txt.out
```

```
connects_R1_R1=false
connects_R1_R2=true
connects_R1_R3=false
connects_R1_R4=false
connects_R2_R1=false
connects_R2_R2=false
connects_R2_R3=true
connects_R2_R4=false
connects_R3_R1=false
connects_R3_R2=false
connects_R3_R3=false
connects_R3_R4=true
connects_R4_R1=true
connects_R4_R2=false
connects_R4_R3=false
connects_R4_R4=false
legal_T1_R1_R1=false
legal_T1_R1_R2=true
legal_T1_R1_R3=false
legal_T1_R1_R4=true
legal_T1_R2_R1=false
legal_T1_R2_R2=true
legal_T1_R2_R3=false
legal_T1_R2_R4=true
legal_T1_R3_R1=false
legal_T1_R3_R2=true
legal_T1_R3_R3=false
legal_T1_R3_R4=true
legal_T1_R4_R1=false
legal_T1_R4_R2=true
legal_T1_R4_R3=false
legal_T1_R4_R4=true
legal_T2_R1_R1=false
legal_T2_R1_R2=true
legal_T2_R1_R3=false
legal_T2_R1_R4=true
legal_T2_R2_R1=false
legal_T2_R2_R2=true
legal_T2_R2_R3=false
legal_T2_R2_R4=true
legal_T2_R3_R1=false
legal_T2_R3_R2=true
legal_T2_R3_R3=false
legal_T2_R3_R4=true
legal_T2_R4_R1=false
legal_T2_R4_R2=true
legal_T2_R4_R3=false
legal_T2_R4_R4=true
on_T1_R1_S1=true
on_T1_R1_S2=false
on_T1_R2_S1=false
on_T1_R2_S2=true
on_T1_R3_S1=false
on_T1_R3_S2=false
on_T1_R4_S1=false
on_T1_R4_S2=false
on_T2_R1_S1=false
on_T2_R1_S2=false
on_T2_R2_S1=false
on_T2_R2_S2=false
on_T2_R3_S1=true
on_T2_R3_S2=false
on_T2_R4_S1=false
on_T2_R4_S2=true
safe_S1=true
safe_S2=true
```

"on" Assignments:

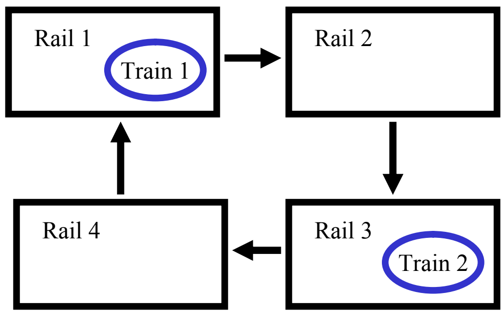
Where are the trains in situation 1 and situation2?

In situation 1,

train 1 is on rail 1 and
train 2 is on rail 3.

```
on_T1_R1_S1=true
on_T2_R3_S1=true
```

Diagram : Layout 1 : Situation 1

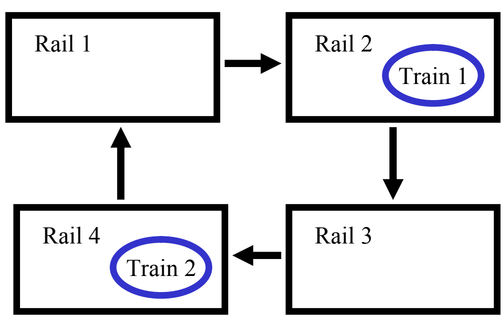


In situation 2,

train 1 is on rail 2 and
train 2 is on rail 4.

```
on_T1_R2_S2=true
on_T2_R4_S2=true
```

Diagram : Layout 1 : Situation 2



The location of the trains is reasonable.

The “dynamics” axiom (Formula 5) asserts that if a train is on a given rail in situation 2, then it was on a connecting rail in situation 1. The “connects” axioms assert that an axiom is not connected to itself. Therefore, the train must move (be on a different rail) from situation 1 to situation 2.

The legal assertion says that a train can only move to an empty rail. Therefore, in order for both trains to be able to move, they must be spaced out, with one rail in separating them. That way, they will both have an empty rail ahead of them.

The location of the trains satisfies this set of assertions and is therefore reasonable.

All of the other “on” assignments are false, as they should be.

```
on_T1_R1_S2=false
on_T1_R2_S1=false
on_T1_R3_S1=false
on_T1_R3_S2=false
on_T1_R4_S1=false
on_T1_R4_S2=false
on_T2_R1_S1=false
on_T2_R1_S2=false
on_T2_R2_S1=false
on_T2_R2_S2=false
on_T2_R3_S2=false
on_T2_R4_S1=false
```

Legal Assignments:

The definition of legal does not take into consideration whether or not the location of the train for which we are defining legality. It only considers the possible destination that we are considering. Therefore, it is legal for any train to move to an empty rail.

The definition of legal only pertains to situation 1.

It is legal for train 1 to move to rail 2, from any rail because rail 2 is not occupied it situation 1.

```
legal_T1_R1_R2=true
legal_T1_R2_R2=true
legal_T1_R3_R2=true
legal_T1_R4_R2=true
```

It is legal for train 2 to move to rail 2, from any rail because rail 2 is not occupied it situation 1.

```
legal_T2_R1_R2=true
legal_T2_R2_R2=true
legal_T2_R3_R2=true
legal_T2_R4_R2=true
```

It is legal for train 1 to move to rail 4, from any rail because rail 4 is not occupied it situation 1.

```
legal_T1_R1_R4=true
legal_T1_R2_R4=true
legal_T1_R3_R4=true
legal_T1_R4_R4=true
```

It is legal for train 2 to move to rail 4, from any rail because rail 4 is not occupied it situation 1.

```
legal_T2_R1_R4=true
legal_T2_R2_R4=true
legal_T2_R3_R4=true
legal_T2_R4_R4=true
```

It is not legal for any train to move to rail 1 or rail 3 because those rails are occupied it situation 1.

```
legal_T1_R1_R1=false
legal_T1_R1_R3=false
legal_T1_R2_R1=false
legal_T1_R2_R3=false
legal_T1_R3_R1=false
legal_T1_R3_R3=false
legal_T1_R4_R1=false
legal_T1_R4_R3=false
legal_T2_R1_R1=false
legal_T2_R1_R3=false
legal_T2_R2_R1=false
legal_T2_R2_R3=false
legal_T2_R3_R1=false
legal_T2_R3_R3=false
legal_T2_R4_R1=false
legal_T2_R4_R3=false
```

These 4 connections are true:

```
connects_R1_R2=true
connects_R2_R3=true
connects_R3_R4=true
connects_R4_R1=true
```

All of the other connections are false:

```
connects_R1_R1=false
connects_R1_R3=false
connects_R1_R4=false
connects_R2_R1=false
connects_R2_R2=false
connects_R2_R4=false
connects_R3_R1=false
connects_R3_R2=false
connects_R3_R3=false
connects_R4_R2=false
connects_R4_R3=false
connects_R4_R4=false
```

Safe Assignments:

The following are the safe assignments. Situation 1 is safe because we explicitly assigned it to be. Situation 2 is safe because there or no 2 trains on the same rail. While, the sat solver explicitly enforced that situation 1 would be safe, situation 2 is safe because of the rules that we axiomized. In other words, our rule cause situation 2 to be safe, assuming that situation 1 was safe. Task 4 proves this to be so.

```
safe_S1=true
safe_S2=true
```

Task 4: Now, add the assertion that there's a possible train wreck, ¬safe(S2), and use the code to show that there are no possible crashes in this domain. Would it have been okay to use WalkSAT for this job? Why or why not?

Prove no wrecks:

Method:
In order to show that there can be no crashes, I included the following assertion into the axiom set:

Assertion (Task 4) : There is a possible train wreck in situation 2.

```
~safe(S2)^
```

I then ran the sat solver (DPLL) on the axiom set. The result was null, which means that the sat solver is unable to find a satisfying solution of the previous rule set and the new axiom. Therefore, it is not possible for a crash to occur given the rule set.

To clarify that point; the new assertion says that there is a wreck in situation 2. However, the rule set that I created makes sure that there is no wreck in situation 2. Therefore, finding a satisfying assignment is impossible. Since DPLL will search the entire assignment space (complete), we know that there is no satisfying assignment if DPLL returns `null`.

Output:

```
task4.txt.out
```

```
null
```

Why not use WalkSAT?

It would not have been OK to use WalkSAT for this task because WalkSAT does not check every possible path to a solution and therefore cannot guarantee that a solution does not exist, even when it does not find one.

Likewise, WalkSAT is not guaranteed to find a solution if one exists. WalkSAT's advantage is that it is fast at finding a satisfying solution, in certain situations, if one exists. WalkSAT is fast if a satisfying solution correlates well with it's objective function (the number of satisfied clauses). If that is the case, then WalkSAT's downward search path towards more satisfied clauses will lead to a satisfying assignment.

WalkSAT can be initialized and terminated in a variety of ways. Some result in a more complete and robust search of the space. However, none of them guarantee a complete search of the space.

Task 5: Modify the definition of legal to include allowing the train to stay on the rail it's currently on. You may also need to modify the "connects" relation. Is this domain still safe? Demonstrate using the java code.

New Definition of Legal:

It is legal for a given train to move to a given rail if there is no train on the given rail, in situation S1, other than itself (the given train).

```
(
 all t1 all r1 all r2 (
      legal(t1,r1,r2) <-> (
            ~(
                  exists t2 (
                        on(t2,r2,S1) ^ ~Equals(t1,t2)
                  )
            )
      )
 )
)^
```

Connects Relations:

I modified the "connects" relation to allow all trains to be connected to themselves. First I removed the negation of this relation (all r ~connects(r,r)) from Formula 2.
Then I added this relation (all r connects(r,r)) to Formula 1. The result is as follows:

```
connects(R1, R2) ^ connects(R2, R3) ^
connects(R3, R4) ^ connects(R4, R1) ^
(all r connects(r,r)) ^

~connects(R1, R3) ^ ~connects(R1, R4) ^
~connects(R2, R4) ^ ~connects(R2, R1) ^
~connects(R3, R1) ^ ~connects(R3, R2) ^
~connects(R4, R2) ^ ~connects(R4, R3) ^
```

Is the domain is still safe?

Yes, there are many possible safe configurations.

How you used the code to show safeness?

I put the assertion that situation 2 is unsafe into the axiom set to see if it could generate an unsafe situation for situation 2.

Assertion : There is a train wreck in situation 2.

```
~safe(S2)^
```

I ran the DPLL sat solver with the "`~safe(S2)^`" assertion, and it returned "null," indicating that no solution was found. Since DPL is guaranteed to find a satisfying solution if one exists, then it is not possible to produce an unsafe condition in situation 2, given the axiom set.

Output:

```
task5nowrecks.txt.out

null
```

Finding a safe situation:

I then ran the sat solver without the "`~safe(S2)^`" assertion, and it found a safe situation where the trains were on rails 1 and 4 in situation 1, and did not move in situation 2. This makes sense; because it is now OK for 2 trains to be on consecutive rails due to the fact that it is legal for a train to "move" to a rail that it is currently on (AKA stay on the same rail) and all rails are connected to themselves. These to notions, together satisfy the dynamics axiom.

"on" assignments:

```
on_T1_R1_S1=true
on_T1_R1_S2=true
on_T2_R4_S1=true
on_T2_R4_S2=true
```

The following connects assignments reflect the new topology.

These reflect the original connections:

```
connects_R1_R2=true
connects_R2_R3=true
connects_R3_R4=true
connects_R4_R1=true
```

These reflect the new connections that a rail is connected to itself:

```
connects_R1_R1=true
connects_R2_R2=true
connects_R3_R3=true
connects_R4_R4=true
```

All of the other connections are false:

```
connects_R1_R3=false
connects_R1_R4=false
connects_R2_R1=false
connects_R2_R4=false
connects_R3_R1=false
connects_R3_R2=false
connects_R4_R2=false
connects_R4_R3=false
```

The following "legal" assignments show that it is legal for train 1 to move to any rail other than rail 4, which is the location of train 2 (in situation 1).

```
legal_T1_R1_R1=true
legal_T1_R1_R2=true
legal_T1_R1_R3=true
legal_T1_R1_R4=false
legal_T1_R2_R1=true
legal_T1_R2_R2=true
legal_T1_R2_R3=true
legal_T1_R2_R4=false
legal_T1_R3_R1=true
legal_T1_R3_R2=true
legal_T1_R3_R3=true
legal_T1_R3_R4=false
legal_T1_R4_R1=true
legal_T1_R4_R2=true
legal_T1_R4_R3=true
legal_T1_R4_R4=false
```

The following "legal" assignments show that it is legal for train 2 to move to any rail other than rail 1, which is the location of train 1 (in situation 1).

```
legal_T2_R1_R1=false
legal_T2_R1_R2=true
legal_T2_R1_R3=true
legal_T2_R1_R4=true
legal_T2_R2_R1=false
legal_T2_R2_R2=true
legal_T2_R2_R3=true
legal_T2_R2_R4=true
legal_T2_R3_R1=false
legal_T2_R3_R2=true
legal_T2_R3_R3=true
legal_T2_R3_R4=true
legal_T2_R4_R1=false
legal_T2_R4_R2=true
legal_T2_R4_R3=true
legal_T2_R4_R4=true
```

Safe assignment:

```
task5safe.txt.out

connects_R1_R1=true
connects_R1_R2=true
connects_R1_R3=false
connects_R1_R4=false
connects_R2_R1=false
connects_R2_R2=true
connects_R2_R3=true
connects_R2_R4=false
connects_R3_R1=false
connects_R3_R2=false
connects_R3_R3=true
connects_R3_R4=true
connects_R4_R1=true
connects_R4_R2=false
connects_R4_R3=false
connects_R4_R4=true
legal_T1_R1_R1=true
legal_T1_R1_R2=true
legal_T1_R1_R3=true
legal_T1_R1_R4=false
legal_T1_R2_R1=true
legal_T1_R2_R2=true
legal_T1_R2_R3=true
legal_T1_R2_R4=false
legal_T1_R3_R1=true
legal_T1_R3_R2=true
legal_T1_R3_R3=true
legal_T1_R3_R4=false
legal_T1_R4_R1=true
legal_T1_R4_R2=true
legal_T1_R4_R3=true
legal_T1_R4_R4=false
legal_T2_R1_R1=false
legal_T2_R1_R2=true
legal_T2_R1_R3=true
legal_T2_R1_R4=true
legal_T2_R2_R1=false
legal_T2_R2_R2=true
legal_T2_R2_R3=true
legal_T2_R2_R4=true
legal_T2_R3_R1=false
legal_T2_R3_R2=true
legal_T2_R3_R3=true
legal_T2_R3_R4=true
legal_T2_R4_R1=false
legal_T2_R4_R2=true
legal_T2_R4_R3=true
legal_T2_R4_R4=true
on_T1_R1_S1=true
on_T1_R1_S2=true
on_T1_R2_S1=false
on_T1_R2_S2=false
on_T1_R3_S1=false
on_T1_R3_S2=false
on_T1_R4_S1=false
on_T1_R4_S2=false
on_T2_R1_S1=false
on_T2_R1_S2=false
on_T2_R2_S1=false
on_T2_R2_S2=false
on_T2_R3_S1=false
on_T2_R3_S2=false
on_T2_R4_S1=true
on_T2_R4_S2=true
safe_S1=true
safe_S2=true
```

Task 6: Consider Layout 2, shown in figure 4. Demonstrate that it is unsafe, given your definition of legal from item 5.

The following is the homework language description of layout 2:

```
 connects(R1, R1) ^  connects(R1, R2) ^ ~connects(R1, R3) ^  connects(R1, R4) ^
 connects(R2, R1) ^  connects(R2, R2) ^ ~connects(R2, R3) ^ ~connects(R2, R4) ^
~connects(R3, R1) ^  connects(R3, R2) ^  connects(R3, R3) ^  connects(R3, R4) ^
~connects(R4, R1) ^ ~connects(R4, R2) ^  connects(R4, R3) ^  connects(R4, R4) ^
```

How do you show it is not safe?

Preliminary Test:

I tested this first without asserting that there will be a wreck in S2.
It generated a safe scenario where T1 started on R1 and T2 started on R4. Both trains stayed were they were. This is a safe assignment, however, this does not tell us if it is possible to have an unsafe assignment.

Conclusive Test:

I then added the assertion that there is a train wreck in S2.

```
~safe(S2)^
```

I ran the DPLL sat solver on it and it found an unsafe situation is S2.

In this unsafe assignment, T1 started on R1, T2 started on R3, and both trains proceeded to R2, producing a train wreck. This proves that the situation is not guaranteed to be safe.

```
on_T1_R1_S1=true
on_T2_R3_S1=true

on_T1_R2_S2=true
on_T2_R2_S2=true
```

It is legal for both trains to move to rail 2, since there is no train on that rail in situation 1.

```
legal_T1_R1_R2=true
legal_T2_R3_R2=true
```

Therefore, an unsafe situation was created (situation 2).

```
safe_S1=true
safe_S2=false
```

The following is the unsafe assignment that was produced, which demonstrates that this setup is not guaranteed to be safe:

```
task6_wreck.txt.out

connects_R1_R2=true
connects_R1_R3=false
connects_R1_R4=true
connects_R2_R1=true
connects_R2_R2=true
connects_R2_R3=false
connects_R2_R4=false
connects_R3_R1=false
connects_R3_R2=true
connects_R3_R3=true
connects_R3_R4=true
connects_R4_R1=false
connects_R4_R2=false
connects_R4_R3=true
connects_R4_R4=true
legal_T1_R1_R1=true
legal_T1_R1_R2=true
legal_T1_R1_R3=false
legal_T1_R1_R4=true
legal_T1_R2_R1=true
legal_T1_R2_R2=true
legal_T1_R2_R3=false
legal_T1_R2_R4=true
legal_T1_R3_R1=true
legal_T1_R3_R2=true
legal_T1_R3_R3=false
legal_T1_R3_R4=true
legal_T1_R4_R1=true
legal_T1_R4_R2=true
legal_T1_R4_R3=false
legal_T1_R4_R4=true
legal_T2_R1_R1=false
legal_T2_R1_R2=true
legal_T2_R1_R3=true
legal_T2_R1_R4=true
legal_T2_R2_R1=false
legal_T2_R2_R2=true
legal_T2_R2_R3=true
legal_T2_R2_R4=true
legal_T2_R3_R1=false
legal_T2_R3_R2=true
legal_T2_R3_R3=true
legal_T2_R3_R4=true
legal_T2_R4_R1=false
legal_T2_R4_R2=true
legal_T2_R4_R3=true
legal_T2_R4_R4=true
on_T1_R1_S1=true
on_T1_R1_S2=false
on_T1_R2_S1=false
on_T1_R2_S2=true
on_T1_R3_S1=false
on_T1_R3_S2=false
on_T1_R4_S1=false
on_T1_R4_S2=false
on_T2_R1_S1=false
on_T2_R1_S2=false
on_T2_R2_S1=false
on_T2_R2_S2=true
on_T2_R3_S1=true
on_T2_R3_S2=false
on_T2_R4_S1=false
on_T2_R4_S2=false
safe_S1=true
safe_S2=false
```

Task 7: Write a new definition of legal, which outlaws the bad behavior demonstrated in the previous task.
Prove that there will be no crashes in this domain.
Show that there are safe configurations for S1 and S2. What are they?
Your new definition should not involve knowledge of the legality of other trains' possible moves (that is, you shouldn't have legal on both the left and right hand side of your axiom).

Present the new definition of "legal" in the homework language and in English.

The following is a new definition of legal which outlaws the bad behavior demonstrated in Task 6.

The bad behavior demonstrated in task 6 was that the trains collided. The trains can collide because the legal rule allowed only checked to see if a destination the rail of a given train was empty (or the given train was not on it).

The new rule will also check to see if there is a train (other than the given train) on any of the rails that connect to the possible destination rail.

It is legal for a given train (t1) to move from a given rail (r1) to a possible destination rail (r2) if and only if the given train is on the given rail and the possible destination rail, or there is no train on any of the rails that connect to the possible destination rail (r2) other than the given train (t1).

Notes:

The possible destination rail (r2) also connects to the possible destination rail (r2), so this legal definition implicitly requires that the possible destination rail is empty of the given train (t1) is on it.

All of these rules assume that situation 1 is safe.

```
(
  all t1 all r1 all r2(
      legal(t1,r1,r2)<->( on(t1,r1,S1)^on(t1,r2,S1) v
            (all r3 all t2 (
                  (connects(r3,r2) ^ on(t2,r3,S1))
                  -> Equals(t1,t2)
            ))


      )
  )
)^
```

Is the domain safe?

Yes, the domain is safe. In order to prove this, I ran Layout 2 with the assertion that there will be a wreck in situation 2(~safe(S2)).
I tried to solve for all of the axioms using DPLL. It was impossible to find a satisfying solution for the axioms, in conjunction with the not safe in S2 assertion. Therefore, no solution was found, returning null.

```
task7_prove_no_wrecks.txt.out

null
```

Therefore, it is not possible to generate an unsafe situation, which is proof that the domain is safe.

Show a safe configuration that you found (or more, if you found more than one) for S1 and S2.

For this layout, there are essentially 12 ways permutations of where the trains can be located. However, due to symmetries in the layout and in how the trains operate, only 3 are logically distinct. I created all 3 in order to show that the rules work properly in all 3.

There is also the issue of all the permutations of were each train could move to in situation 2. This, however, doesn't really matter, because the legal assignments tell us where it is legal for the trains to move, which is what we are interested in.

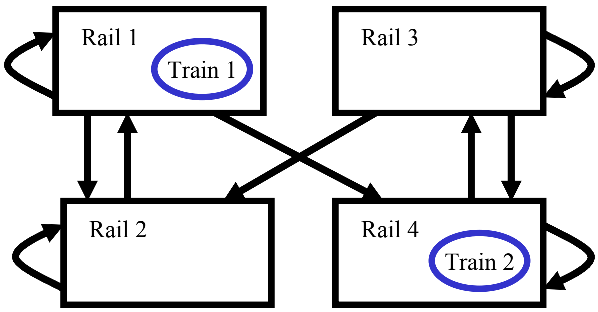
The first interpretation was produced by the axioms, without dictating where each train should start. The second and third interpretations dictated where the trains should start, so that we could look at the legal assignments in those interpretations.

First train location assignment:

Produced, T1 on R1, and T2 on R4.

```
on_T1_R1_S1=true
on_T2_R4_S1=true
```

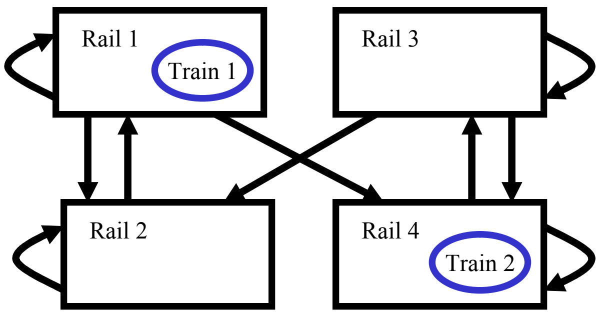
Task 4 Diagram : Layout 2 : Situation 1



Both trains stayed put.

```
on_T1_R1_S2=true
on_T2_R4_S2=true
```

Task 4 Diagram : Layout 2 : Situation 2



Legal Assignments:

It is legal for train 1 to move to Rail 2 or stay put.
Train 1 could not move to Rail 4 because train 2 is there.

```
legal_T1_R1_R1=true
legal_T1_R1_R2=true
legal_T1_R1_R3=false
legal_T1_R1_R4=false
```

It is legal for train 2 to move to Rail 3 or stay put.

```
legal_T2_R4_R1=false
legal_T2_R4_R2=false
legal_T2_R4_R3=true
legal_T2_R4_R4=true
```

The following is the entire assignment:

```
task7_find_safe_configuration2.txt.out
```

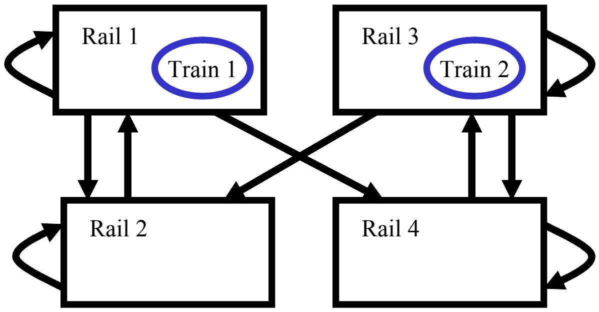
```
connects_R1_R1=true
connects_R1_R2=true
connects_R1_R3=false
connects_R1_R4=true
connects_R2_R1=true
connects_R2_R2=true
connects_R2_R3=false
connects_R2_R4=false
connects_R3_R1=false
connects_R3_R2=true
connects_R3_R3=true
connects_R3_R4=true
connects_R4_R1=false
connects_R4_R2=false
connects_R4_R3=true
connects_R4_R4=true
legal_T1_R1_R1=true
legal_T1_R1_R2=true
legal_T1_R1_R3=false
legal_T1_R1_R4=false
legal_T1_R2_R1=true
legal_T1_R2_R2=true
legal_T1_R2_R3=false
legal_T1_R2_R4=false
legal_T1_R3_R1=true
legal_T1_R3_R2=true
legal_T1_R3_R3=false
legal_T1_R3_R4=false
legal_T1_R4_R1=true
legal_T1_R4_R2=true
legal_T1_R4_R3=false
legal_T1_R4_R4=false
legal_T2_R1_R1=false
legal_T2_R1_R2=false
legal_T2_R1_R3=true
legal_T2_R1_R4=false
legal_T2_R2_R1=false
legal_T2_R2_R2=false
legal_T2_R2_R3=true
legal_T2_R2_R4=false
legal_T2_R3_R1=false
legal_T2_R3_R2=false
legal_T2_R3_R3=true
legal_T2_R3_R4=false
legal_T2_R4_R1=false
legal_T2_R4_R2=false
legal_T2_R4_R3=true
legal_T2_R4_R4=true
on_T1_R1_S1=true
on_T1_R1_S2=true
on_T1_R2_S1=false
on_T1_R2_S2=false
on_T1_R3_S1=false
on_T1_R3_S2=false
on_T1_R4_S1=false
on_T1_R4_S2=false
on_T2_R1_S1=false
on_T2_R1_S2=false
on_T2_R2_S1=false
on_T2_R2_S2=false
on_T2_R3_S1=false
on_T2_R3_S2=false
on_T2_R4_S1=true
on_T2_R4_S2=true
safe_S1=true
safe_S2=true
```

Second train location assignment:

Produced, T1 on R1, and T2 on R3.
T1 stayed on R1 and T2 stayed on R4.

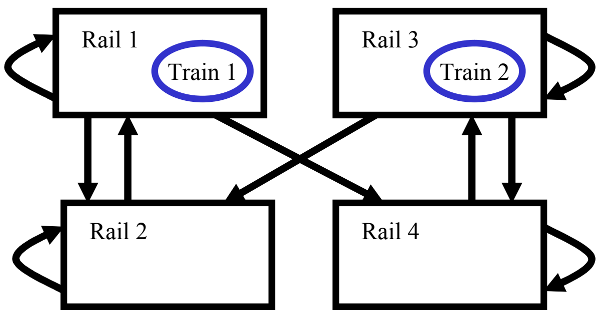
```
on_T1_R1_S1=true
on_T2_R3_S1=true
```

Task 4 Diagram : Layout 2 : Situation 1




```
on_T1_R1_S2=true
on_T2_R3_S2=true
```

Task 4 Diagram : Layout 2 : Situation 1



Legal Assignments:

Since, T1 is on R1 in S1 and T2 is on R3 in S1, the following are the only legal assignments that matter:

```
legal_T1_R1_R1=true
legal_T1_R1_R2=false
legal_T1_R1_R3=false
legal_T1_R1_R4=false
```

```
legal_T2_R3_R1=false
legal_T2_R3_R2=false
legal_T2_R3_R3=true
legal_T2_R3_R4=false
```

As you can see, in this setup, the only legal move for either train is to stay put. This is because, the other train (respectively) is also connected to R2 or R4, which are possible destination rails.

The following is the entire assignment:

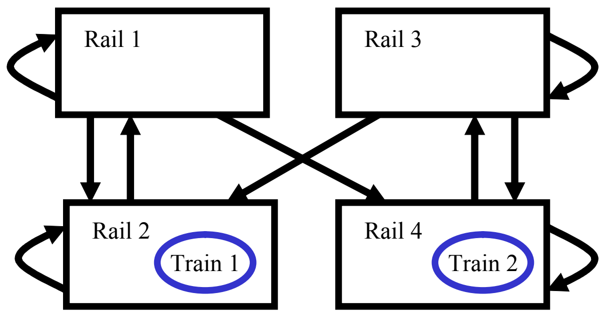
```
task7_find_safe_configuration2.txt.out

connects_R1_R1=true
connects_R1_R2=true
connects_R1_R3=false
connects_R1_R4=true
connects_R2_R1=true
connects_R2_R2=true
connects_R2_R3=false
connects_R2_R4=false
connects_R3_R1=false
connects_R3_R2=true
connects_R3_R3=true
connects_R3_R4=true
connects_R4_R1=false
connects_R4_R2=false
connects_R4_R3=true
connects_R4_R4=true
legal_T1_R1_R1=true
legal_T1_R1_R2=false
legal_T1_R1_R3=false
legal_T1_R1_R4=false
legal_T1_R2_R1=true
legal_T1_R2_R2=false
legal_T1_R2_R3=false
legal_T1_R2_R4=false
legal_T1_R3_R1=true
legal_T1_R3_R2=false
legal_T1_R3_R3=false
legal_T1_R3_R4=false
legal_T1_R4_R1=true
legal_T1_R4_R2=false
legal_T1_R4_R3=false
legal_T1_R4_R4=false
legal_T2_R1_R1=false
legal_T2_R1_R2=false
legal_T2_R1_R3=true
legal_T2_R1_R4=false
legal_T2_R2_R1=false
legal_T2_R2_R2=false
legal_T2_R2_R3=true
legal_T2_R2_R4=false
legal_T2_R3_R1=false
legal_T2_R3_R2=false
legal_T2_R3_R3=true
legal_T2_R3_R4=false
legal_T2_R4_R1=false
legal_T2_R4_R2=false
legal_T2_R4_R3=true
legal_T2_R4_R4=false
on_T1_R1_S1=true
on_T1_R1_S2=true
on_T1_R2_S1=false
on_T1_R2_S2=false
on_T1_R3_S1=false
on_T1_R3_S2=false
on_T1_R4_S1=false
on_T1_R4_S2=false
on_T2_R1_S1=false
on_T2_R1_S2=false
on_T2_R2_S1=false
on_T2_R2_S2=false
on_T2_R3_S1=true
on_T2_R3_S2=true
on_T2_R4_S1=false
on_T2_R4_S2=false
safe_S1=true
safe_S2=true
```

Third train location assignment:

Produced, T1 on R2, and T2 on R4.
T1 moved to R1, T2 stayed on R4.

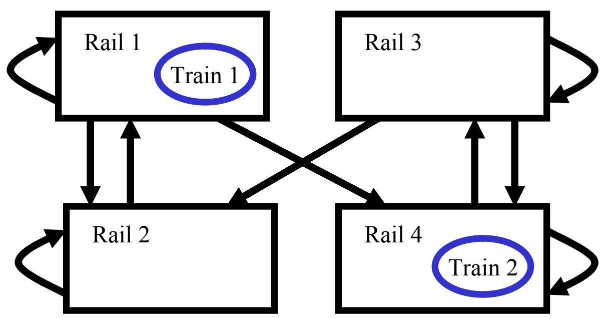
```
on_T1_R2_S1=true
on_T2_R4_S1=true
```

Task 4 Diagram : Layout 2 : Situation 1




```
on_T1_R1_S2=true
on_T2_R4_S2=true
```

Task 4 Diagram : Layout 2 : Situation 2



Legal Assignments:

Since, T1 is on R2 in S1 and T2 is on R4 in S1, the trains can move to any connected nodes. It is not legal to move to the other 2 rails, because the other train is wither on that rail, or on a rail connected to that rail.

```
legal_T1_R2_R1=true
legal_T1_R2_R2=true
legal_T1_R2_R3=false
legal_T1_R2_R4=false
```

```
legal_T2_R4_R1=false
legal_T2_R4_R2=false
legal_T2_R4_R3=true
legal_T2_R4_R4=true
```

The following is the entire assignment:

`task7_find_safe_configuration2.txt.out`

```
connects_R1_R1=true
connects_R1_R2=true
connects_R1_R3=false
connects_R1_R4=true
connects_R2_R1=true
connects_R2_R2=true
connects_R2_R3=false
connects_R2_R4=false
connects_R3_R1=false
connects_R3_R2=true
connects_R3_R3=true
connects_R3_R4=true
connects_R4_R1=false
connects_R4_R2=false
connects_R4_R3=true
connects_R4_R4=true
legal_T1_R1_R1=true
legal_T1_R1_R2=true
legal_T1_R1_R3=false
legal_T1_R1_R4=false
legal_T1_R2_R1=true
legal_T1_R2_R2=true
legal_T1_R2_R3=false
legal_T1_R2_R4=false
legal_T1_R3_R1=true
legal_T1_R3_R2=true
legal_T1_R3_R3=false
legal_T1_R3_R4=false
legal_T1_R4_R1=true
legal_T1_R4_R2=true
legal_T1_R4_R3=false
legal_T1_R4_R4=false
legal_T2_R1_R1=false
legal_T2_R1_R2=false
legal_T2_R1_R3=true
legal_T2_R1_R4=true
legal_T2_R2_R1=false
legal_T2_R2_R2=false
legal_T2_R2_R3=true
legal_T2_R2_R4=true
legal_T2_R3_R1=false
legal_T2_R3_R2=false
legal_T2_R3_R3=true
legal_T2_R3_R4=true
legal_T2_R4_R1=false
legal_T2_R4_R2=false
legal_T2_R4_R3=true
legal_T2_R4_R4=true
on_T1_R1_S1=false
on_T1_R1_S2=true
on_T1_R2_S1=true
on_T1_R2_S2=false
on_T1_R3_S1=false
on_T1_R3_S2=false
on_T1_R4_S1=false
on_T1_R4_S2=false
on_T2_R1_S1=false
on_T2_R1_S2=false
on_T2_R2_S1=false
on_T2_R2_S2=false
on_T2_R3_S1=false
on_T2_R3_S2=false
on_T2_R4_S1=true
on_T2_R4_S2=true
safe_S1=true
safe_S2=true
```

Task 8: One way to avoid crashes is never to move.
So, we'd also like to show that the system doesn't become deadlocked.
The system is deadlocked if there is no train that can move.
Add an axiom that constrains S1 to be deadlocked.
Using your definition of legal from item 7 show that there is no deadlock in layout 1.

The following is the axiom which asserts that S1 is deadlocked:

```
(
 all t1 all r1 (
      on(t1,r1,S1) ->   (
            all r2 (
                  (connects(r1,r2)^~Equals(r1,r2)) -> (
                        exists t2 exists r3 (
                          (connects(r3,r2)^on(t2,r3,S1)^~Equals(t1,t2))
                              )
                        )
                  )
            )
      )
)^
```

This works because the following must be true for all trains and all rails:

> If a given train is on a given rail is S1,
> then all rails which it is connected to (other than itself) there is another rail that connects to it, with another train (not itself) on it.

Notes:

If there is no train, on a given rail, then this axiom is satisfied (true) for that particular rail.

If there is a train on a given rail, but the rail is not connected to the particular r2 being considered, then the axiom is satisfied (true) for that particular t1, r1 and r2.

If there is a train on a given rail, and the rail is connected to the particular r2 being considered, but the r2 is really just r1, then the axiom is satisfied (true) for that particular t1, r1 and r2.

If there is a train on a given rail, and the rail is connected to the particular r2 being considered, and the r2 is not r1, then the axiom is satisfied (true) only if there exists a train (t2) on one of the rails that leads to r2 which is not the given train (t1).

Method for showing no deadlock:

The following is a description of the method I used to show no deadlock in layout 1:

I ran the DPLL sat solver on the domain to see if it could find a satisfying assignment. If it found a satisfying assignment, which would mean that it could create a situation that is deadlocked.

A satisfying assignment could not be found, therefore, there is no configuration that produces a deadlock.

```
task8_reduce.txt.out

null
```

This is correct because layout 1 is 4 rails in a cycle. Therefore, the trains are either next to each other or separated by 1 rail. Therefore, at least 1 of the trains will be able to move forward.

Diagram : Layout 1

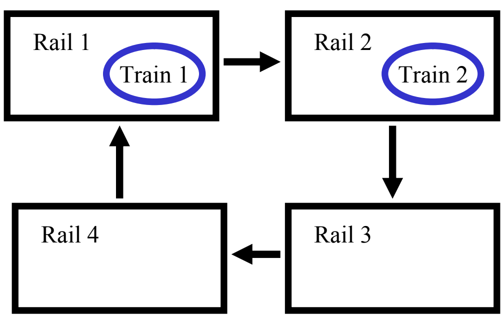
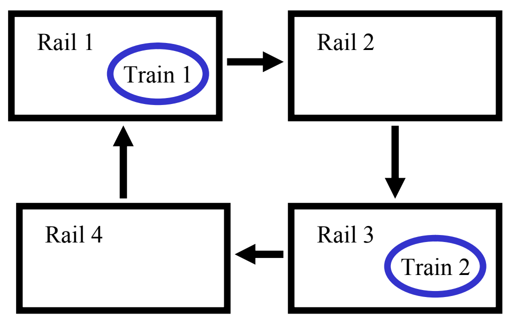


Diagram : Layout 1



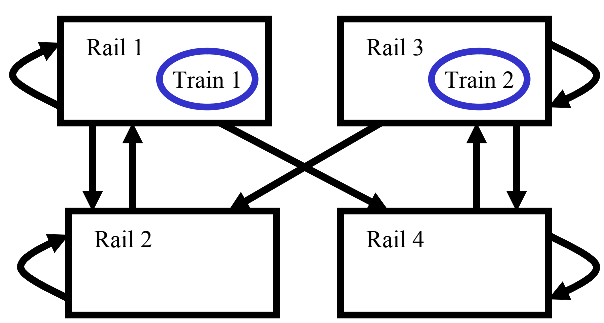
Task 9: Show that there can be a deadlock in layout 2. What is the on relation in your example?

```
To show that there can be a deadlock in layout 2, I ran the DPLL sat
solver with the axiom (from Task 8) that asserts that situation 1 is
deadlocked.  A satisfying deadlocked assignment was found which shows
that the domain can be deadlocked.
```

The following are the members of the ON relation, if your proof method produced an assignment.

```
on_T1_R1_S1=true
on_T1_R1_S2=true
on_T1_R2_S1=false
on_T1_R2_S2=false
on_T1_R3_S1=false
on_T1_R3_S2=false
on_T1_R4_S1=false
on_T1_R4_S2=false
on_T2_R1_S1=false
on_T2_R1_S2=false
on_T2_R2_S1=false
on_T2_R2_S2=false
on_T2_R3_S1=true
on_T2_R3_S2=true
on_T2_R4_S1=false
on_T2_R4_S2=false
```

Task 9 Diagram : Layout 2 : Situation 1



The sat solver produced and assignment with T1 on R1, and T2 on R3.

Since, T1 is on R1 in S1 and T2 is on R3 in S1, the only legal moves are for both trains to stay put:

```
legal_T1_R1_R1=true
legal_T1_R1_R2=false
legal_T1_R1_R3=false
legal_T1_R1_R4=false
legal_T2_R3_R1=false
legal_T2_R3_R2=false
legal_T2_R3_R3=true
legal_T2_R3_R4=false
```

As you can see, in this setup, the only legal move for either train is to stay put. This is because, the other train (respectively) is also connected to R2 or R4, which are possible destination rails.

A printout of the assignment, if you have one.

task9test.txt.out

```
connects_R1_R1=true
connects_R1_R2=true
connects_R1_R3=false
connects_R1_R4=true
connects_R2_R1=true
connects_R2_R2=true
connects_R2_R3=false
connects_R2_R4=false
connects_R3_R1=false
connects_R3_R2=true
connects_R3_R3=true
connects_R3_R4=true
connects_R4_R1=false
connects_R4_R2=false
connects_R4_R3=true
connects_R4_R4=true
legal_T1_R1_R1=true
legal_T1_R1_R2=false
legal_T1_R1_R3=false
legal_T1_R1_R4=false
legal_T1_R2_R1=true
legal_T1_R2_R2=false
legal_T1_R2_R3=false
legal_T1_R2_R4=false
legal_T1_R3_R1=true
legal_T1_R3_R2=false
legal_T1_R3_R3=false
legal_T1_R3_R4=false
legal_T1_R4_R1=true
legal_T1_R4_R2=false
legal_T1_R4_R3=false
legal_T1_R4_R4=false
legal_T2_R1_R1=false
legal_T2_R1_R2=false
legal_T2_R1_R3=true
legal_T2_R1_R4=false
legal_T2_R2_R1=false
legal_T2_R2_R2=false
legal_T2_R2_R3=true
legal_T2_R2_R4=false
legal_T2_R3_R1=false
legal_T2_R3_R2=false
legal_T2_R3_R3=true
legal_T2_R3_R4=false
legal_T2_R4_R1=false
legal_T2_R4_R2=false
legal_T2_R4_R3=true
legal_T2_R4_R4=false
on_T1_R1_S1=true
on_T1_R1_S2=true
on_T1_R2_S1=false
on_T1_R2_S2=false
on_T1_R3_S1=false
on_T1_R3_S2=false
on_T1_R4_S1=false
on_T1_R4_S2=false
on_T2_R1_S1=false
on_T2_R1_S2=false
on_T2_R2_S1=false
on_T2_R2_S2=false
on_T2_R3_S1=true
on_T2_R3_S2=true
on_T2_R4_S1=false
on_T2_R4_S2=false
safe_S1=true
safe_S2=true
```

Task 10: Revise your definition of legal so that you no longer can have a deadlock in layout 2. (Hint: you may need to add an extra relation on trains, which can mention them by name. However, your axioms for train movement, legality, etc. should not mention trains by name.) Give a satisfying assignment for the situation in which T1 is on R1 and T2 is on R3 in S1.

New definition of legal:

In my new definition of legal, it is legal for a given train to move to a given destination rail if and only if there are no trains on any of the rails which connect to the given destination rail, other than the given train, or there in no train on the given destination rail, and the given train has precedence (`iamit`).

```
(
 all t1 all r1 all r2(
      legal(t1,r1,r2)<->(
            all r3 (
                  connects(r3,r2)->(
                        ~(
                              exists t2 (
                                    on(t2,r3,S1)^~Equals(t1,t2)
                              )
                         )
                  )
            ) v
            (
                  all t3 ((~on(t3,r2,S1)) ^ deadlocked(S1) ^ iamit(t1))
            )
      )
 )
)^
```

Other new axioms or relations that I added:

In order to make the new definition of legal work, I gave train 1 precedence. In other words, if none of the trains have moves that would definitely be safe, train 1 has precedence to move to an empty rail. In configuration 2, this will work, if neither train has a move that is definitely safe, both trains are connected to a rail that is currently empty.

A more generic way to award precedence is to construct an axiom that says one train or the other has precedence. This technique, however, is flawed, because it leaves the decision up to the sat solver (DPLL algorithm) which then acts as an omniscient scheduler.

It would also be more generic to cause the precedence to change from situation to situation while defining the precedence for only situation 1. However, the legal definition does not take situation into consideration. It is applicable, really, only to situation 1.

Therefore, there is no more appropriate way to set precedence then to just define it for situation 1.

In order to be generic with respect to the number of trains, I set the precedence of T1 (to true), and require that the precedence of all of the other trains (really there is just 1 other train) to the opposite (false). That way, this would work, no matter how many trains there are.

The axiom works as follows. If `iamit(t1)` is false, then we don't care what `iamit(t2)` is. However, if `iamit(t1)` is true (i.e. it is T1), then all of the others will have to be false, for the axiom to be satisfied.

```
iamit(T1)^

(
      all t1 all t2 (
            ( iamit(t1)->~iamit(t2) ) v Equals(t1,t2)
      )
)^
```

Enforce movement

In order to make sure that the moves would be executed appropriately, I added an axiom that requires the train that has precedence must move. This was just for testing purposes. I ran the program with and without this axiom. The variables are assigned appropriately in both situations. When movement is not enforced, train 1 (with precedence) has legal moves, but chooses not to move. When movement is enforced, train 1 (with precedence) has legal moves, and chooses to move to rail 4.

```
(all t1 all r1 (
                  iamit(t1) -> (
                        on(t1,r1,S1) -> ~on(t1,r1,S2)
                  )
            )
)^
```

I rearranged my deadlock assertion for this task so that I could use it more easily. I named it, so that it could be used in the legal axiom and so that I could turn it on and off for testing.

The "nomoves" predicate defines whether or not a given train can move to a given rail in a given situation.

```
(
 all t1 all r1 all s nomoves(t1,r1,s) <-> (
      on(t1,r1,s) -> (
            all r2 (
                  ( connects(r1,r2)^~Equals(r1,r2) ) -> (
                        exists t2 exists r3 (
                        (connects(r3,r2)^on(t2,r3,s)^~Equals(t1,t2))
                        )
                  )
            )
      )
 )
)^
```

I named the "deadlocked" assertion, so that I could switch it on and off. The "deadlocked" assertion pertains to the entire situation.

```
(
      all s deadlocked(s) <-> (
            all t1 all r1 nomoves(t1,r1,s)
      )
)^
```

Here I switch it on, for situation 1.

```
deadlocked(S1)^
```

A printout of the assignment for the given arrangement.

The results are in the table below.
The results show that the trains started in R1 and R3.
Train 1 was given precedence; therefore, it was legal for Train 1 to move to R2 or R4 or R1.

When movement was enforced, certain interpretation variables changed for situation 1. Since train 1 moved to rail 4, train 2, on rail 3 had a legal move (to rail2) in situation 2. Therefore, situation 2 is not deadlocked.

Without enforcing movement	**With enforcing movement:**
task10v3.txt.out connects_R1_R1=true connects_R1_R2=true connects_R1_R3=false connects_R1_R4=true connects_R2_R1=true connects_R2_R2=true	task10v3.txt.out connects_R1_R1=true connects_R1_R2=true connects_R1_R3=false connects_R1_R4=true connects_R2_R1=true connects_R2_R2=true

connects_R2_R3=false	connects_R2_R3=false
connects_R2_R4=false	connects_R2_R4=false
connects_R3_R1=false	connects_R3_R1=false
connects_R3_R2=true	connects_R3_R2=true
connects_R3_R3=true	connects_R3_R3=true
connects_R3_R4=true	connects_R3_R4=true
connects_R4_R1=false	connects_R4_R1=false
connects_R4_R2=false	connects_R4_R2=false
connects_R4_R3=true	connects_R4_R3=true
connects_R4_R4=true	connects_R4_R4=true
deadlocked_S1=true	deadlocked_S1=true
deadlocked_S2=true	deadlocked_S2=false
iamit_T1=true	iamit_T1=true
iamit_T2=false	iamit_T2=false
legal_T1_R1_R1=true	legal_T1_R1_R1=true
legal_T1_R1_R2=true	legal_T1_R1_R2=true
legal_T1_R1_R3=false	legal_T1_R1_R3=false
legal_T1_R1_R4=true	legal_T1_R1_R4=true
legal_T1_R2_R1=true	legal_T1_R2_R1=true
legal_T1_R2_R2=true	legal_T1_R2_R2=true
legal_T1_R2_R3=false	legal_T1_R2_R3=false
legal_T1_R2_R4=true	legal_T1_R2_R4=true
legal_T1_R3_R1=true	legal_T1_R3_R1=true
legal_T1_R3_R2=true	legal_T1_R3_R2=true
legal_T1_R3_R3=false	legal_T1_R3_R3=false
legal_T1_R3_R4=true	legal_T1_R3_R4=true
legal_T1_R4_R1=true	legal_T1_R4_R1=true
legal_T1_R4_R2=true	legal_T1_R4_R2=true
legal_T1_R4_R3=false	legal_T1_R4_R3=false
legal_T1_R4_R4=true	legal_T1_R4_R4=true
legal_T2_R1_R1=false	legal_T2_R1_R1=false
legal_T2_R1_R2=false	legal_T2_R1_R2=false
legal_T2_R1_R3=true	legal_T2_R1_R3=true
legal_T2_R1_R4=false	legal_T2_R1_R4=false
legal_T2_R2_R1=false	legal_T2_R2_R1=false
legal_T2_R2_R2=false	legal_T2_R2_R2=false
legal_T2_R2_R3=true	legal_T2_R2_R3=true
legal_T2_R2_R4=false	legal_T2_R2_R4=false
legal_T2_R3_R1=false	legal_T2_R3_R1=false
legal_T2_R3_R2=false	legal_T2_R3_R2=false
legal_T2_R3_R3=true	legal_T2_R3_R3=true
legal_T2_R3_R4=false	legal_T2_R3_R4=false
legal_T2_R4_R1=false	legal_T2_R4_R1=false
legal_T2_R4_R2=false	legal_T2_R4_R2=false
legal_T2_R4_R3=true	legal_T2_R4_R3=true
legal_T2_R4_R4=false	legal_T2_R4_R4=false
nomoves_T1_R1_S1=true	nomoves_T1_R1_S1=true
nomoves_T1_R1_S2=true	nomoves_T1_R1_S2=true
nomoves_T1_R2_S1=true	nomoves_T1_R2_S1=true
nomoves_T1_R2_S2=true	nomoves_T1_R2_S2=true
nomoves_T1_R3_S1=true	nomoves_T1_R3_S1=true
nomoves_T1_R3_S2=true	nomoves_T1_R3_S2=true
nomoves_T1_R4_S1=true	nomoves_T1_R4_S1=true
nomoves_T1_R4_S2=true	nomoves_T1_R4_S2=true
nomoves_T2_R1_S1=true	nomoves_T2_R1_S1=true
nomoves_T2_R1_S2=true	nomoves_T2_R1_S2=true

nomoves_T2_R2_S1=true nomoves_T2_R2_S2=true nomoves_T2_R3_S1=true nomoves_T2_R3_S2=true nomoves_T2_R4_S1=true nomoves_T2_R4_S2=true on_T1_R1_S1=true on_T1_R1_S2=true on_T1_R2_S1=false on_T1_R2_S2=false on_T1_R3_S1=false on_T1_R3_S2=false on_T1_R4_S1=false on_T1_R4_S2=false on_T2_R1_S1=false on_T2_R1_S2=false on_T2_R2_S1=false on_T2_R2_S2=false on_T2_R3_S1=true on_T2_R3_S2=true on_T2_R4_S1=false on_T2_R4_S2=false safe_S1=true safe_S2=true	nomoves_T2_R2_S1=true nomoves_T2_R2_S2=true nomoves_T2_R3_S1=true nomoves_T2_R3_S2=false nomoves_T2_R4_S1=true nomoves_T2_R4_S2=true on_T1_R1_S1=true on_T1_R1_S2=false on_T1_R2_S1=false on_T1_R2_S2=false on_T1_R3_S1=false on_T1_R3_S2=false on_T1_R4_S1=false on_T1_R4_S2=true on_T2_R1_S1=false on_T2_R1_S2=false on_T2_R2_S1=false on_T2_R2_S2=false on_T2_R3_S1=true on_T2_R3_S2=true on_T2_R4_S1=false on_T2_R4_S2=false safe_S1=true safe_S2=true

Task 11: It might seem like the implication in formula 5 should really be a bi-conditional. Explain in English what that would mean, and why it would cause problems in our axiomatization.

The following is Formula 5 expressed in the homework language.

Dynamics: Only make legal moves to connected rails.

```
(
 all t all r2 (
      on(t,r2,S2) -> exists r1 (
            on(t,r1,S1) ^ connects(r1,r2) ^ legal(t,r1,r2)
      )
 )
) ^
```

The above axiom uses implication, rather than a bi-conditional. The original axiom requires that if a train is on a given rail in situation 2, then it was on a rail that is connected to the given rail is situation 1 and it is legal to move to from the connected rail to the given rail.

Changing Formula 5 to a bi-conditional changes its' meaning. Its' new meaning includes the former meaning. However, the bi-conditional also requires that if a train is on a given rail in situation 1, and it is connected to another rail, and it is legal to move to that rail, then it will be on that rail in situation 2. In other words, any legal move to a connected rail <u>will</u> be made.

Therefore, a given train will be on multiple rails in situation 2, if it has more than 1 possible move. This is actually impossible to satisfy, unless we allow deadlock, because of this axiom:

Axiom 2 (Task 2) : No train is on two rails at the same time.

```
(
 all t all s all r1 (
      on(t,r1,s) -> (
            all r2 (
                  on(t,r2,s)->Equals(r1,r2)
            )
      )
 )
)^
```

If deadlock was allowed, then the trains could stay on the same rail, which would be the one and only legal move to a connected rail. However, deadlock is not allowed, therefore, this would also cause unsafe situations for many assignments.

Our axioms are meant to represent something in the real world (i.e. a simulation). This change is akin to the manifestation of multiple futures in a single space-time.

Task 12 : Extra Credit: Our definition of liveness (lack of deadlock) in section 8 is weak, because there still might be some train that never gets to move. But requiring every train to be able to move on every step is too strong. How could you axiomatize the notion that every train eventually gets to move? Would it work to try to prove or disprove such a liveness condition using the methods of this assignment? Why or why not?

My answer to this question assumes that the axiom for liveness should be in a form akin to the way deadlock was axiomatized. In other words, I would require that a train will not always be in the same place, over all of the situations.

```
(all t1 all r1 all s1(
                        on(t1,r1,s1) -> (
                              exists s2 ~on(t1,r1,s2)
                        )
            )
)^
```

In order to test this in our situation, I would then include the negation of this axiom, in the axiom set, and attempt to solve in using DPLL sat solver. The sat solver could return null, which would mean that all trains eventually get to move. Otherwise, it could return a satisfying assignment where at least one train never gets to move.

This would not be appropriate if using the given methods for 2 reasons. First, since there are just 2 situations, this requirement is the same as requiring that every train move in every situation. This is explicitly not the goal, and this would also restrict the train locations, to locations where they could simultaneously move. For example, it would not be possible for a train to be on R1 and R3.

In sum, testing on just 2 situations would make the dynamics very constrained, and it just doesn't accomplish our goal.

Second, in order to test this we would therefore need to add multiple situations with an ordering (a time frame). The legal definition would need to be able to impose an ordering on the situations. Perhaps they could be joined in a series as follows:

```
next(S1,S2) ^
next(S2,S3) ^
next(S3,S4) ^
```

Then the legal axiom could use this ordering to determine if a move is legal in a given situation.

In theory, however, you would need an infinite number of states. You cannot have a universe of infinite size in the system that we are using. Even if you could, you could never return a satisfying assignment (where some train never gets to move) because you would never know if nobody ever gets to move.

For practical purposes, in our small layout, we would be able to prove that it worked (every train eventually gets to move) for a small number of situations. But we would not be able to be sure that it does not work (no one ever gets to move).

Project 1, Part 2

Task1:

Formalized Axioms:

```
(all x man(x) -> mortal(x))
(all x mortal(x) -> boring(x))
~boring(Hera)
~(exists x ~man(x))
```

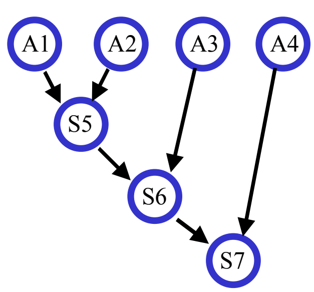
Proof, Version 1:

```
A1: ~man(x2) v mortal(x2)
A2: ~mortal(x4) v boring(x4)
A3: ~boring(CHera)
A4:  man(x6)
S5: ~man(x7) v boring(x7)      Res(1,2) {x2/x4}
S6: ~man(CHera)                Res(5,3) {x7/CHera}
S7:  F                         Res(6,4) {x6/CHera}
```

Proof Sketch:

Proving forward from "all men are mortal" to "Hera is not boring" can solve this proof. First, we resolve, "all men are mortal", with "all mortals are boring", to get "all men are boring." This is logically stated as "if you are a man then you are boring." We then resolve, "if you are a man then you are boring" with "Hera is not boring" to get "Hera is not a man." We conclude by resolving, "Hera is not a man" with the negation of the axiom "There exists someone who is not a man."

Diagram : Task 1 Proof Version 1



Key Steps:

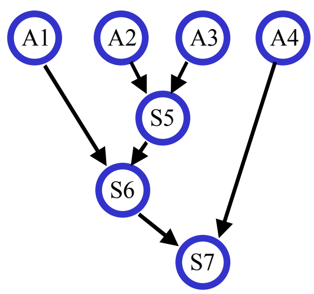
This proof is entirely linear. None of the proof steps are key.

Proof, Version 2:

```
A1: ~man(x2) v mortal(x2)
A2: ~mortal(x4) v boring(x4)
A3: ~boring(CHera)
A4:  man(x6)
S5: ~mortal(CHera)       Res(2,3) {x4/CHera}
S6: ~man(CHera)          Res(5,1) {x2/CHera}
S7:  F                   Res(6,4) {x6/CHera}
```

This can also be done, proving backward from to "Hera is not boring" to "all men are mortal." First, we resolve, "all mortals are boring" with "Hera is not boring" to get "Hera is not mortal." We then resolve "Hera is not mortal" with "all men are mortal" to get "Hera is not a man." We conclude by resolving, "Hera is not a man" with the negation of the axiom "There exists someone who is not a man."

Diagram : Task 1 Proof Version 2



Task 2:

Proof:

```
A1:  son(Father(CA), Father(CMe))
A2: ~sib(x14, CMe)
A3: ~sib(x18, y17) v Equals(Father(x18), Father(y17))
A4: ~sib(x20, y19) v ~Equals(x20, y19)
A5: ~Equals(Father(x22), Father(y21)) v Equals(x22, y21) v
      sib(x22, y21)
A6: ~son(x26, y25) v Equals(Father(x26), y25)
A7: ~Equals(Father(x28), y27) v son(x28, y27)
A8: ~son(CA, CMe)
S9: Equals(Father(Father(CA)), Father(CMe))
      Res(1,6) {y25/Father(CMe), x26/Father(CA)}
S10: Equals(Father(CA), CMe) v sib(Father(CA), CMe)
      Res(9,5) {y21/CMe, x22/Father(CA)}
S11: Equals(Father(CA), CMe)        Res(10,2) {x14/Father(CA)}
S12: son(CA, CMe)                   Res(11,7) {y27/CMe, x28/CA}
S13: F                              Res(12,8) {}
```

In this proof, “that man” is represented by the constant “A.”

Since that man’s father is my father’s son, and I don’t have any siblings, then that man’s father must be me, and that man must be my son. However, we have to use the sibling relationship to connect the son relationship with the fact that I have no siblings.

To resolve this proof, I started with the first axiom, “That man’s father is my father’s son.” This can only be resolved with A6: “If x is the son of y, then the father of x is y.” These resolve to “That man’s Grand father is my father.”

I then resolve, “That man’s Grand father is my father” with “if x and y have the same father, then x and y are siblings or x and y are the same person.” I then get, either “that man’s father is me,” or “that man’s father is my sibling.”

I then eliminate the possibility that that man’s father is my sibling by resolving it with “I don’t have any siblings” to get “that man’s father is me.” I then resolve this with “ if the father of x is y then x is the son of y” to get “that man is my son!”

This resolves with the conclusion of the proof.

Another way to show this is to first use the sibling relationship and the fact that I have no siblings to show that the only one who can have my father as their father is me.

```
S9: ~Equals(Father(x25), Father(CMe)) v Equals(x25, CMe)
      Res(2,5) {x14/x22, y21/CMe}
```

I then show that my son's grandfather is my father.

```
S10: ~Equals(Father(Father(x31)), Father(CMe)) v son(x31, CMe)
Res(9,7) {y27/CMe, x25/Father(x28)}
```

I then show that my son's father is the son of my father.

```
S11: son(x32, CMe) v ~son(Father(x32), Father(CMe))      Res(10,6)
{y25/Father(CMe), x26/Father(x31)}

S12: son(CA, CMe)      Res(11,1) {x32/CA}
```

I then show that A is my son which resolves the proof.

```
S13: F      Res(12,8) {}
```

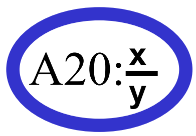
Task 3.

Printout of the Proof :

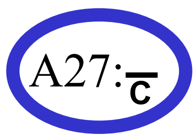
```
A1: ~Equals(Ca, Cb)
A2: ~Equals(Ca, Cc)
A3: ~Equals(Cb, Ca)
A4: ~Equals(Cb, Cc)
A5: ~Equals(Cc, Ca)
A6: ~Equals(Cc, Cb)
A7: ~Equals(Ca, Ctable)
A8: ~Equals(Cb, Ctable)
A9: ~Equals(Cc, Ctable)
A10: block(Ca)
A11: block(Cb)
A12: block(Cc)
A13: ~block(Ctable)
A14: ~on(x12, y11, s10) v ~on(x12, z13, s10) v Equals(y11, z13)
A15: ~on(x20, z19, s18) v ~on(y21, z19, s18) v Equals(x20, y21) v
Equals(z19, Ctable)
A16: ~clear(x27, s26) v ~block(y28) v ~on(y28, x27, s26) v Equals(x27,
Ctable)
A17: block(F_1(y31, x30, s29)) v clear(x30, s29)
A18: on(F_1(y24, x33, s32), x33, s32) v clear(x33, s32)
A19: ~Equals(x34, Ctable) v clear(x34, s35)
A20: ~clear(x41, s40) v ~clear(y42, s40) v ~on(x41, z43, s40) v on(x41,
y42, result(move(x41, y42), s40))
A21: ~clear(x45, s44) v ~clear(y46, s44) v ~on(x45, z47, s44) v
clear(z47, result(move(x45, y46), s44))
A22: ~on(x55, y54, s53) v Equals(x55, z56) v on(x55, y54,
result(move(z56, w52), s53))
A23: ~clear(x62, s61) v Equals(x62, z63) v clear(x62, result(move(y64,
z63), s61))
A24: on(Ca, Ctable, Cs0)
A25: on(Cc, Ca, Cs0)
A26: on(Cb, Ctable, Cs0)
A27: clear(Cc, Cs0)
A28: clear(Cb, Cs0)
A29: clear(Ctable, s66)
A30: ~on(Cb, Cc, s68) v answer(s68)
S31: ~clear(x69, Cs0) v ~on(x69, z70, Cs0) v on(x69, Cc,
result(move(x69, Cc), Cs0))      Res(20,27) {s40/Cs0, y42/Cc}
S32: ~on(Cb, z72, Cs0) v on(Cb, Cc, result(move(Cb, Cc), Cs0))
Res(31,28) {x69/Cb}
S33: on(Cb, Cc, result(move(Cb, Cc), Cs0))      Res(32,26) {z72/Ctable}
S34: answer(result(move(Cb, Cc), Cs0))      Res(33,30)
{s68/result(move(Cb, Cc), Cs0)}
```

Proof Description:

To resolve the proof, we need to refute the assertion that B cannot be on C in some situation. The following diagram outlines the process. The diagram shows a brief representation of what a given axiom says about the world. The For example,



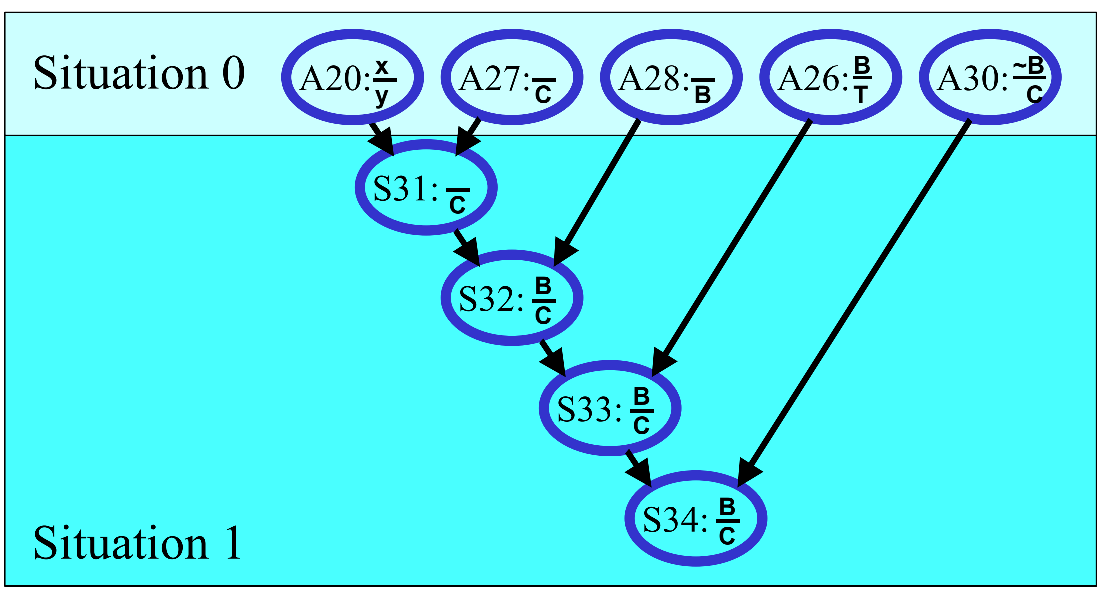
means that variable x is on variable y and



means that C is clear.

This is a shorthand representation, which aides in following the proof. To get the full and accurate meaning of a give axiom, you have to look at the axiom.

Diagram : Task 3



Ultimately, we want to prove Axiom 30:

```
Axiom 30 : ~exists s on(b,c,s) ^ ~answer(s)
```

The proof checker represents it as follows:

```
A30: ~on(Cb, Cc, s68) v answer(s68)
```

To do this, we satisfy the following axiom.

Axiom 20-21

If x and y are clear, and x is on z, then moving x to y results in x being on y and z being clear.

```
all s all x all y all z clear(x,s) ^ clear(y,s) ^ on (x,z,s) ->
    on(x,y,result(move(x,y),s)) ^ clear(z,result(move(x,y),s))
```

In other words, we are going to move B onto C. If we do, then the result is "B is on C." The axioms say that this can only be done if there is nothing on top of B or C and B is on z, where z can be anything. In our case, B is on the table.

The proof checker splits this axiom up into:

```
A20: ~clear(x41, s40) v ~clear(y42, s40) v ~on(x41, z43, s40) v on(x41,
y42, result(move(x41, y42), s40))
```

and

```
A21: ~clear(x45, s44) v ~clear(y46, s44) v ~on(x45, z47, s44) v
clear(z47, result(move(x45, y46), s44))
```

We only need to satisfy A20, because we want to show that x is on y (B is on C in particular). A21 would resolve to z is clear.

We have to show that B and C are both clear, in order to make the move.

```
A27: clear(Cc, Cs0)
A28: clear(Cb, Cs0)
```

However, we also have to show that B is on Z (in this case the table).

```
A26: on(Cb, Ctable, Cs0)
```

This seems unnecessary, but this requirement comes from the original axiom so that it can simultaneously declare that Z is clear (if we wanted that). In our case, we chose the subsequent clause that shows that B will be on C.

In essence, we have to resolve A26, A27 and A28 with A20.

A20 is the actual move. It is contingent on A26, A27 and A28.

```
A20: ~clear(x99, s98) v ~clear(y100, s98) v ~on(x99, z101, s98) v
on(x99, y100, result(move(x99, y100), s98))
```

The order doesn't matter, but the contingencies need to be resolved with the action.

Therefore, I resolved A20 with A27 to imply the move and show that C is clear. You have to make sure to choose the move which results in moving something to C.

```
S31: ~clear(x69, Cs0) v ~on(x69, z70, Cs0) v on(x69, Cc,
result(move(x69, Cc), Cs0))      Res(20,27) {s40/Cs0, y42/Cc}
```

I then resolved S31 with A28 to show that C is clear.

```
S32: ~on(Cb, z72, Cs0) v on(Cb, Cc, result(move(Cb, Cc), Cs0))
Res(31,28) {x69/Cb}
```

I then resolved S32 with A26 to show that B is on something (in this case the table).

```
S33: on(Cb, Cc, result(move(Cb, Cc), Cs0))      Res(32,26) {z72/Ctable}
```

I then resolved S33 with S30 (the answer) to get rid of the on statement and tuck the description of the move into the answer.

```
S34: answer(result(move(Cb, Cc), Cs0))      Res(33,30)
{s68/result(move(Cb, Cc), Cs0)}
```

Task 3 Extra Credit:

Printout of the proof:

```
A1: ~Equals(Ca, Cb)
A2: ~Equals(Ca, Cc)
A3: ~Equals(Cb, Ca)
A4: ~Equals(Cb, Cc)
A5: ~Equals(Cc, Ca)
A6: ~Equals(Cc, Cb)
A7: ~Equals(Ca, Ctable)
A8: ~Equals(Cb, Ctable)
A9: ~Equals(Cc, Ctable)
A10: block(Ca)
A11: block(Cb)
A12: block(Cc)
A13: ~block(Ctable)
A14: ~on(x168, y167, s166) v ~on(x168, z169, s166) v Equals(y167, z169)
A15: ~on(x176, z175, s174) v ~on(y177, z175, s174) v Equals(x176, y177)
v Equals(z175, Ctable)
A16: ~clear(x183, s182) v ~block(y184) v ~on(y184, x183, s182)
A17: ~clear(x186, s185) v ~Equals(x186, Ctable)
A18: block(F_3(y189, x188, s187)) v Equals(x188, Ctable) v clear(x188,
s187)
A19: on(F_3(y180, x191, s190), x191, s190) v Equals(x191, Ctable) v
clear(x191, s190)
A20: ~clear(x197, s196) v ~clear(y198, s196) v ~on(x197, z199, s196) v
on(x197, y198, result(move(x197, y198), s196))
A21: ~clear(x201, s200) v ~clear(y202, s200) v ~on(x201, z203, s200) v
clear(z203, result(move(x201, y202), s200))
A22: ~clear(x205, s204) v ~clear(y206, s204) v ~on(x205, z207, s204) v
clear(x205, result(move(x205, y206), s204))
A23: ~on(x215, y214, s213) v Equals(x215, z216) v on(x215, y214,
result(move(z216, w212), s213))
A24: ~clear(x222, s221) v Equals(x222, z223) v clear(x222,
result(move(y224, z223), s221))
A25: on(Ca, Ctable, Cs0)
A26: on(Cc, Ca, Cs0)
A27: on(Cb, Ctable, Cs0)
A28: clear(Cc, Cs0)
A29: clear(Cb, Cs0)
A30: clear(Ctable, s226)
A31: ~on(Ca, Cb, s228) v ~on(Cb, Cc, s228) v answer(s228)
S32: Equals(Ca, z45) v on(Ca, Ctable, result(move(z45, w46), Cs0))
Res(23,25) {y214/Ctable, x215/Ca, s213/Cs0}
S33: on(Ca, Ctable, result(move(Cc, w54), Cs0))      Res(32,2) {z45/Cc}
S34: Equals(Cb, z1) v on(Cb, Ctable, result(move(z1, w2), Cs0))
Res(23,27) {x215/Cb, s213/Cs0, y214/Ctable}
S35: on(Cb, Ctable, result(move(Cc, w3), Cs0))      Res(34,4) {z1/Cc}
S36: ~clear(Cc, Cs0) v ~clear(y4, Cs0) v clear(Ca, result(move(Cc, y4),
Cs0))      Res(21,26) {x201/Cc, z203/Ca, s200/Cs0}
S37: ~clear(y5, Cs0) v clear(Ca, result(move(Cc, y5), Cs0))
Res(36,28) {}
```

```
S38: clear(Ca, result(move(Cc, Ctable), Cs0))      Res(37,30)
{y5/Ctable, s226/Cs0}
S39: Equals(Cb, z6) v clear(Cb, result(move(y7, z6), Cs0))
Res(24,29) {s221/Cs0, x222/Cb}
S40: clear(Cb, result(move(y8, Ctable), Cs0))      Res(39,8)
{z6/Ctable}
S41: ~clear(x10, s9) v ~on(x10, z11, s9) v clear(x10, result(move(x10,
Ctable), s9))      Res(22,30) {s204/s226, y206/Ctable}
S42: ~on(Cc, z12, Cs0) v clear(Cc, result(move(Cc, Ctable), Cs0))
Res(41,28) {s9/Cs0, x10/Cc}
S43: clear(Cc, result(move(Cc, Ctable), Cs0))      Res(42,26) {z12/Ca}
S44: Equals(Ca, z13) v on(Ca, Ctable, result(move(z13, w212),
result(move(Cc, w14), Cs0)))      Res(33,23) {x215/Ca,
s213/result(move(Cc, w54), Cs0), y214/Ctable}
S45: on(Ca, Ctable, result(move(Cb, w16), result(move(Cc, w15), Cs0)))
Res(44,1) {z13/Cb}
S46: ~clear(Cb, result(move(Cc, w17), Cs0)) v ~clear(y18,
result(move(Cc, w17), Cs0)) v on(Cb, y18, result(move(Cb, y18),
result(move(Cc, w17), Cs0)))      Res(35,20) {z199/Ctable,
s196/result(move(Cc, w3), Cs0), x197/Cb}
S47: ~clear(Cb, result(move(Cc, Ctable), Cs0)) v on(Cb, Cc,
result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)))      Res(43,46)
{w17/Ctable, y18/Cc}
S48: on(Cb, Cc, result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)))
Res(40,47) {y8/Cc}
S49: Equals(Ca, z19) v clear(Ca, result(move(y20, z19), result(move(Cc,
Ctable), Cs0)))      Res(38,24) {s221/result(move(Cc, Ctable), Cs0),
x222/Ca}
S50: clear(Ca, result(move(y21, Cc), result(move(Cc, Ctable), Cs0)))
Res(49,2) {z19/Cc}
S51: ~clear(x22, result(move(Cc, Ctable), Cs0)) v ~on(x22, z23,
result(move(Cc, Ctable), Cs0)) v clear(x22, result(move(x22, Cc),
result(move(Cc, Ctable), Cs0)))      Res(43,22) {s204/result(move(Cc,
Ctable), Cs0), y206/Cc}
S52: ~clear(Cb, result(move(Cc, Ctable), Cs0)) v clear(Cb,
result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)))      Res(35,51)
{z23/Ctable, x22/Cb, w3/Ctable}
S53: clear(Cb, result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)))
Res(40,52) {y8/Cc}
S54: ~clear(Ca, result(move(Cb, w25), result(move(Cc, w24), Cs0))) v
~clear(y26, result(move(Cb, w25), result(move(Cc, w24), Cs0))) v on(Ca,
y26, result(move(Ca, y26), result(move(Cb, w25), result(move(Cc, w24),
Cs0))))      Res(45,20) {z199/Ctable, s196/result(move(Cb, w16),
result(move(Cc, w15), Cs0)), x197/Ca}
S55: ~clear(y27, result(move(Cb, Cc), result(move(Cc, Ctable), Cs0))) v
on(Ca, y27, result(move(Ca, y27), result(move(Cb, Cc), result(move(Cc,
Ctable), Cs0))))      Res(50,54) {w24/Ctable, y21/Cb, w25/Cc}
S56: on(Ca, Cb, result(move(Ca, Cb), result(move(Cb, Cc),
result(move(Cc, Ctable), Cs0))))      Res(53,55) {y27/Cb}
S57: Equals(Cb, z28) v on(Cb, Cc, result(move(z28, w29),
result(move(Cb, Cc), result(move(Cc, Ctable), Cs0))))      Res(48,23)
{x215/Cb, s213/result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)),
y214/Cc}
S58: on(Cb, Cc, result(move(Ca, w30), result(move(Cb, Cc),
result(move(Cc, Ctable), Cs0))))      Res(57,3) {z28/Ca}
```

```
S59: ~on(Cb, Cc, result(move(Ca, Cb), result(move(Cb, Cc),
result(move(Cc, Ctable), Cs0)))) v answer(result(move(Ca, Cb),
result(move(Cb, Cc), result(move(Cc, Ctable), Cs0))))      Res(56,31)
{s228/result(move(Ca, Cb), result(move(Cb, Cc), result(move(Cc,
Ctable), Cs0))))}
S60: answer(result(move(Ca, Cb), result(move(Cb, Cc), result(move(Cc,
Ctable), Cs0))))      Res(58,59) {w30/Cb}
```

Proof Explanation: The following is an explanation of the proof steps.

We start in Situation 0 with the facts that:

```
A is on the table,
C is on A,
B is on the table,
C is clear,
B is clear,
The table is clear.
```

Note: The the table is always considered clear.

These facts are expressed in the following axioms:

```
A25: on(Ca, Ctable, Cs0)
A26: on(Cc, Ca, Cs0)
A27: on(Cb, Ctable, Cs0)
A28: clear(Cc, Cs0)
A29: clear(Cb, Cs0)
A30: clear(Ctable, s226)
```

We then rearrange the blocks. When we rearrange the blocks, we end up in a different situation. For convenience, I number the situations from 0 (initial situation) to 3 (final situation).

Really, we are just proving what the result of moving the blocks would be, based on the axioms. If we wanted to prove that the result of moving C onto the table is that C is on the table, we could do that. That was the result of the previous proof. However, that result is not something that we need to prove in this proof. Rather, we want to prove the following results (outlined below), which are needed later in the proof. For example, in order to prove that B is on C in Situation 2, we need to have already proven that B is clear, B is on the table, and C is clear in Situation 1.

Outline:

Move C to Table (Situation 1):
- Prove: A is on the table
- Prove: B is on the table
- Prove: A is clear
- Prove: B is clear
- Prove: C is clear

Move B to C (Situation 2):
- Prove: A is on the table
- Prove: B is on C
- Prove: A is clear
- Prove: B is clear

Move A onto B (Situation 3):

- Prove: A is on B
- Prove: B is on C

Resolve Answer (A is on B ^ B is on C):
- Prove: The final resolved answer.

The following is a diagram of the steps:

Diagram : Task 3 Extra Credit : Situation 0

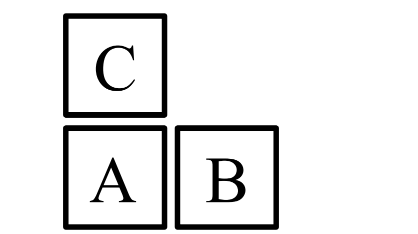


Diagram : Task 3 Extra Credit : Situation 1

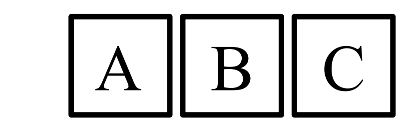


Diagram : Task 3 Extra Credit : Situation 2

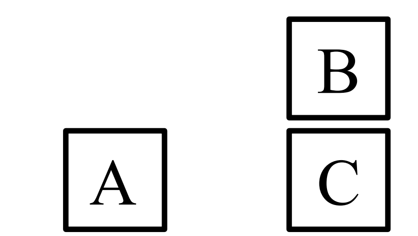
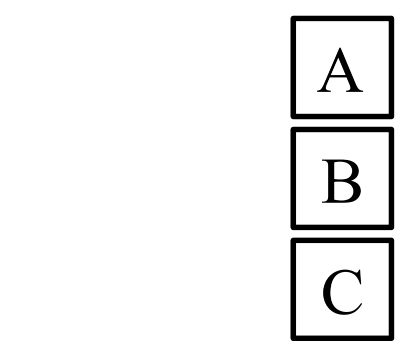


Diagram : Task 3 Extra Credit : Situation 3



Move C to Table (Situation 1):

If we move C to the table, we need to prove the following:

Prove: A is on the table

S32: Equals(Ca, z45) v on(Ca, Ctable, result(move(z45, w46), Cs0))
Res(23,25) {y214/Ctable, x215/Ca, s213/Cs0}

S33: on(Ca, Ctable, result(move(Cc, w54), Cs0)) Res(32,2) {z45/Cc}

Prove: B is on the table

S34: Equals(Cb, z1) v on(Cb, Ctable, result(move(z1, w2), Cs0))
Res(23,27) {x215/Cb, s213/Cs0, y214/Ctable}

S35: on(Cb, Ctable, result(move(Cc, w3), Cs0)) Res(34,4) {z1/Cc}

Prove: A is clear

S36: ~clear(Cc, Cs0) v ~clear(y4, Cs0) v clear(Ca, result(move(Cc, y4), Cs0)) Res(21,26) {x201/Cc, z203/Ca, s200/Cs0}

S37: ~clear(y5, Cs0) v clear(Ca, result(move(Cc, y5), Cs0))
Res(36,28) {}

S38: clear(Ca, result(move(Cc, Ctable), Cs0)) Res(37,30)
{y5/Ctable, s226/Cs0}

Prove: B is clear

S39: Equals(Cb, z6) v clear(Cb, result(move(y7, z6), Cs0))
Res(24,29) {s221/Cs0, x222/Cb}

S40: clear(Cb, result(move(y8, Ctable), Cs0)) Res(39,8)
{z6/Ctable}

Prove: C is clear

S41: ~clear(x10, s9) v ~on(x10, z11, s9) v clear(x10, result(move(x10, Ctable), s9)) Res(22,30) {s204/s226, y206/Ctable}

S42: ~on(Cc, z12, Cs0) v clear(Cc, result(move(Cc, Ctable), Cs0))
Res(41,28) {s9/Cs0, x10/Cc}

S43: clear(Cc, result(move(Cc, Ctable), Cs0)) Res(42,26) {z12/Ca}

Move B to C (Situation 2):

If we move B onto C, we need to prove the following:

Prove: A is on the table

S44: Equals(Ca, z13) v on(Ca, Ctable, result(move(z13, w212), result(move(Cc, w14), Cs0))) Res(33,23) {x215/Ca, s213/result(move(Cc, w54), Cs0), y214/Ctable}

S45: on(Ca, Ctable, result(move(Cb, w16), result(move(Cc, w15), Cs0))) Res(44,1) {z13/Cb}

Prove: B is on C

S46: ~clear(Cb, result(move(Cc, w17), Cs0)) v ~clear(y18, result(move(Cc, w17), Cs0)) v on(Cb, y18, result(move(Cb, y18), result(move(Cc, w17), Cs0))) Res(35,20) {z199/Ctable, s196/result(move(Cc, w3), Cs0), x197/Cb}

S47: ~clear(Cb, result(move(Cc, Ctable), Cs0)) v on(Cb, Cc, result(move(Cb, Cc), result(move(Cc, Ctable), Cs0))) Res(43,46) {w17/Ctable, y18/Cc}

S48: on(Cb, Cc, result(move(Cb, Cc), result(move(Cc, Ctable), Cs0))) Res(40,47) {y8/Cc}

Prove: A is clear

S49: Equals(Ca, z19) v clear(Ca, result(move(y20, z19), result(move(Cc, Ctable), Cs0))) Res(38,24) {s221/result(move(Cc, Ctable), Cs0), x222/Ca}

S50: clear(Ca, result(move(y21, Cc), result(move(Cc, Ctable), Cs0))) Res(49,2) {z19/Cc}

Prove: B is clear

S51: ~clear(x22, result(move(Cc, Ctable), Cs0)) v ~on(x22, z23, result(move(Cc, Ctable), Cs0)) v clear(x22, result(move(x22, Cc), result(move(Cc, Ctable), Cs0))) Res(43,22) {s204/result(move(Cc, Ctable), Cs0), y206/Cc}

S52: ~clear(Cb, result(move(Cc, Ctable), Cs0)) v clear(Cb, result(move(Cb, Cc), result(move(Cc, Ctable), Cs0))) Res(35,51) {z23/Ctable, x22/Cb, w3/Ctable}

S53: clear(Cb, result(move(Cb, Cc), result(move(Cc, Ctable), Cs0))) Res(40,52) {y8/Cc}

Move A onto B (Situation 3):

If we move A onto B, we need to prove the following:

Prove: A is on B

S54: ~clear(Ca, result(move(Cb, w25), result(move(Cc, w24), Cs0))) v ~clear(y26, result(move(Cb, w25), result(move(Cc, w24), Cs0))) v on(Ca, y26, result(move(Ca, y26), result(move(Cb, w25), result(move(Cc, w24), Cs0)))) Res(45,20) {z199/Ctable, s196/result(move(Cb, w16), result(move(Cc, w15), Cs0)), x197/Ca}

S55: ~clear(y27, result(move(Cb, Cc), result(move(Cc, Ctable), Cs0))) v on(Ca, y27, result(move(Ca, y27), result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)))) Res(50,54) {w24/Ctable, y21/Cb, w25/Cc}

S56: on(Ca, Cb, result(move(Ca, Cb), result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)))) Res(53,55) {y27/Cb}

Prove: B is on C

S57: Equals(Cb, z28) v on(Cb, Cc, result(move(z28, w29), result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)))) Res(48,23) {x215/Cb, s213/result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)), y214/Cc}

S58: on(Cb, Cc, result(move(Ca, w30), result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)))) Res(57,3) {z28/Ca}

We then fill in the answer by resolving A is on B, and B is on C with the "Answer" axiom.

S59: ~on(Cb, Cc, result(move(Ca, Cb), result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)))) v answer(result(move(Ca, Cb), result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)))) Res(56,31) {s228/result(move(Ca, Cb), result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)))}

Resolve Answer (A is on B ^ B is on C):

Prove: The final resolved answer.

The following is the final resolved answer:

S60: answer(result(move(Ca, Cb), result(move(Cb, Cc), result(move(Cc, Ctable), Cs0)))) Res(58,59) {w30/Cb}

Task 4.

Proof Printout:

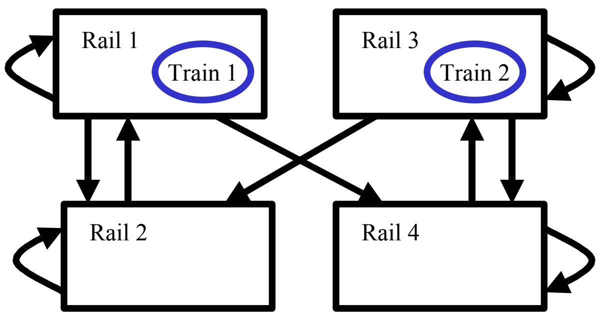
```
A1: rail(CR1)
A2: rail(CR2)
A3: rail(CR3)
A4: rail(CR4)
A5: ~rail(x2) v Equals(x2, CR1) v Equals(x2, CR2) v Equals(x2, CR3) v
Equals(x2, CR4)
A6: ~Equals(CR1, CR2)
A7: ~Equals(CR1, CR3)
A8: ~Equals(CR1, CR4)
A9: ~Equals(CR2, CR1)
A10: ~Equals(CR2, CR3)
A11: ~Equals(CR2, CR4)
A12: ~Equals(CR3, CR1)
A13: ~Equals(CR3, CR2)
A14: ~Equals(CR3, CR4)
A15: ~Equals(CR4, CR1)
A16: ~Equals(CR4, CR2)
A17: ~Equals(CR4, CR3)
A18: situation(CS1)
A19: situation(CS2)
A20: ~Equals(CS1, CS2)
A21: ~Equals(CS2, CS1)
A22: train(CT1)
A23: train(CT2)
A24: ~Equals(CT1, CT2)
A25: ~Equals(CT2, CT1)
A26: ~train(t4) v Equals(t4, CT1) v Equals(t4, CT2)
A27: connects(CR1, CR2)
A28: connects(CR3, CR2)
A29: connects(CR1, CR4)
A30: connects(CR3, CR4)
A31: connects(CR2, CR1)
A32: connects(CR4, CR3)
A33: ~connects(CR1, CR3)
A34: connects(CR1, CR1)
A35: ~connects(CR2, CR3)
A36: ~connects(CR2, CR4)
A37: connects(CR2, CR2)
A38: ~connects(CR3, CR1)
A39: connects(CR3, CR3)
A40: ~connects(CR4, CR2)
A41: ~connects(CR4, CR1)
A42: connects(CR4, CR4)
A43: ~on(t11, r110, s9) v ~on(t11, r212, s9) v Equals(r110, r212)
A44: rail(F_1(t17, s16))
A45: on(t19, F_1(t19, s18), s18)
A46: ~safe(s27) v ~on(t129, r28, s27) v ~on(t230, r28, s27) v
Equals(t129, t230)
A47: on(F_3(t223, t122, r21, s31), F_2(t223, t122, r21, s31), s31) v
safe(s31)
```

```
A48: on(F_4(t223, t122, r21, s32), F_2(t223, t122, r21, s32), s32) v
safe(s32)
A49: ~Equals(F_3(t236, t135, r34, s33), F_4(t236, t135, r34, s33)) v
safe(s33)
A50: ~on(t41, r240, CS2) v on(t41, F_5(r240, t41), CS1)
A51: ~on(t43, r242, CS2) v connects(F_5(r242, t43), r242)
A52: ~on(t45, r244, CS2) v legal(t45, F_5(r244, t45), r244)
A53: ~legal(t153, r152, r251) v Equals(r152, r251) v ~train(t254) v
~on(t254, r251, CS1)
A54: ~Equals(r156, r255) v legal(t157, r156, r255)
A55: train(F_6(t261, r260, r159, t158)) v legal(t158, r159, r260)
A56: on(F_6(t249, r262, r163, t164), r262, CS1) v legal(t164, r163,
r262)
A57: safe(CS1)
A58: on(CT1, CR1, CS1)
A59: on(CT2, CR3, CS1)
A60: ~legal(CT1, CR1, CR2)
S61: ~train(F_6(t212, r211, r110, t19)) v Equals(F_6(t212, r211, r110,
t19), CT2) v on(CT1, r211, CS1) v legal(t19, r110, r211)
Para(26,56) {t4/F_6(t249, r262, r163, t164)}
S62: ~train(F_6(t266, r265, r164, t163)) v on(CT1, r265, CS1) v
legal(t163, r164, r265) v on(CT2, r265, CS1) v legal(t163, r164, r265)
Para(61,56) {t19/t164, r211/r262, r110/r163, t212/t249}
S63: on(CT1, r267, CS1) v legal(t169, r168, r267) v on(CT2, r267, CS1)
v legal(t169, r168, r267) v legal(t169, r168, r267)      Res(62,55)
{r164/r159, t266/t261, r265/r260, t163/t158}
S64: on(CT1, r21, CS1) v on(CT2, r21, CS1) v legal(t13, r12, r21) v
legal(t13, r12, r21)      Fact(63) {}
S65: on(CT1, r24, CS1) v on(CT2, r24, CS1) v legal(t16, r15, r24)
Fact(64) {}
S66: ~on(t12, CR3, s11) v ~on(t12, CR2, s11)      Res(13,43) {r212/CR2,
r110/CR3}
S67: ~on(CT2, CR2, CS1)      Res(66,59) {t12/CT2, s11/CS1}
S68: ~on(t14, CR1, s13) v ~on(t14, CR2, s13)      Res(6,43) {r212/CR2,
r110/CR1}
S69: ~on(CT1, CR2, CS1)      Res(68,58) {s13/CS1, t14/CT1}
S70: on(CT1, CR2, CS1) v legal(t116, r115, CR2)      Res(65,67)
{r24/CR2}
S71: legal(t118, r117, CR2)      Res(69,70) {}
S72: F      Res(71,60) {t118/CT1, r117/CR1}
```

Proof Explanation:

In this proof, we wish to prove that it is legal for T1 to move to R2.
We know that Train 1 is on Rail 1 and Train 2 is on Rail 3. We also know that there are no other trains. The initial setup looks as follows:

Diagram : Initial Setup



This proof is broken into 2 main parts.

In the first part, we use the fact that there are only 2 trains (A26),

```
all t train(t) -> Equals(t,T1) v Equals(t,T2)
```

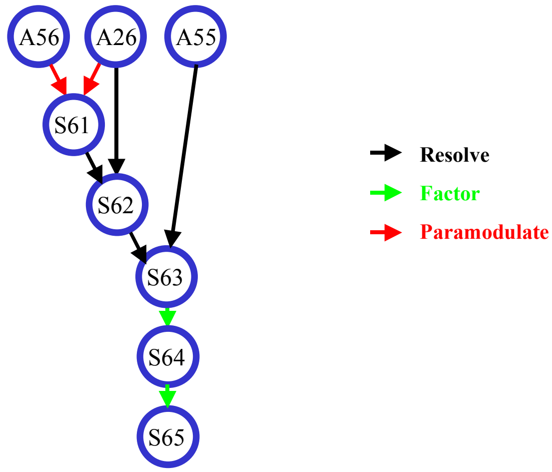
and if there is no train on a given track, then it is legal to move there,

```
all t1 all r1 all r2
    legal(t1,r1,r2) <->
     (Equals(r1,r2) v (~ exists t2 train(t2) ^ on(t2,r2,S1)))
```

to deduce that if neither T1 or T2 are on a given track, then it is legal to move there.

```
S65: on(CT1, r24, CS1) v on(CT2, r24, CS1) v legal(t16, r15, r24)
```

Diagram : Task 4 Part 1



The proof checker steps are as follows:

```
all t1 all r1 all r2
    legal(t1,r1,r2) <->
     (Equals(r1,r2) v (~ exists t2 train(t2) ^ on(t2,r2,S1)))
```

is broken up into A55 and A56 and is connected by the skolem funcion F_6.

```
A55: train(F_6(t261, r260, r159, t158)) v legal(t158, r159, r260)
A56: on(F_6(t249, r262, r163, t164), r262, CS1) v legal(t164, r163,
r262)

A26 says that there are only 2 trains.
A26: ~train(t4) v Equals(t4, CT1) v Equals(t4, CT2)
```

We paramodulate 56 with 26 twice because there are 2 trains and we will have to show that neither is on rail 2.

```
S61: ~train(F_6(t212, r211, r110, t19)) v Equals(F_6(t212, r211, r110,
t19), CT2) v on(CT1, r211, CS1) v legal(t19, r110, r211)
Para(26,56) {t4/F_6(t249, r262, r163, t164)}


S62: ~train(F_6(t266, r265, r164, t163)) v on(CT1, r265, CS1) v
legal(t163, r164, r265) v on(CT2, r265, CS1) v legal(t163, r164, r265)
Para(61,56) {t19/t164, r211/r262, r110/r163, t212/t249}
```

We then resolve this with A55, and factor it twice, to show that it is legal to move to a given rail if T1 and T2 are not on it.

```
S63: on(CT1, r267, CS1) v legal(t169, r168, r267) v on(CT2, r267, CS1)
v legal(t169, r168, r267) v legal(t169, r168, r267)      Res(62,55)
{r164/r159, t266/t261, r265/r260, t163/t158}


S64: on(CT1, r21, CS1) v on(CT2, r21, CS1) v legal(t13, r12, r21) v
legal(t13, r12, r21)      Fact(63) {}


S65: on(CT1, r24, CS1) v on(CT2, r24, CS1) v legal(t16, r15, r24)
Fact(64) {}
```

This result will be resolved after we generate the negation of the 2 "on" relations.

In the second part, we use the fact that 2 trains cannot be on the same track (A43), to prove that neither Train 1 nor Train 2 is on Rail 2 (the desired destination).

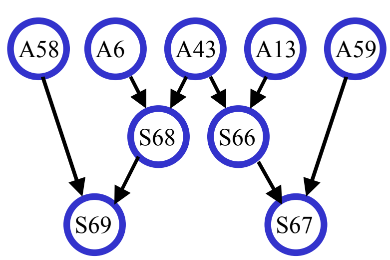
```
all s all t all r1 all r2 on(t, r1, s) ^ on(t, r2, s) -> Equals(r1, r2)
A43: ~on(t11, r110, s9) v ~on(t11, r212, s9) v Equals(r110, r212)
```

We know that Rail 1 and Rail 2 are not the same rail, and that Train 1 is on Rail 1 and Train 2 is on Rail 3.

```
A6:  ~Equals(CR1, CR2)
A13: ~Equals(CR3, CR2)
A58: on(CT1, CR1, CS1)
A59: on(CT2, CR3, CS1)
```

The proof checker steps are as follows:

Diagram : Task 4 Part 2




```
S66: ~on(t12, CR3, s11) v ~on(t12, CR2, s11)      Res(13,43) {r212/CR2,
r110/CR3}
S67: ~on(CT2, CR2, CS1)      Res(66,59) {t12/CT2, s11/CS1}
```

```
S68: ~on(t14, CR1, s13) v ~on(t14, CR2, s13)      Res(6,43) {r212/CR2,
r110/CR1}
S69: ~on(CT1, CR2, CS1)      Res(68,58) {s13/CS1, t14/CT1}
```

We then use these newly proven facts (that neither Train 1 nor Train 2 is on Rail 2) to resolve S65 from above.

```
S70: on(CT1, CR2, CS1) v legal(t116, r115, CR2)   Res(65,67) {r24/CR2}
S71: legal(t118, r117, CR2)      Res(69,70) {}
```

This result can resolve our proof:

```
S72: F      Res(71,60) {t118/CT1, r117/CR1}
```

Task 5:

The file trains2.txt contains the same axioms as the previous problem, but the conclusion is:

~(on(T1,R1,S1) ^ on(T2,R3,S1) -> ~safe(S2))

Now, we're asking you to show that, for this particular choice of initial conditions, the second situation is not safe.

Proof:

```
A1: rail(CR1)
A2: rail(CR2)
A3: rail(CR3)
A4: rail(CR4)
A5: ~rail(x2) v Equals(x2, CR1) v Equals(x2, CR2) v Equals(x2, CR3) v
Equals(x2, CR4)
A6: ~Equals(CR1, CR2)
A7: ~Equals(CR1, CR3)
A8: ~Equals(CR1, CR4)
A9: ~Equals(CR2, CR1)
A10: ~Equals(CR2, CR3)
A11: ~Equals(CR2, CR4)
A12: ~Equals(CR3, CR1)
A13: ~Equals(CR3, CR2)
A14: ~Equals(CR3, CR4)
A15: ~Equals(CR4, CR1)
A16: ~Equals(CR4, CR2)
A17: ~Equals(CR4, CR3)
A18: situation(CS1)
A19: situation(CS2)
A20: ~Equals(CS1, CS2)
A21: ~Equals(CS2, CS1)
A22: train(CT1)
A23: train(CT2)
A24: ~Equals(CT1, CT2)
A25: ~Equals(CT2, CT1)
A26: ~train(t4) v Equals(t4, CT1) v Equals(t4, CT2)
A27: connects(CR1, CR2)
A28: connects(CR3, CR2)
A29: connects(CR1, CR4)
A30: connects(CR3, CR4)
A31: connects(CR2, CR1)
A32: connects(CR4, CR3)
A33: ~connects(CR1, CR3)
A34: connects(CR1, CR1)
A35: ~connects(CR2, CR3)
A36: ~connects(CR2, CR4)
A37: connects(CR2, CR2)
A38: ~connects(CR3, CR1)
A39: connects(CR3, CR3)
```

```
A40: ~connects(CR4, CR2)
A41: ~connects(CR4, CR1)
A42: connects(CR4, CR4)
A43: ~on(t9, r18, CS1) v ~on(t9, r210, CS1) v Equals(r18, r210)
A44: rail(F_1(t15, s14))
A45: on(t17, F_1(t17, s16), s16)
A46: ~safe(s25) v ~on(t127, r26, s25) v ~on(t228, r26, s25) v
Equals(t127, t228)
A47: on(F_3(t221, t120, r19, s29), F_2(t221, t120, r19, s29), s29) v
safe(s29)
A48: on(F_4(t221, t120, r19, s30), F_2(t221, t120, r19, s30), s30) v
safe(s30)
A49: ~Equals(F_3(t234, t133, r32, s31), F_4(t234, t133, r32, s31)) v
safe(s31)
A50: ~on(t40, r239, CS2) v on(t40, F_5(r239, t40), CS1)
A51: ~on(t42, r241, CS2) v connects(F_5(r241, t42), r241)
A52: ~on(t44, r243, CS2) v legal(t44, F_5(r243, t44), r243)
A53: ~on(t46, r145, CS1) v ~connects(r145, r247) v ~legal(t46, r145,
r247) v on(t46, r247, CS2)
A54: ~legal(t155, r154, r253) v Equals(r154, r253) v ~train(t256) v
~on(t256, r253, CS1)
A55: ~Equals(r158, r257) v legal(t159, r158, r257)
A56: train(F_6(t263, r262, r161, t160)) v legal(t160, r161, r262)
A57: on(F_6(t251, r264, r165, t166), r264, CS1) v legal(t166, r165,
r264)
A58: safe(CS1)
A59: on(CT1, CR1, CS1)
A60: on(CT2, CR3, CS1)
A61: safe(CS2)
S62: ~train(F_6(t294, r293, r192, t191)) v Equals(F_6(t294, r293, r192,
t191), CT2) v on(CT1, r293, CS1) v legal(t191, r192, r293)
Para(26,57) {t4/F_6(t251, r264, r165, t166)}
S63: ~train(F_6(t2102, r2101, r1100, t199)) v on(CT1, r2101, CS1) v
legal(t199, r1100, r2101) v on(CT2, r2101, CS1) v legal(t199, r1100,
r2101)      Para(62,57) {t191/t166, r293/r264, t294/t251, r192/r165}
S64: on(CT1, r2103, CS1) v legal(t1105, r1104, r2103) v on(CT2, r2103,
CS1) v legal(t1105, r1104, r2103) v legal(t1105, r1104, r2103)
Res(63,56) {r1100/r161, t199/t160, r2101/r262, t2102/t263}
S65: ~on(t106, CR1, CS1) v ~on(t106, CR2, CS1)      Res(6,43)
{r210/CR2, r18/CR1}
S66: ~on(CT1, CR2, CS1)      Res(65,59) {t106/CT1}
S67: ~on(t107, CR3, CS1) v ~on(t107, CR2, CS1)      Res(13,43)
{r210/CR2, r18/CR3}
S68: ~on(CT2, CR2, CS1)      Res(67,60) {t107/CT2}
S69: legal(t1109, r1108, CR2) v on(CT2, CR2, CS1) v legal(t1109, r1108,
CR2) v legal(t1109, r1108, CR2)      Res(64,66) {r2103/CR2}
S70: legal(t1111, r1110, CR2) v legal(t1111, r1110, CR2) v legal(t1111,
r1110, CR2)      Res(68,69) {}
S71: legal(t1113, r1112, CR2) v legal(t1113, r1112, CR2)      Fact(70)
{}
S72: legal(t1115, r1114, CR2)      Fact(71) {}
S73: ~on(t117, r1116, CS1) v ~connects(r1116, CR2) v on(t117, CR2, CS2)
Res(72,53) {r247/CR2, r1114/r145, t1115/t46}
S74: ~connects(CR1, CR2) v on(CT1, CR2, CS2)      Res(73,59) {t117/CT1,
r1116/CR1}
```

```
S75: ~connects(CR3, CR2) v on(CT2, CR2, CS2)      Res(73,60) {t117/CT2,
r1116/CR3}
S76: on(CT1, CR2, CS2)      Res(74,27) {}
S77: on(CT2, CR2, CS2)      Res(75,28) {}
S78: ~safe(CS2) v ~on(t2118, CR2, CS2) v Equals(CT1, t2118)
Res(76,46) {s25/CS2, t127/CT1, r26/CR2}
S79: ~safe(CS2) v Equals(CT1, CT2)      Res(77,78) {t2118/CT2}
S80: ~safe(CS2)      Res(79,24) {}
S81: F      Res(80,61) {}
```

English explanation of the Proof:

For starters, the axioms in Trains1.txt and Trains2.txt differ for this particular axiom:

InTrains1.txt :

```
all s all t all r1 all r2 on(t, r1, s) ^ on(t, r2, s) -> Equals(r1, r2)
A43: ~on(t11, r110, s9) v ~on(t11, r212, s9) v Equals(r110, r212)

all t all r2
    on(t,r2,S2) ->
       exists r1 on(t,r1,S1) ^ connects(r1,r2) ^ legal(t,r1,r2)

A50: ~on(t41, r240, CS2) v on(t41, F_5(r240, t41), CS1)
A51: ~on(t43, r242, CS2) v connects(F_5(r242, t43), r242)
A52: ~on(t45, r244, CS2) v legal(t45, F_5(r244, t45), r244)
```

In Trains2.txt :

This relation is restricted to S1, which means that the trains are not restricted to being on only 1 rail in S2.

```
all t all r1 all r2 on(t, r1, S1) ^ on(t, r2, S1) -> Equals(r1, r2)
A43: ~on(t9, r18, CS1) v ~on(t9, r210, CS1) v Equals(r18, r210)
```

This relation is made into a bi-conditional.

```
all t all r2
    on(t,r2,S2) <->
       exists r1 on(t,r1,S1) ^ connects(r1,r2) ^ legal(t,r1,r2)
```

The bi-conditionality causes it to generate the following axioms, which includes an additional last "on" relation (A53),

```
A50: ~on(t40, r239, CS2) v on(t40, F_5(r239, t40), CS1)
A51: ~on(t42, r241, CS2) v connects(F_5(r241, t42), r241)
A52: ~on(t44, r243, CS2) v legal(t44, F_5(r243, t44), r243)
A53: ~on(t46, r145, CS1) v ~connects(r145, r247) v ~legal(t46, r145,
r247) v on(t46, r247, CS2)
```

Axioms 50 - 52 essentially say that if a train is on a given node in situation 2, then it was on connecting rail in situation 1, and the move was legal. These 3 axioms are connected by the skolem function F_5.

The additional axiom 53 says that if a train is on a given rail in situation 1, and it is connected to another rail, and it is legal to move to that rail, then it will be on that rail in situation 2. In other words, any legal move to a connected rail <u>will</u> be made.

Axiom 54 combined with axiom 43 mean that a given train will be on multiple rails in situation 2, if it has more than 1 possible moves. This really doesn't make any sense. However, this is why we can prove that train 1 will be on rail 2 and rail 4 in situation 2, and train 2 will be on rail 2 and rail 4 in situation 2.

This set of axioms, also allows us to prove that situation 2 is not safe. Without the bi-conditional, trains <u>could</u> move to the same rail, but they wouldn't have to.

Proof:

I constructed the proof as follows:

Steps 62 – 72, essentially mirror the proof from Task 4, which results in the axiom that it is legal to move to Rail 2.

```
S62: ~train(F_6(t294, r293, r192, t191)) v Equals(F_6(t294, r293, r192,
t191), CT2) v on(CT1, r293, CS1) v legal(t191, r192, r293)
Para(26,57) {t4/F_6(t251, r264, r165, t166)}
S63: ~train(F_6(t2102, r2101, r1100, t199)) v on(CT1, r2101, CS1) v
legal(t199, r1100, r2101) v on(CT2, r2101, CS1) v legal(t199, r1100,
r2101)      Para(62,57) {t191/t166, r293/r264, t294/t251, r192/r165}
S64: on(CT1, r2103, CS1) v legal(t1105, r1104, r2103) v on(CT2, r2103,
CS1) v legal(t1105, r1104, r2103) v legal(t1105, r1104, r2103)
Res(63,56) {r1100/r161, t199/t160, r2101/r262, t2102/t263}
S65: ~on(t106, CR1, CS1) v ~on(t106, CR2, CS1)      Res(6,43)
{r210/CR2, r18/CR1}
S66: ~on(CT1, CR2, CS1)      Res(65,59) {t106/CT1}
S67: ~on(t107, CR3, CS1) v ~on(t107, CR2, CS1)      Res(13,43)
{r210/CR2, r18/CR3}
S68: ~on(CT2, CR2, CS1)      Res(67,60) {t107/CT2}
```

```
S69: legal(t1109, r1108, CR2) v on(CT2, CR2, CS1) v legal(t1109, r1108,
CR2) v legal(t1109, r1108, CR2)      Res(64,66) {r2103/CR2}
S70: legal(t1111, r1110, CR2) v legal(t1111, r1110, CR2) v legal(t1111,
r1110, CR2)      Res(68,69) {}
S71: legal(t1113, r1112, CR2) v legal(t1113, r1112, CR2)      Fact(70)
{}
S72: legal(t1115, r1114, CR2)      Fact(71) {}
```

Next, we resolve this with A53 to show that the train that made this legal move was on a connecting rail, in S1.

```
S73: ~on(t117, r1116, CS1) v ~connects(r1116, CR2) v on(t117, CR2, CS2)
Res(72,53) {r247/CR2, r1114/r145, t1115/t46}
```

We then resolve this (separately) with the axioms that Train 1 was on Rail 1 and Train 2 was on Rail 3.

```
S74: ~connects(CR1, CR2) v on(CT1, CR2, CS2)      Res(73,59) {t117/CT1,
r1116/CR1}
S75: ~connects(CR3, CR2) v on(CT2, CR2, CS2)      Res(73,60) {t117/CT2,
r1116/CR3}
```

We then resolve these, respectively, with the axioms that Rail 1 is connected to Rail 2 and Rail 3 is connected to Rail 2.

```
S76: on(CT1, CR2, CS2)      Res(74,27) {}
S77: on(CT2, CR2, CS2)      Res(75,28) {}
```

These 2 results can then be used to resolve the "save" axiom.

```
S78: ~safe(CS2) v ~on(t2118, CR2, CS2) v Equals(CT1, t2118)
Res(76,46) {s25/CS2, t127/CT1, r26/CR2}
S79: ~safe(CS2) v Equals(CT1, CT2)      Res(77,78) {t2118/CT2}
S80: ~safe(CS2)      Res(79,24) {}
```

This result resolves our negation to False.

```
S81: F      Res(80,61) {}
```

6. Extra Credit: Estimate the size of the search space in the last problem. Can you think of an example where theorem proving would be a better approach than model-checking and vice versa?

The size of the search space is exponential, with regard to the number of axioms. The depth of my proof is 9. In other words, if an automatic theorem prover used breadth first search, it would solve the proof in the 9^{th} layer of the search tree.

The size of the tree depends on how many new axioms can be created from a pair of axioms. In our proof, there were situations that could resolve to 2 different axioms. This could be higher. If that factor is B, then the size of the search space is:

O((B^9) * (31^9))

This is a worst-case scenario because many of the axioms don't resolve.
something like the number of each type of variable (rails, trains, situations) times the number of instances of that type of variable (4, 2, 2 respectively).

A theorem prover is better than a model checker when there are a small set of axioms, a large universe and a short proof. For example, in the "all men are mortal" proof, if there was a large universe (more than just Hera), the model checker must consider all people, in order to show conclusively that there is no x that is not a man. In this case, x could include all men, animals, objects, and etcetera. However, the theorem prover would solve it in just a few steps.

This is because the search space for a model checker is 2^number of variables, rather than (number of axioms) ^ (depth of tree).

A model checker is better when there are many axioms, but a small universe.
The train example is a case in point. It has only 4 rails, 2 trains and 2 situations. However, it has many axioms. For this situation, it would be more efficient to test it using a theorem prover.

7. Extra Credit: Suppose you were writing a fully automatic resolution-refutation theorem prover that tries to derive a contradiction from the entire space of axioms. What kinds of heuristics might be useful for such a theorem-prover in searching for clauses to resolve with each other?

First we need to define what we mean by fully automatic. The input axioms for a theorem prover can be divided into sets, one set, could be important facts about the problem. In this case, every resolution step would resolve a member of this set, against another axiom. This is a forward search strategy. However, you could consider this not to be fully automatic, because the prover knows what facts about the problem are important.

Another form would be to use a backward search from the axiom that we are trying to refute. This does not require knowledge of what axioms are important.

We need a heuristic function that eliminates some sub goals that are less interesting. To do this, the heuristic could correlate to the size of the clause or the difficulty in resolving it. For example, a unit clause would be resolved first and the longest clause would be resolved last. This makes sense because we are trying to end up with an empty clause that resolves to false.
The prover would work something like this. Start with the axiom, which is the refutation of what we want to prove. Then, put the other axioms into a map, where the axiom has a key that corresponds to each negated sub clause. This will allow us to pick out the axioms that will resolve with the negated axiom that we wish to refute. This is not the primary heuristic, however.

We then, resolve the negated axiom with all of the other axioms which are keyed to the negated sub clause. The axioms that are produced are put into the map of axioms. We get the smallest axiom and attempt to resolve it against all of the other axioms. We set aside axioms, as they cannot be resolved. We terminate when we get an empty axiom that resolves to false.

This strategy, while does not address paramodulation, attempts to avoid those axioms that just get longer an longer. For example, in the siblings proof, we can generate very long axioms, by combining siblings, their father, their father's father, etc. that just get farther and farther from resolving the proof