Monolithic Compiler Experiments Using C++ Expression Templates*

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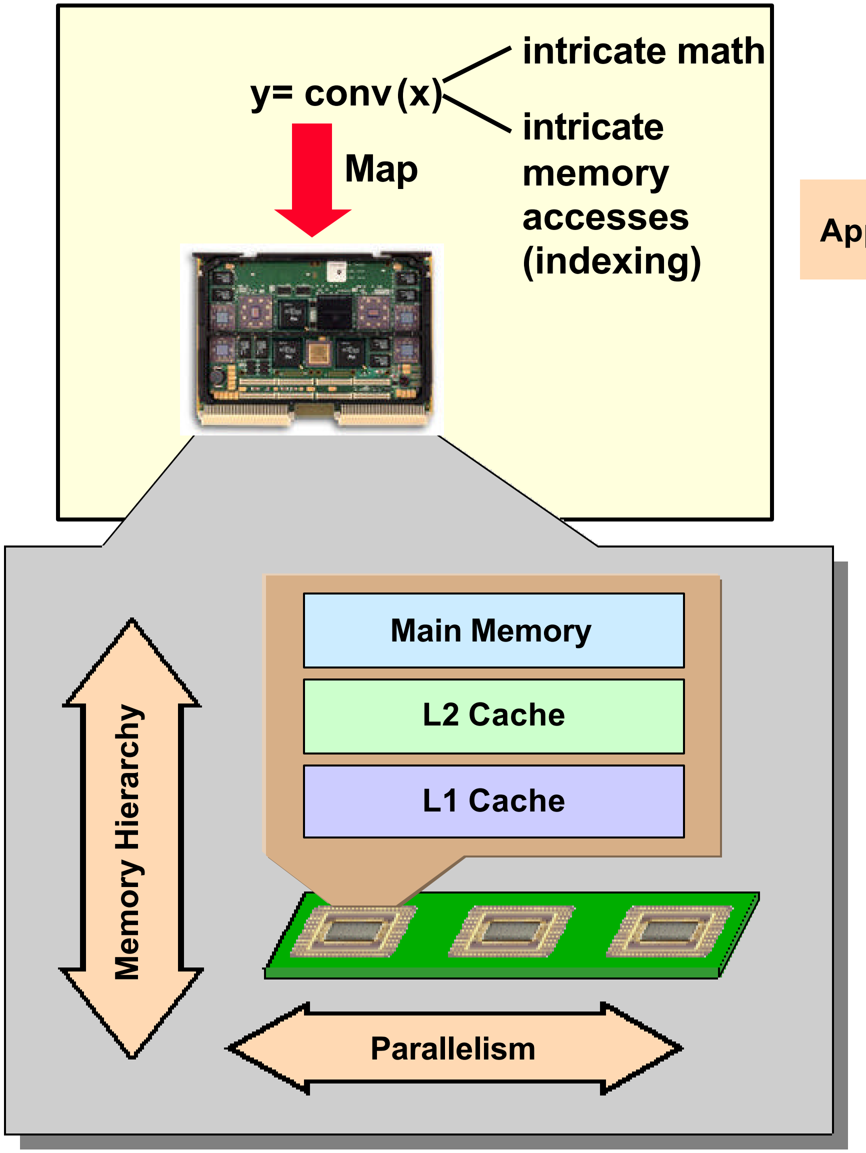
** Dr. Mullin participated in this work while on sabbatical leave from the Dept. of Computer Science, University of Albany, State University of New York, Albany, NY.

Outline

- **Overview**
  - Motivation
  - The Psi Calculus
  - Expression Templates
- Implementing the Psi Calculus with Expression Templates
- Experiments
- Future Work and Conclusions

Motivation: The Mapping Problem



Approach

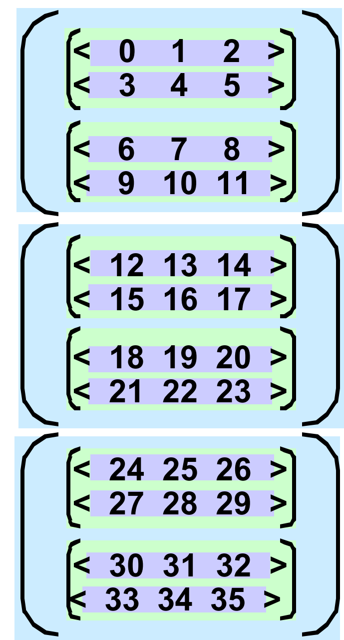
Mathematics of Arrays

- Math and indexing operations in same expression
- Framework for design space search
  - Rigorous and provably correct
  - Extensible to complex architectures

Example: "raising" array dimensionality

x: < 0 1 2 … 35 >

Map:



Basic Idea

- **Expression Templates**
  - Efficient high-level container operations
  - C++
- **Psi Calculus**
  - Array operations that compose efficiently
  - Minimum number of memory reads/writes

Implementation

Theory

PETE Style Array Operations

Benefits

- Theory based
- High level API
- Efficient

Combining Expression Templates and Psi Calculus yields an optimal implementation of array operations

Psi Calculus[1] Key Concepts

Denotational Normal Form (DNF):
- Minimum number of memory reads/writes for a given array expression
- Independent of data storage order

Gamma function: Specifies data storage order

Operational Normal Form (ONF):
- Like DNF, but takes data storage into account
- For 1-d expressions, consists of one or more loops of the form:
  $x[i]=y[\mathrm{stride}*i+\mathrm{offset}],\ l \leq i < u$
- Easily translated into an efficient implementation

- Psi Calculus rules are applied mechanically to produce the DNF, which is optimal in terms of memory accesses
- The Gamma function is applied to the DNF to produce the ONF, which is easily translated to an efficient implementation

[1] L. M. R. Mullin. A Mathematics of Arrays. PhD thesis, Syracuse University, December 1988.

Some Psi Calculus Operations

Operations	Arguments	Definition
take	*Vector* A, *int* N	Forms a Vector of the first N elements of A
drop	*Vector* A, *int* N	Forms a Vector of the last (A.size-N) elements of A
rotate	*Vector* A, *int* N	Forms a Vector of the last N elements of A concatenated to the other elements of A
cat	*Vector* A, *Vector* B	Forms a Vector that is the concatenation of A and B
unaryOmega	*Operation* Op, *dimension* D, *Array* A	Applies unary operator Op to D-dimensional components of A (like a for all loop)
binaryOmega	*Operation* Op, *Dimension* Adim. *Array* A, *Dimension* Bdim, *Array* B	Applies binary operator Op to Adim-dimensional components of A and Bdim-dimensional components of B (like a for all loop)
reshape	*Vector* A, *Vector* B	Reshapes B into an array having A.size dimensions, where the length in each dimension is given by the corresponding element of A
iota	*int* N	Forms a vector of size N, containing values 0 . . N-1

= index permutation   = operators   = restructuring   = index generation

Convolution: Psi Calculus Decomposition

Definition of y=conv(h,x)			
	y[n]= $\sum_{k=0}^{M-1} h[k]x'[n-k]$	where *x* has *N* elements, *h* has *M* elements, *0≤n<N+M-1*, and *x'* is *x* padded by *M-1* zeros on either end	

Algorithm and Psi Calculus Decomposition

Algorithm step	Psi Calculus	
Initial step	x= < 1 2 3 4 > h= < 5 6 7 >	x= < 1 2 3 4 > h= < 5 6 7 >
Form x'	x'=cat(reshape(<k-1>, <0>), cat(x, reshape(<k-1>,<0>)))=	x'= < 0 0 1 . . . 4 0 0 >
rotate x' (N+M-1) times	x'_{rot}=binaryOmega(rotate,0,iota(N+M-1), 1 x')	x'_{rot}= < 0 0 1 2 . . . > < 0 1 2 3 . . . > < 1 2 3 4 . . . > ⋮
take the "interesting" part of x'_{rot}	x'_{final}=binaryOmega(take,0,reshape(<N+M-1>,<M>),1,x'_{rot})	x'_{final}= < 0 0 1 > < 0 1 2 > < 1 2 3 > ⋮
multiply	Prod=binaryOmega (*,1, h,1,x'_{final})	Prod= < 0 0 7 > < 0 6 14 > < 5 12 21 > ⋮
sum	Y=unaryOmega (sum, 1, Prod)	Y= < 7 20 38 . . . >

Psi Calculus reduces this to DNF with minimum memory accesses

Typical C++ Operator Overloading

Example: A=B+C vector add

Main

1. Pass B and C references to operator +

B&, C&

Operator +

2. Create temporary result vector — temp
3. Calculate results, store in temporary — B+C → temp
4. Return copy of temporary

temp copy

5. Pass results reference to operator=

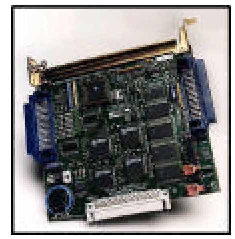
temp copy &

Operator =

6. Perform assignment — temp copy → A

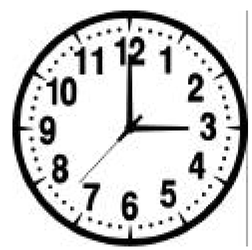
2 temporary vectors created

Additional Memory Use



- Static memory
- Dynamic memory (also affects execution time)

Additional Execution Time



- Cache misses/ page faults
- Time to create a new vector
- Time to create a copy of a vector
- Time to destruct both temporaries

C++ Expression Templates and PETE

Expression		Parse Tree	Expression Type
A=B+C	Expression Templates →	+ (B, C)	BinaryNode<OpAdd, Reference<Vector>, Reference<Vector > >

Main

1. Pass B and C references to operator +

B&, C&

Operator +

2. Create expression parse tree

+ (B&, C&)

3. Return expression parse tree

copy

4. Pass expression tree reference to operator

copy &

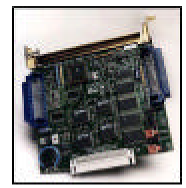
Operator =

5. Calculate result and perform assignment

B+C → A

Parse trees, not vectors, created

Reduced Memory Use



- Parse tree only contains references

Reduced Execution Time

- Better cache use
- Loop fusion style optimization
- Compile-time expression tree manipulation

- PETE, the Portable Expression Template Engine, is available from the Advanced Computing Laboratory at Los Alamos National Laboratory
- PETE provides:
  - Expression template capability
  - Facilities to help navigate and evaluating parse trees

PETE: http://www.acl.lanl.gov/pete

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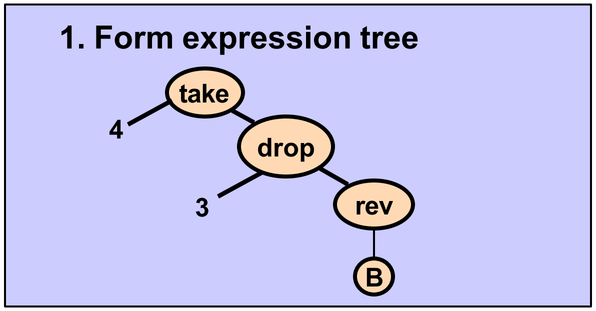
Implementing Psi Calculus with Expression Templates

Example:
A=take(4,drop(3,rev(B)))

B=<1 2 3 4 5 6 7 8 9 10>
A=<7 6 5 4>



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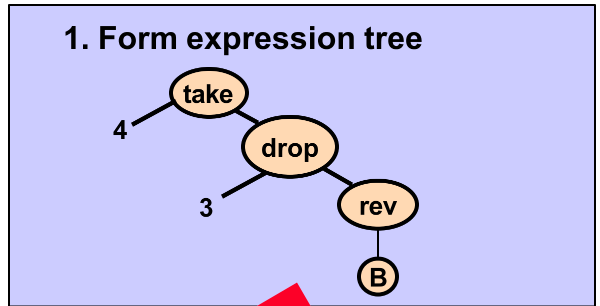

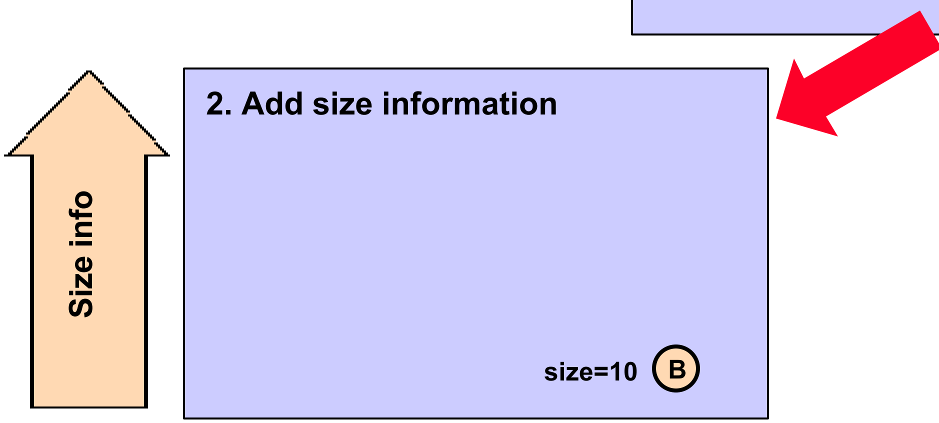

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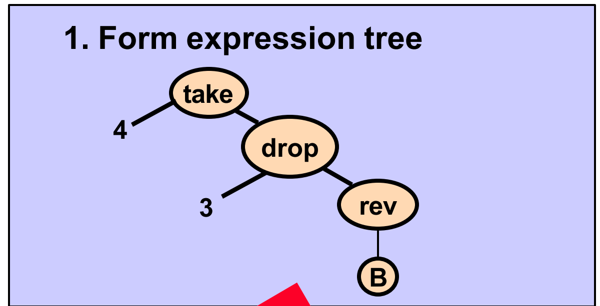

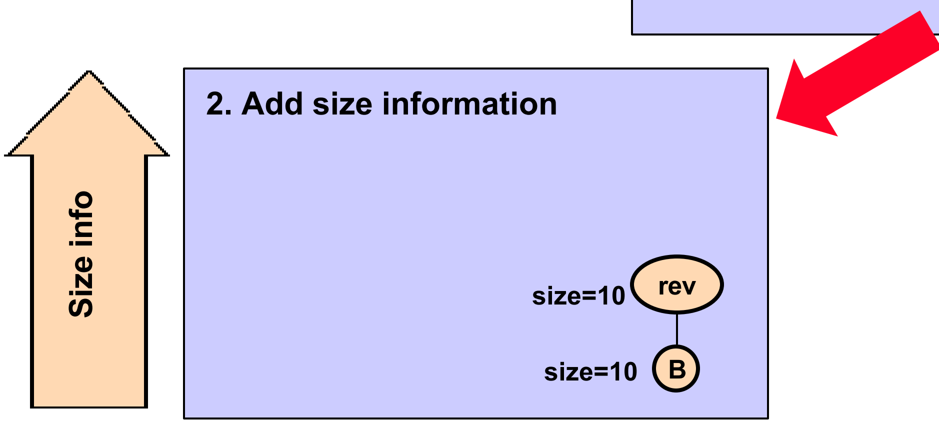

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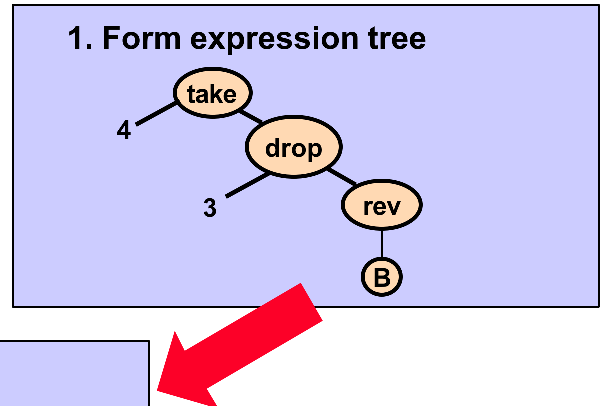

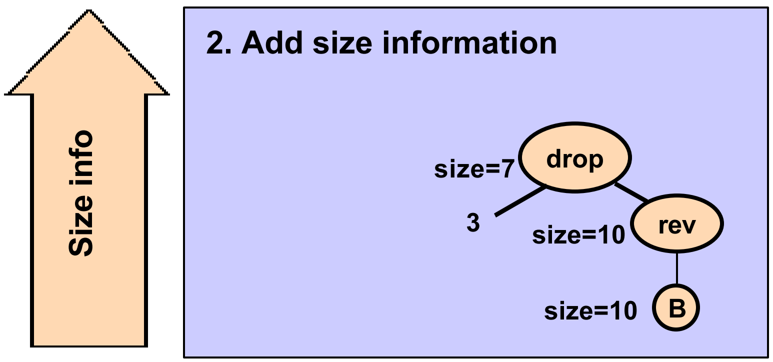

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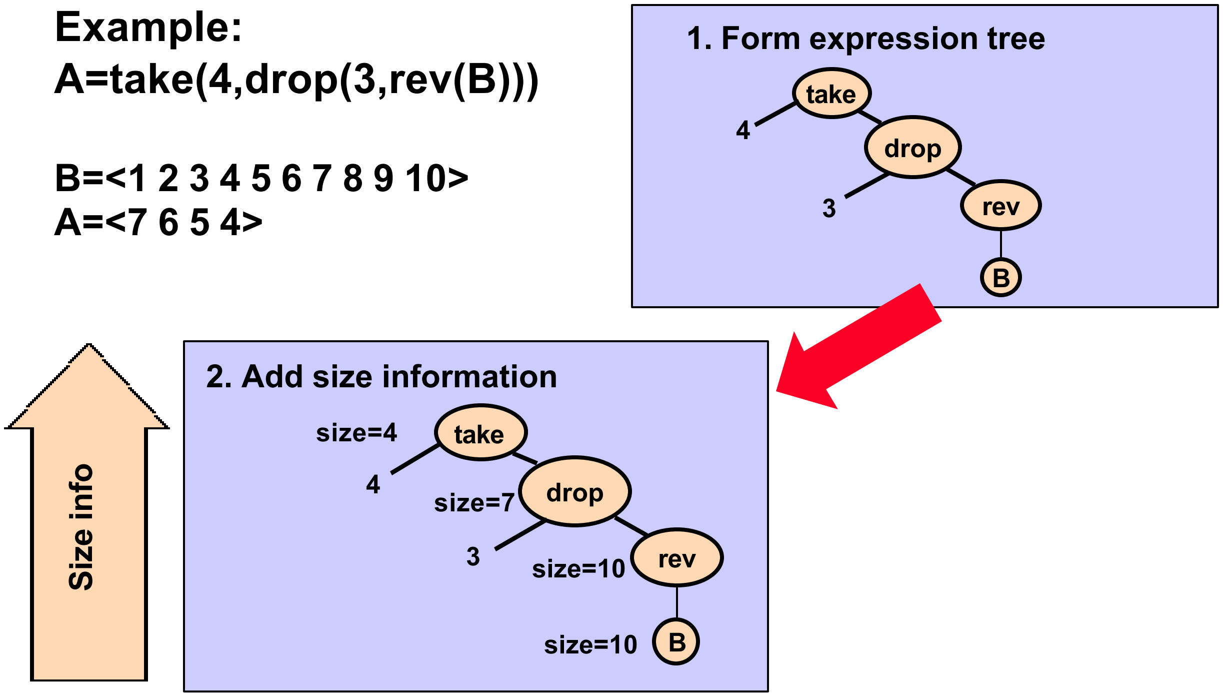

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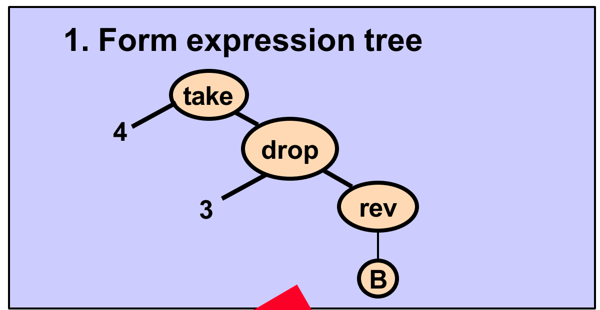

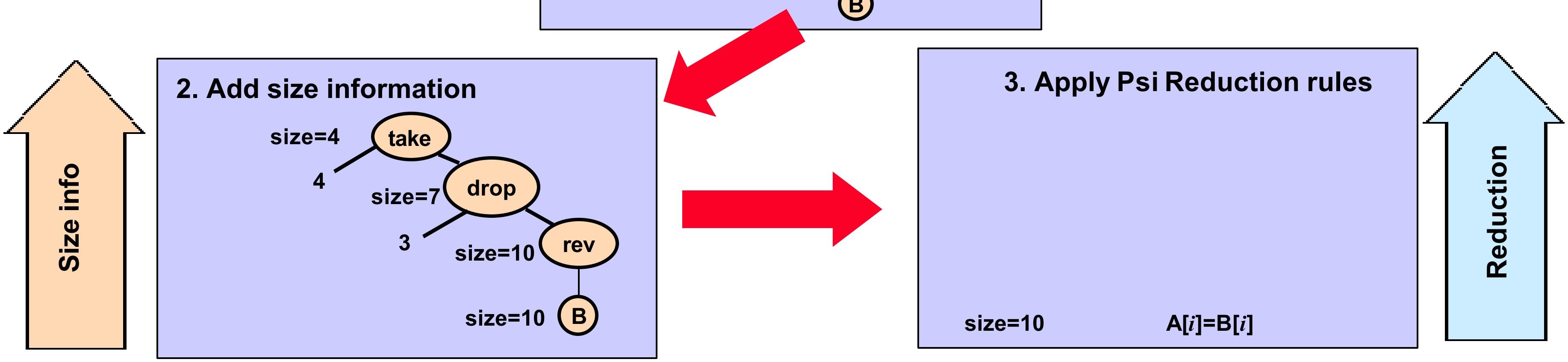

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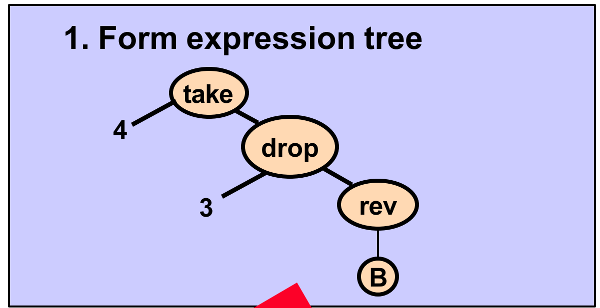

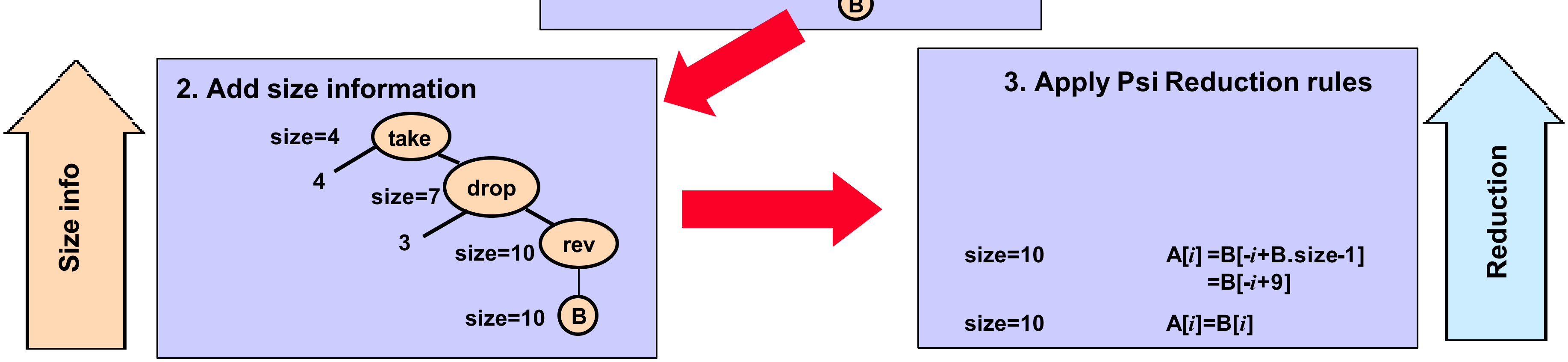

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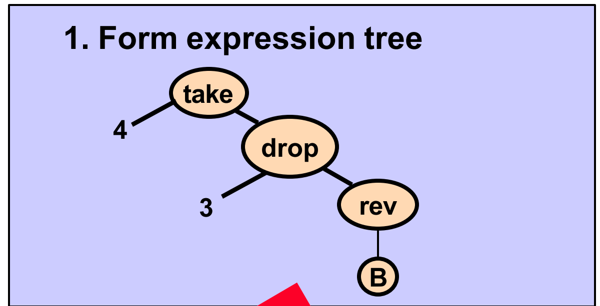

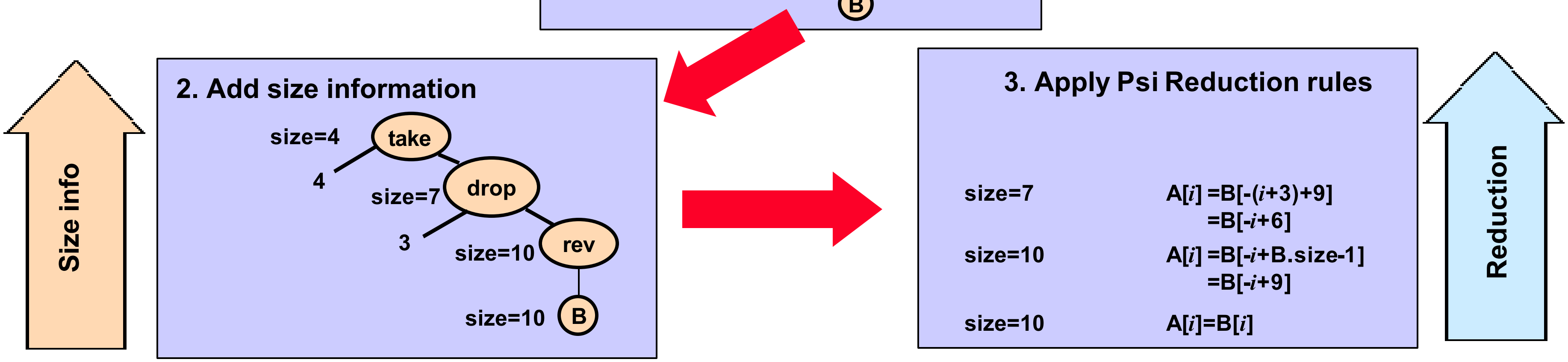

Implementing Psi Calculus with Expression Templates

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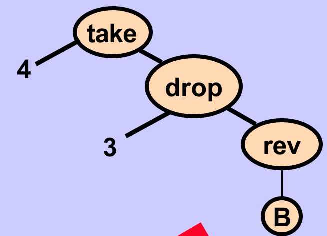
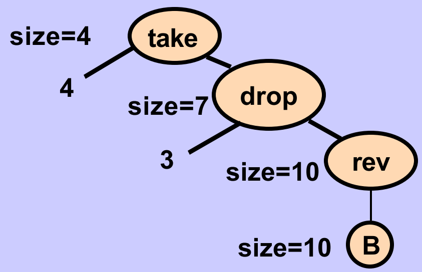
1. Form expression tree



Recall:
Psi Reduction for 1-d arrays always yields one or more expressions of the form:
x[*i*]=y[stridei*+ offset]**
$l \le i < u$

2. Add size information



Size info

3. Apply Psi Reduction rules

size=4	A[*i*]=B[-*i*+6]
size=7	A[*i*] =B[-(*i*+3)+9] =B[-*i*+6]
size=10	A[*i*] =B[-*i*+B.size-1] =B[-*i*+9]
size=10	A[*i*]=B[*i*]

Reduction

Implementing Psi Calculus with Expression Templates

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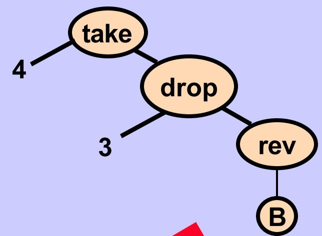

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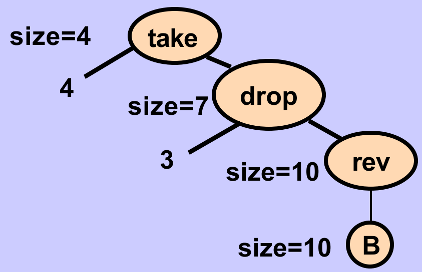

3. Apply Psi Reduction rules

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size=10	A[*i*] =B[-*i*+B.size-1] =B[-*i*+9]
size=10	A[*i*]=B[*i*]

Reduction

4. Rewrite as sub-expressions with iterators at the leaves, and loop bounds information at the root

size=4

iterator:
offset=6
stride=-1

- **Iterators used for efficiency, rather than recalculating indices for each *i***
- **One "for" loop to evaluate each sub-expression**

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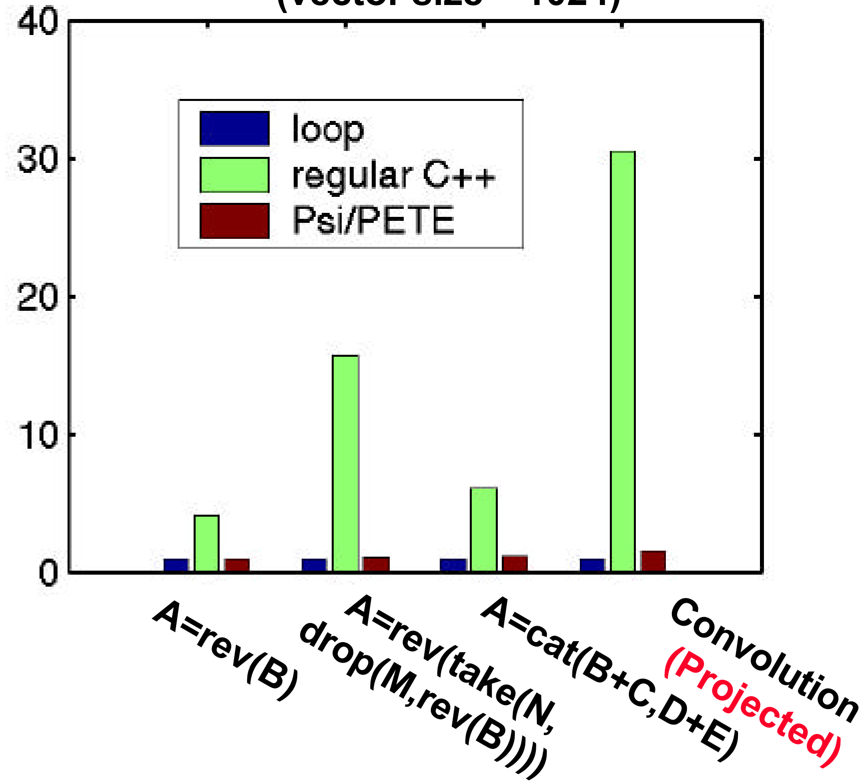
Experiments

Results

- Loop implementation achieves good performance, but is problem specific and low level
- Traditional C++ operator implementation is general and high level, but performs poorly when composing many operations
- PETE/Psi array operators perform almost as well as the loop implementation, compose well, are general, and are high level

Execution Time Normalized to Loop Implementation (vector size = 1024)

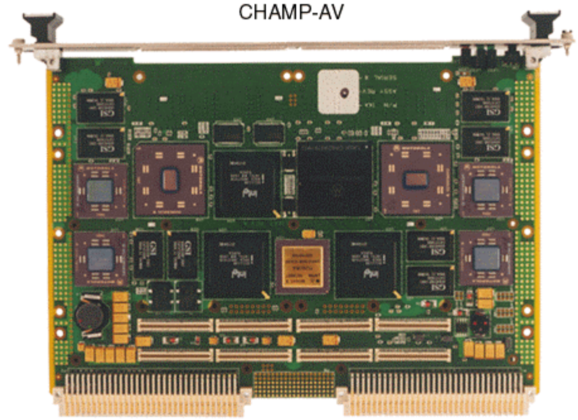


Test ability to compose operations

Experimental Platform and Method

Hardware
- DY4 CHAMP-AV Board
  - Contains 4 MPC7400's and 1 MPC 8420
- MPC7400 (G4)
  - 450 MHz
  - 32 KB L1 data cache
  - 2 MB L2 cache
  - 64 MB memory/processor

CHAMP-AV

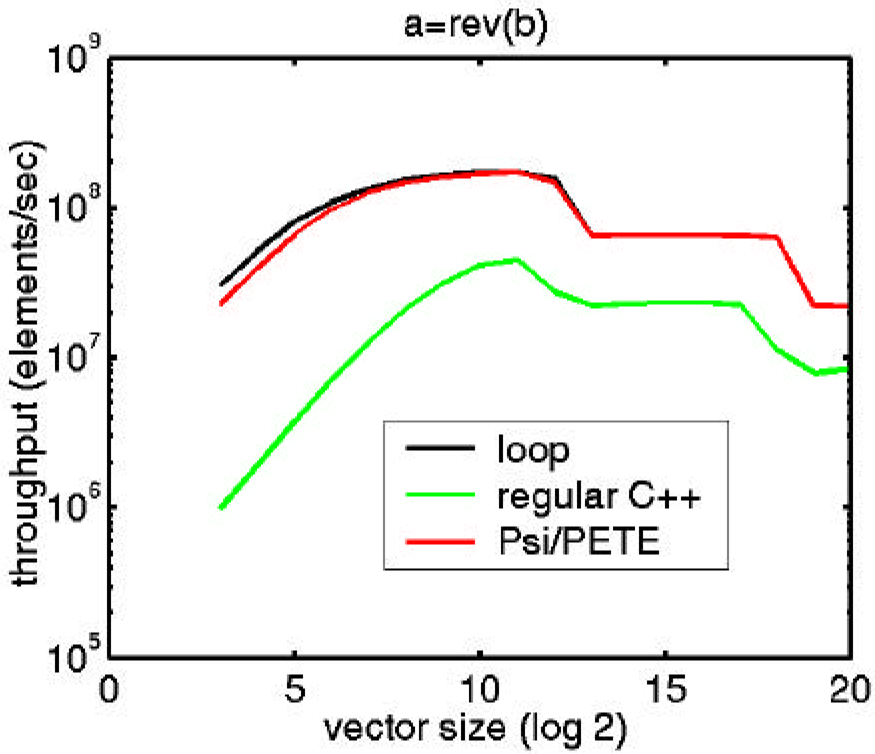


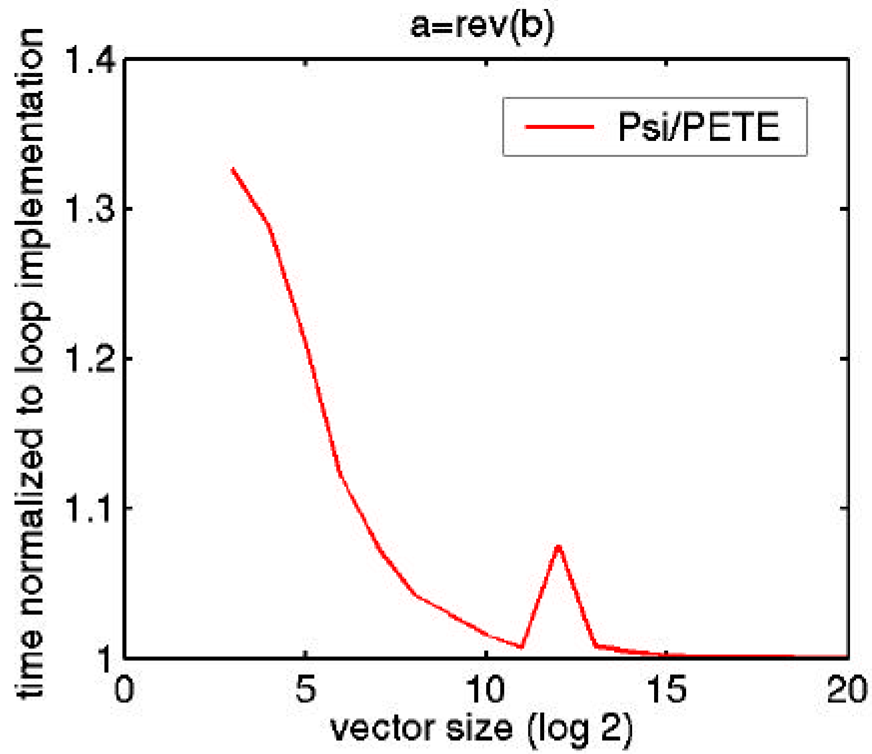
Software
- VxWorks 5.2
  - Real-time OS
- GCC 2.95.4 (non-official release)
  - GCC 2.95.3 with patches for VxWorks
  - Optimization flags:
    -O3 -funroll-loops -fstrict-aliasing

Method
- Run many iterations, report average, minimum, maximum time
  - From 10,000,000 iterations for small data sizes, to 1000 for large data sizes
- All approaches run on same data
- Only average times shown here
- Only one G4 processor used

> • Use of the VxWorks OS resulted in very low variability in timing
> • High degree of confidence in results

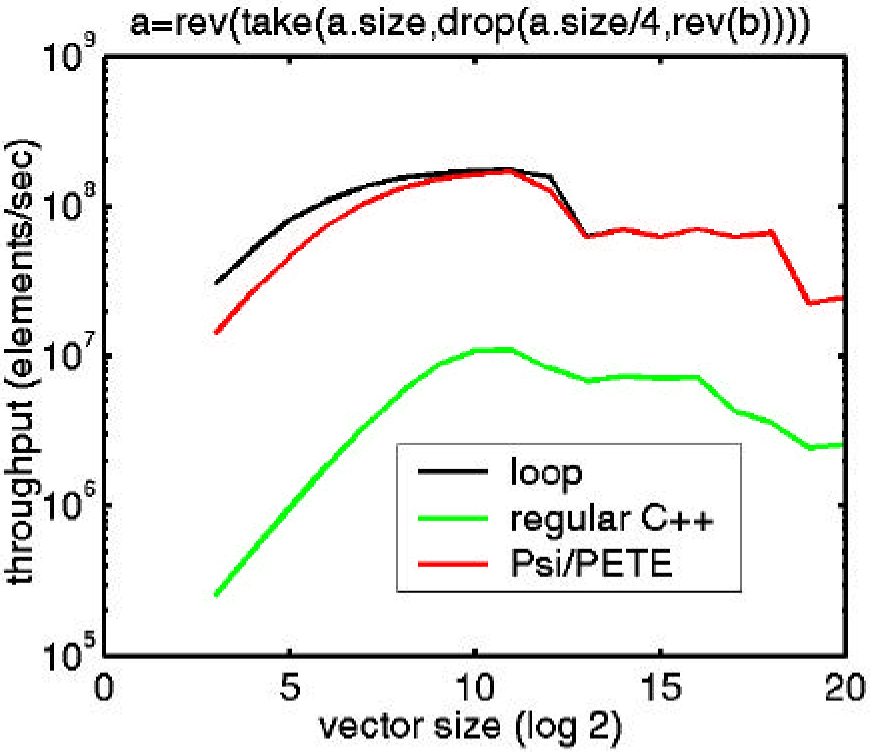
Experiment 1: A=rev(B)

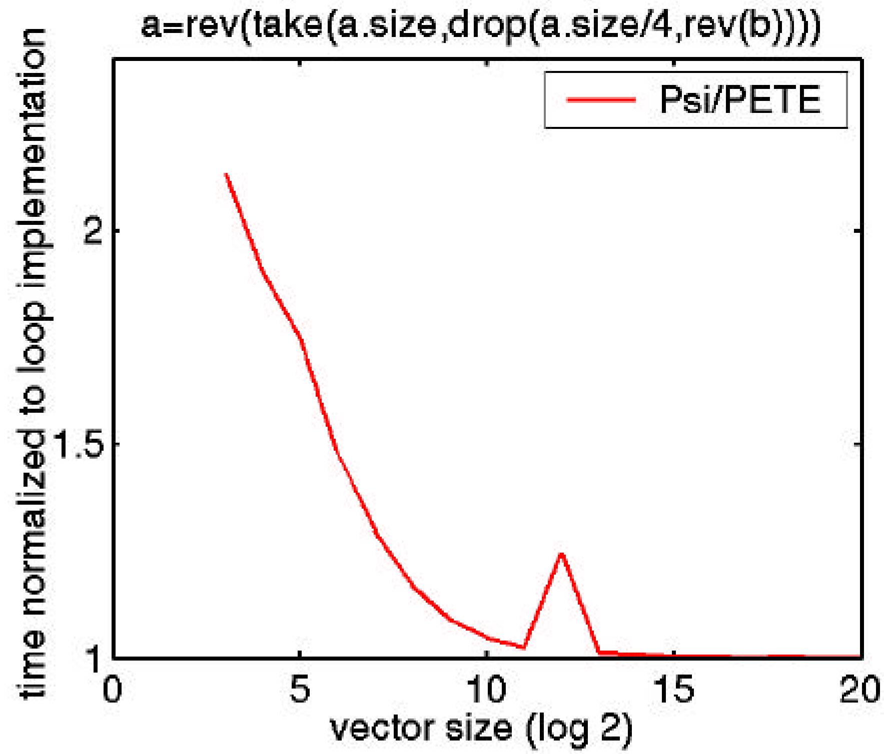




- **PETE/Psi implementation performs nearly as well as hand coded loop, and much better than regular C++ implementation**
- **Some overhead associated with expression tree manipulation**

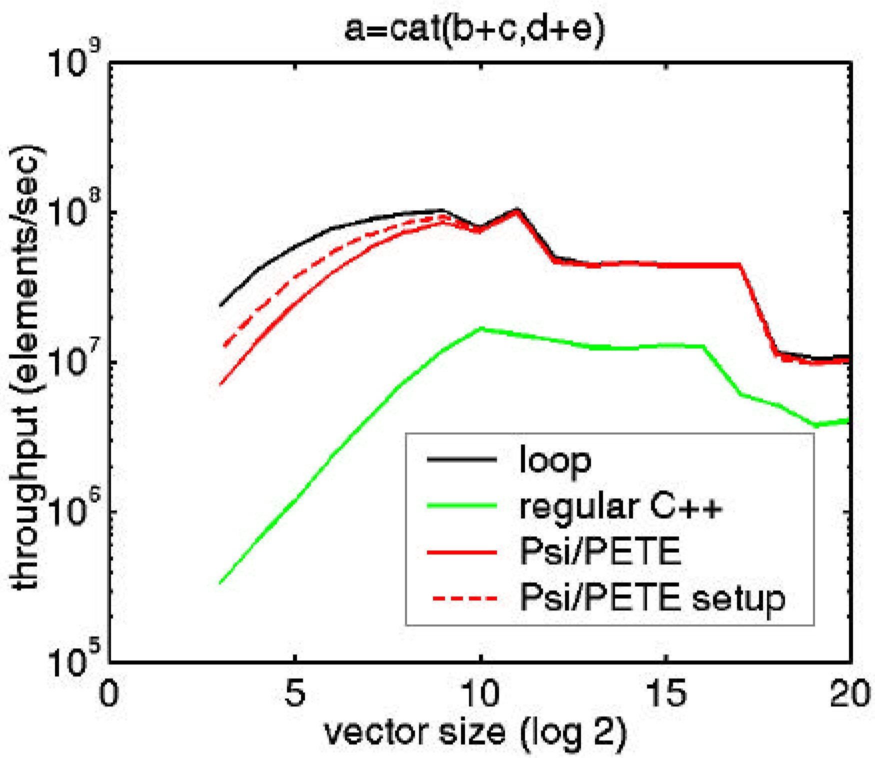
Experiment 2: a=rev(take(N,drop(M,rev(b)))

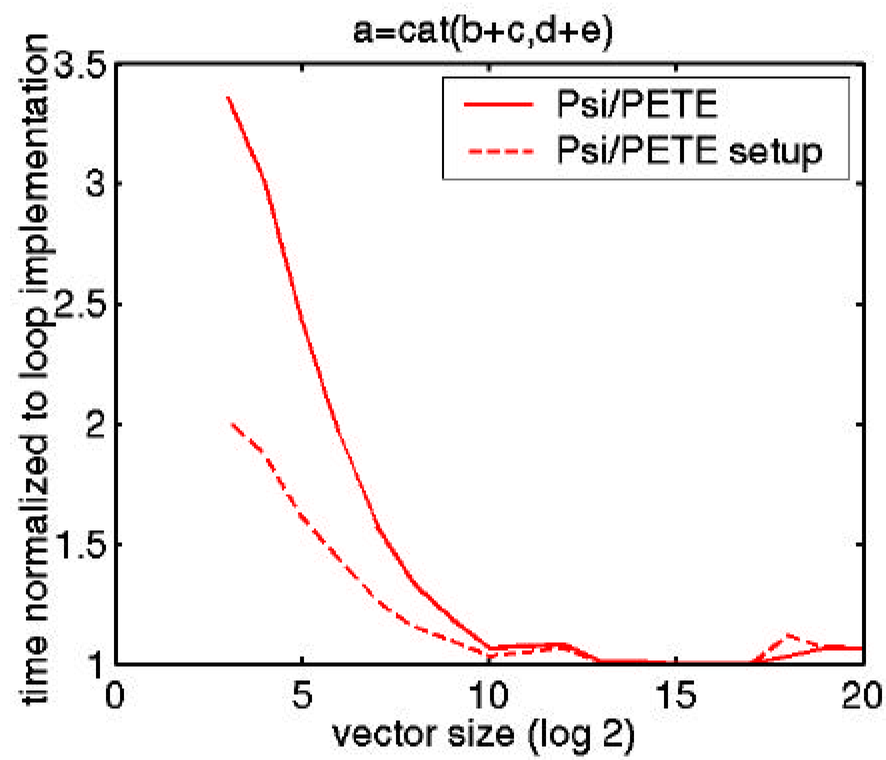




- Larger gap between regular C++ performance and performance of other implementations → regular C++ operators do not compose efficiently
- Larger overhead associated with expression-tree manipulation due to more complex expression

Experiment 3: a=cat(b+c, d+e)





- **Still larger overhead associated with tree manipulation due to cat()**
- **Overhead can be mitigated by “setup” step prior to assignment**

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Future Work

- **Multiple Dimensions:** Extend this work to N-dimensional arrays (N is any non-negative integer)
- **Parallelism:** Explore dimension lifting to exploit multiple processors
- **Memory Hierarchy:** Explore dimension lifting to exploit levels of memory
- **Mechanize Index Decomposition:** Currently a time consuming process done by hand
- **Program Block Optimizations:** PETE-style optimizations across statements to eliminate unnecessary temporaries

Conclusions

- Psi calculus provides rules to reduce array expressions to the minimum of number of reads and writes
- Expression templates provide the ability to perform compiler preprocessor-style optimizations (expression tree manipulation)
- Combining Psi calculus with expression templates results in array operators that
  - Compose efficiently
  - Are high performance
  - Are high level
- The C++ template mechanism can be applied to a wide variety of problems (e.g. tree traversal ala PETE, graph traversal, list traversal) to gain run-time speedup at the expense of compile time/space