

# Appendix A: Implementation Code

---

## 1. Utilities

```

Off[General::spell1]

Disp[x_] := (Print[x]; x)

Disp[x_, f_] := (Print[f[x]]; x)

DispConstraints[cc_] := Disp[cc, TableForm@Map[MatrixForm, #, {2}] &]

Pairs[x_] := Subsets[x, {2}]

MapGroup[expr_, f_] :=
  Last@Reap[MapThread[Sow, Part[List @@@ {expr, List /@ #}, All, Ordering[#]] &[f /@ expr]]]

MapGroup[expr_, {f_, m_List}] :=
  With[{r = Range@Length[m]}, With[{s = Sequence@@Flatten[Transpose@{m, r}, 1]},
    Flatten[#, 1] & /@ Last@Reap[Sow[#, List@Switch[f[#, s]] & /@ expr, r]]]

SparseArrayUnion[arrays_] := With[{rules = ArrayRules /@ arrays},
  SparseArray[Join@@Most /@ rules, Dimensions@First[arrays], Max[Last /@ Last /@ rules]]]

```

---

## 2. Example Problem

```

SetDirectory["C:/Documents and Settings/Administrator/Desktop/Documents/aca
  Academic/16412 Cognitive Robotics/aca-16412-ref-ProblemSet02Benchmarks"]

C:\Documents and Settings\Administrator\Desktop\Documents\aca
  Academic\16412 Cognitive Robotics\aca-16412-ref-ProblemSet02Benchmarks

p = q = 0; r = {};

data = Import[".\academics\4wqueens.wcsp", "Table"];

nvars = data[[1, 2]]

4

domains = data[[2]]

{4, 4, 4, 4}

```

```

constraints =
  Normal@{First@First[#], SparseArrayUnion[Last /@ #]} & /@ MapGroup[First@Last@Reap[
    Drop[data, 2] /. {_, v_, d_, l_}, c_] => (Sow@{{v} + 1, SparseArray[Most[#] + 1 ->
      Last[#] & /@ Take[{c}, l], domains[{{v} + 1}], d]}; Drop[{c}, 1])], First];

```

```

DispConstraints[constraints];

```

$$\begin{array}{l}
 (1) \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \\
 (2) \quad \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \\
 (3) \quad \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \\
 (4) \quad \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \\
 \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 5 & 5 & 0 & 0 \\ 5 & 5 & 5 & 0 \\ 0 & 5 & 5 & 5 \\ 0 & 0 & 5 & 5 \end{pmatrix} \\
 \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad \begin{pmatrix} 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \\ 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \end{pmatrix} \\
 \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad \begin{pmatrix} 5 & 0 & 0 & 5 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 5 & 0 & 0 & 5 \end{pmatrix} \\
 \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \begin{pmatrix} 5 & 5 & 0 & 0 \\ 5 & 5 & 5 & 0 \\ 0 & 5 & 5 & 5 \\ 0 & 0 & 5 & 5 \end{pmatrix} \\
 \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad \begin{pmatrix} 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \\ 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \end{pmatrix} \\
 \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \begin{pmatrix} 5 & 5 & 0 & 0 \\ 5 & 5 & 5 & 0 \\ 0 & 5 & 5 & 5 \\ 0 & 0 & 5 & 5 \end{pmatrix}
 \end{array}$$

### 3. Algorithm Implementation

```

Hyperedges[cc_] := First /@ cc

```

```

hyperedges = Hyperedges[constraints]

```

```

{{1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}}

```

```
Edges[hh_] := Union @@ Pairs /@ hh
```

```
edges = Edges[hyperedges]
```

```
{{1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}}
```

```
Neighbors[n_, ee_] := Flatten /@ Last@
```

```
Reap[Sow @@@ Flatten[{ee, Reverse /@ ee}, 1], Range[n], DeleteCases[Union[#2], #] &]
```

```
neighbors = Neighbors[nvars, edges]
```

```
{{2, 3, 4}, {1, 3, 4}, {1, 2, 4}, {1, 2, 3}}
```

```
FillNodeOrdering[bb_] :=
```

```
Ordering[Length@Select[Pairs[#], ! MemberQ[Part[bb, #], #2] & @ #] &] /@ bb]
```

```
FillNodeOrdering[neighbors]
```

```
{1, 2, 3, 4}
```

```
NodeOrder[n_, ee_, bb_] := Module[{v, edges = ee, neighbors = bb},
```

```
Flatten@Last@Reap@Do[
```

```
v = Sow@Take[FillNodeOrdering[neighbors], {i}];
```

```
edges = Union[DeleteCases[edges, {v, _} | {_, v}], Pairs[neighbors[[v]]];
```

```
neighbors = Neighbors[n, edges];
```

```
, {i, n}]
```

```
]
```

```
NodeOrder[nvars, edges, neighbors]
```

```
{1, 2, 3, 4}
```

```
ProjectConstraint[c_, y_] := Module[{vars, inter, domain, codomain},
```

```
vars = First[c];
```

```
inter = Intersection[vars, y];
```

```
domain = Flatten[Position[vars, #] & /@ inter];
```

```
codomain = Complement[Range@Length[vars], domain];
```

```
{inter, Map[Min@{p += Length[#]; #} & @ Flatten[#] &,
```

```
Transpose[Last[c], Ordering@Join[domain, codomain]], {Length[domain]}]}
```

```
DispConstraints@{constraints[[6]]};
```

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad \begin{pmatrix} 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \\ 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \end{pmatrix}$$

```
DispConstraints@{ProjectConstraint[constraints[[6]], {1}]};
```

$$(1) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

```
DispConstraints@{ProjectConstraint[constraints[[6]], {3}]};
```

$$(3) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{DispConstraints@}\left\{\text{ProjectConstraint}\left[\left\{\{2, 3, 4\}, \begin{pmatrix} 5 \\ 1 \\ 1 \\ 5 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 5 \\ 0 \end{pmatrix} \begin{pmatrix} 5 \\ 1 \\ 1 \\ 5 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 5 \\ 0 \end{pmatrix} \right\}, \{3, 4\}\right]\right\};$$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
CombineConstraints[cc : {__}] := First[cc]
```

```
CombineConstraints[cc : {_, _}] := Module[{vars, inter, indices, coindices},
  vars = First /@ cc;
  inter = Intersection@@vars;
  indices = Flatten /@ Outer[Position, vars, inter, 1];
  coindices = MapThread[Delete[Range@Length[#], List /@ #2] &, {vars, indices}];
  {Flatten@{inter, MapThread[Part, {vars, coindices}]},
   (q += Times@@Dimensions[#]; #) &@MapThread[Function[{x, y}, Map[# + y &, x, {-1}]],
    MapThread[Transpose[Last[#], Ordering@Join[##2]] &,
     {cc, indices, coindices}], Length[inter]]]
]
```

```
CombineConstraints[cc : {__}] := Fold[CombineConstraints[{{##}}] &, First[cc], Rest[cc]]
```

**DispConstraints[constraints[{{6}}]];**

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \\ 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \end{pmatrix}$$

**DispConstraints@{CombineConstraints[constraints[{{6}}]]};**

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \\ 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \end{pmatrix}$$

**DispConstraints[constraints[{{6, 8}}]];**

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \\ 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \begin{pmatrix} 5 & 5 & 0 & 0 \\ 5 & 5 & 5 & 0 \\ 0 & 5 & 5 & 5 \\ 0 & 0 & 5 & 5 \end{pmatrix}$$

**DispConstraints@{CombineConstraints[constraints[{{6, 8}}]]};**

$$\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} \begin{pmatrix} 10 \\ 10 \\ 5 \\ 5 \end{pmatrix} & \begin{pmatrix} 5 \\ 5 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 10 \\ 10 \\ 5 \\ 5 \end{pmatrix} & \begin{pmatrix} 5 \\ 5 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 5 \\ 5 \\ 5 \\ 0 \end{pmatrix} & \begin{pmatrix} 10 \\ 10 \\ 10 \\ 5 \end{pmatrix} & \begin{pmatrix} 5 \\ 5 \\ 0 \\ 5 \end{pmatrix} & \begin{pmatrix} 10 \\ 10 \\ 10 \\ 5 \end{pmatrix} \\ \begin{pmatrix} 5 \\ 10 \\ 10 \\ 10 \end{pmatrix} & \begin{pmatrix} 5 \\ 5 \\ 5 \\ 5 \end{pmatrix} & \begin{pmatrix} 5 \\ 10 \\ 10 \\ 10 \end{pmatrix} & \begin{pmatrix} 0 \\ 5 \\ 5 \\ 5 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 5 \\ 5 \end{pmatrix} & \begin{pmatrix} 5 \\ 5 \\ 10 \\ 10 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 5 \\ 5 \end{pmatrix} & \begin{pmatrix} 5 \\ 5 \\ 10 \\ 10 \end{pmatrix} \end{pmatrix}$$

**DispConstraints[constraints[{{3, 6, 8}}]];**

$$\begin{pmatrix} 3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \\ 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \begin{pmatrix} 5 & 5 & 0 & 0 \\ 5 & 5 & 5 & 0 \\ 0 & 5 & 5 & 5 \\ 0 & 0 & 5 & 5 \end{pmatrix}$$

```
DispConstraints@{CombineConstraints[constraints[{{3, 6, 8}}]}];
```

$$\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \left( \begin{pmatrix} 10 \\ 10 \\ 5 \\ 5 \end{pmatrix} \begin{pmatrix} 5 \\ 5 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 10 \\ 10 \\ 5 \\ 5 \end{pmatrix} \begin{pmatrix} 5 \\ 5 \\ 0 \\ 0 \end{pmatrix} \right. \\ \begin{pmatrix} 6 \\ 6 \\ 6 \\ 1 \end{pmatrix} \begin{pmatrix} 11 \\ 11 \\ 11 \\ 6 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \\ 6 \\ 1 \end{pmatrix} \begin{pmatrix} 11 \\ 11 \\ 11 \\ 6 \end{pmatrix} \\ \begin{pmatrix} 6 \\ 11 \\ 11 \\ 11 \end{pmatrix} \begin{pmatrix} 1 \\ 6 \\ 6 \\ 6 \end{pmatrix} \begin{pmatrix} 6 \\ 11 \\ 11 \\ 11 \end{pmatrix} \begin{pmatrix} 1 \\ 6 \\ 6 \\ 6 \end{pmatrix} \\ \left. \begin{pmatrix} 0 \\ 0 \\ 5 \\ 5 \end{pmatrix} \begin{pmatrix} 5 \\ 5 \\ 10 \\ 10 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 5 \\ 5 \end{pmatrix} \begin{pmatrix} 5 \\ 5 \\ 10 \\ 10 \end{pmatrix} \right)$$

```
BucketEliminate[n_, cc_] :=
Module[{constraints = cc, edges, neighbors, order},
First@Last@Reap[
vertices = Range[n];
edges = Edges@Hyperedges[constraints];
neighbors = Neighbors[n, edges];
order = Disp@NodeOrder[n, edges, neighbors];
Fold[
Function[{c, v}, (AppendTo [r, Total[Times @@@ Dimensions /@ Last /@ #]]]; #) &@
(Append[#, ProjectConstraint[Sow@Disp[CombineConstraints[#2], First],
vertices = Complement[vertices, {v}]]] &@@ MapGroup[c,
{MemberQ[First[#, v] &, {False, True}]]], constraints, order];
]]
```

```
buckets = BucketEliminate[nvars, constraints]; buckets // ColumnForm
```

 $\{1, 2, 3, 4\}$  $\{1, 2, 3, 4\}$  $\{2, 3, 4\}$  $\{3, 4\}$ 

{ 4 }

$$\begin{aligned} & \{1, 2, 3, 4\}, \{\{\{15, 10, 10, 15\}, \{10, 5, 5, 10\}, \{15, 10, 10, 15\}, \{10, 5, 5, 10\}\}, \{\{15, 10, \\ & \{2, 3, 4\}, \{\{\{15, 6, 11, 10\}, \{10, 5, 15, 5\}, \{10, 1, 6, 5\}, \{5, 0, 10, 0\}\}, \{\{12, 12, 7, 17\}, \\ & \{3, 4\}, \{\{6, 11, 1, 6\}, \{11, 11, 6, 6\}, \{7, 7, 12, 11\}, \{5, 0, 10, 5\}\}\} \\ & \{4\}, \{6, 0, 2, 5\} \end{aligned}$$

```

BucketExpand[n_, kk_] := Module[{assign, new},
  assign = Table[All, {n}];
  Scan[Function[c,
    new = First@Select[First[c], assign[[#]] == All &, 1];
    assign[[new]] =
      First@Ordering[Part[Last[c], Sequence@@Part[assign, First[c]]], 1];
  ], kk];
  MapThread[Rule, {Range[n], assign}]
]

```

```
BucketExpand[nvars, Reverse[buckets]]
```

```
{1 → 3, 2 → 1, 3 → 4, 4 → 2}
```

```
SolveWCSP[n_, cc_] := BucketExpand[n, Reverse@BucketEliminate[n, cc]]
```

```

SolveWCSP[path_] :=
Module[{data = Import[path, "Table"], nvars, constraints, domains},
  p = q = 0; r = {};
  nvars = data[[1, 2]];
  domains = data[[2]];
  constraints = Normal@{First@First[#], SparseArrayUnion[Last /@ #]} & /@
    MapGroup[First@Last@Reap[Drop[data, 2] /. {_, v_, d_, l_}, c_] =>
      (Sow@{{v} + 1, SparseArray[Most[#] + 1 → Last[#] & /@ Take[{c}, 1],
        domains[[{v} + 1], d]}; Drop[{c}, 1])], First];
  DispConstraints[constraints];
  SolveWCSP[nvars, constraints]
]

```

## 4. Algorithm Testing

For each example, the output shows the original constraints, followed by a sequence of variable subsets corresponding to the new combined constraints generated during at each step of the execution.

```
paths = FileNames["*.wcsp", ".", ∞]; paths // ColumnForm
```

```
.\academics\16wqueens.wcsp
.\academics\4wqueens.wcsp
.\academics\8wqueens.wcsp
.\academics\donald.wcsp
.\academics\send.wcsp
.\academics\zebra.wcsp
.\celar\CELAR6-SUB0.wcsp
.\celar\CELAR6-SUB1-24.wcsp
.\celar\CELAR6-SUB2.wcsp
.\dimacs\ssa0432-003.wcsp
.\dimacs\ssa2670-130.wcsp
.\dimacs\ssa2670-141.wcsp
.\random\denseloose\vcsp30_10_25_48_1.wcsp
.\random\denseloose\vcsp30_10_25_48_2.wcsp
.\random\denseloose\vcsp30_10_25_48_3.wcsp
.\random\denseloose\vcsp30_10_25_48_4.wcsp
.\random\denseloose\vcsp30_10_25_48_5.wcsp
.\random\densetight\vcsp25_10_25_87_1.wcsp
.\random\densetight\vcsp25_10_25_87_2.wcsp
.\random\densetight\vcsp25_10_25_87_3.wcsp
.\random\densetight\vcsp25_10_25_87_4.wcsp
.\random\densetight\vcsp25_10_25_87_5.wcsp
.\random\sparseloose\vcsp40_10_13_60_1.wcsp
.\random\sparseloose\vcsp40_10_13_60_2.wcsp
.\random\sparseloose\vcsp40_10_13_60_3.wcsp
.\random\sparseloose\vcsp40_10_13_60_4.wcsp
.\random\sparseloose\vcsp40_10_13_60_5.wcsp
.\random\sparssetight\vcsp25_10_21_85_1.wcsp
.\random\sparssetight\vcsp25_10_21_85_2.wcsp
.\random\sparssetight\vcsp25_10_21_85_3.wcsp
.\random\sparssetight\vcsp25_10_21_85_4.wcsp
.\random\sparssetight\vcsp25_10_21_85_5.wcsp
```

```
SolveWCSP[".\academics\zebra.wcsp"]
```

```
( 1 )  $\begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ 
```

```
( 22 )  $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 
```

```
 $\begin{pmatrix} 1 \\ 17 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}$ 
```



$$\begin{pmatrix} 23 \\ 16 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

[illegible]



































[illegible]

```

{15, 19, 22, 24, 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 20, 21, 23, 25, 6}

{11, 12, 13, 14, 15}

{16, 19, 17, 18, 20}

{22, 21, 23, 24, 25}

{21, 23, 24, 25}

{1, 17, 2, 3, 4, 5}

{2, 20, 3, 4, 5, 17}

{3, 7, 4, 5, 17, 20}

{4, 14, 5, 7, 17, 20}

{5, 25, 7, 14, 17, 20}

{7, 6, 8, 9, 10, 14, 17, 20, 25}

{8, 12, 6, 9, 10, 14, 17, 20, 25}

{9, 11, 6, 10, 12, 14, 17, 20, 25}

{10, 21, 6, 11, 12, 14, 17, 20, 25}

{11, 12, 14, 13, 6, 17, 20, 21, 25}

{6, 12, 13, 14, 17, 20, 21, 25}

{6, 13, 14, 17, 20, 21, 25}

{6, 14, 17, 20, 21, 25}

{16, 23, 17, 18, 20}

{17, 20, 6, 21, 25, 18, 23}

{6, 18, 20, 21, 23, 25}

{6, 20, 21, 23, 25}

{21, 23, 25, 6}

{6, 23, 25}

{6, 25}

{6}

{1 → 1, 2 → 3, 3 → 5, 4 → 4, 5 → 2, 6 → 1, 7 → 5, 8 → 3, 9 → 2, 10 → 4, 11 → 1, 12 → 3, 13 → 2, 14 → 4,
  15 → 5, 16 → 5, 17 → 2, 18 → 1, 19 → 4, 20 → 3, 21 → 4, 22 → 3, 23 → 5, 24 → 1, 25 → 2}

{p, q, r}

{10438905, 10025650,
 {13435, 10910, 8405, 7905, 7875, 7850, 7825, 7800, 7775, 392150, 392125, 392100,
  392075, 391450, 78950, 16425, 3925, 3900, 15775, 3250, 750, 125, 25, 5, 1}}

```

**SolveWCSP[".\academics\4wqueens.wcsp"]**

$$(1) \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$(2) \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$(3) \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

$$(4) \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} \begin{pmatrix} 5 & 5 & 0 & 0 \\ 5 & 5 & 5 & 0 \\ 0 & 5 & 5 & 5 \\ 0 & 0 & 5 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \begin{pmatrix} 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \\ 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 4 \end{pmatrix} \begin{pmatrix} 5 & 0 & 0 & 5 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 5 & 0 & 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} \begin{pmatrix} 5 & 5 & 0 & 0 \\ 5 & 5 & 5 & 0 \\ 0 & 5 & 5 & 5 \\ 0 & 0 & 5 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 4 \end{pmatrix} \begin{pmatrix} 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \\ 5 & 0 & 5 & 0 \\ 0 & 5 & 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} \begin{pmatrix} 5 & 5 & 0 & 0 \\ 5 & 5 & 5 & 0 \\ 0 & 5 & 5 & 5 \\ 0 & 0 & 5 & 5 \end{pmatrix}$$

{1, 2, 3, 4}

{1, 2, 3, 4}

{2, 3, 4}

{3, 4}

{4}

{1 → 3, 2 → 1, 3 → 4, 4 → 2}

{p, q, r}

{340, 516, {124, 40, 8, 1}}

**SolveWCSP[".\academics\8wqueens.wcsp"]**

$$\begin{array}{l} (1) \quad \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \\ (2) \quad \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \\ (3) \quad \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \\ (4) \quad \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \\ (5) \quad \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \\ (6) \quad \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \\ (7) \quad \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \end{array}$$

[illegible]

$$\begin{pmatrix} 2 \\ 8 \end{pmatrix} \begin{pmatrix} 9 & 0 & 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 9 & 0 & 0 & 0 & 0 & 0 & 9 \\ 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 \\ 9 & 0 & 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 9 & 0 & 0 & 0 & 0 & 0 & 9 \end{pmatrix}$$





$$\begin{pmatrix} 7 \\ 8 \end{pmatrix} \begin{pmatrix} 9 & 9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 9 & 9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 9 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & 9 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 9 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 9 & 9 & 0 \end{pmatrix}$$

{1, 2, 3, 4, 5, 6, 7, 8}

{1, 2, 3, 4, 5, 6, 7, 8}

{2, 3, 4, 5, 6, 7, 8}

{3, 4, 5, 6, 7, 8}

{4, 5, 6, 7, 8}

{5, 6, 7, 8}

{6, 7, 8}

{7, 8}

{8}

{1 → 2, 2 → 5, 3 → 7, 4 → 4, 5 → 1, 6 → 8, 7 → 6, 8 → 3}

**{p, q, r}**

{19173960, 24309768, {2098552, 263152, 33448, 4512, 728, 144, 16, 1}}

**SolveWCSP[".\academics\16queens.wcsp"]**

**SolveWCSP[".\academics\send.wcsp"]**

**SolveWCSP[".\dimacs\ssa0432-003.wcsp"]**

**SolveWCSP[".\random\sparseloose\vcsp40\_10\_13\_60\_1.wcsp"]**

**SolveWCSP[".\random\sparsetight\vcsp25\_10\_21\_85\_1.wcsp"]**

**SolveWCSP[".\random\densetight\vcsp25\_10\_25\_87\_1.wcsp"]**

**SolveWCSP[".\random\denseloose\vcsp30\_10\_25\_48\_1.wcsp"]**