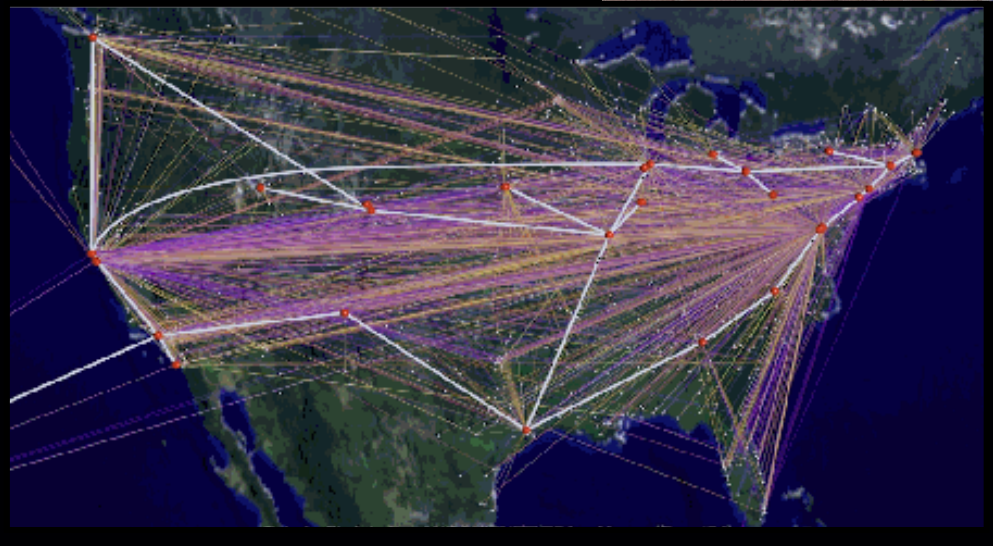
6.896: Topics in Algorithmic Game Theory

Lecture 13b

Constantinos Daskalakis

Markets



"Economics is a science which studies human behavior as a relationship between ends and scarce means which have alternative uses."

Lionel Robbins (1898 – 1984)

How are scarce resources assigned
to alternative uses?


i'm in everything

How are scarce resources assigned
to alternative uses?

Prices!

Parity between demand and supply

equilibrium prices

the beginnings of a mathematical theory

Augustin Cournot (1801-1877)

RESEARCHES

INTO THE

MATHEMATICAL PRINCIPLES

OF THE

THEORY OF WEALTH

BY

AUGUSTIN COURNOT

1838

TRANSLATED BY NATHANIEL T. BACON

WITH A

BIBLIOGRAPHY OF MATHEMATICAL ECONOMICS

BY IRVING FISHER

New York

THE MACMILLAN COMPANY

LONDON: MACMILLAN & CO., LTD.

1897

standpoint, a subject of general interest which has so many different sides.

But the title of this work sets forth not only theoretical researches; it shows also that I intend to apply to them the forms and symbols of mathematical analysis. This is a plan likely, I confess, to draw on me at the outset the condemnation of theorists of repute. With one accord they have set themselves against the use of mathematical forms, and it will doubtless be difficult to overcome to-day a prejudice which thinkers, like Smith and other more modern writers, have contributed to strengthen. The reasons for this prejudice seem to be, on the one hand, the false point of view from which theory has been regarded by the small number of those who have thought of applying mathematics to it; and, on the other hand, the false notion which has been formed of this analysis by men otherwise judicious and well versed in the subject of Political Economy, but to whom the mathematical sciences are unfamiliar.

Cournot's Contributions:

notion of a *demand function*

$D = F(p)$, where p is the price; F(.) is assumed continuous, and it is taken as an empirical proposition that the demand function is decreasing

analysis of a *monopoly*:

- profit-maximizing producer with *cost* f(D), for production D; discusses decreasing, constant and increasing cost functions

- equations determining *equilibrium price*

duopoly model:

two rival producers of a homogeneous product

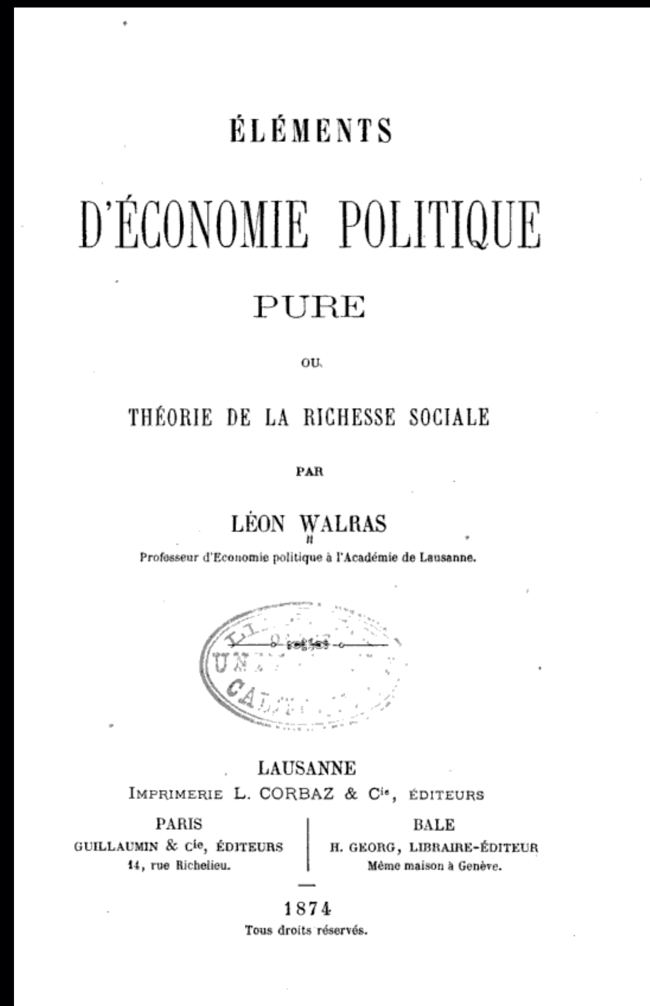
unlimited competition,

communication of markets on single commodity, ...

question left unanswered by Cournot…

react on each other. An increase in the income of the producers of commodity *A* will affect the demand for commodities *B*, *C*, etc., and the incomes of their producers, and, by its reaction, will involve a change in the demand for commodity *A*. It seems, therefore, as if, for a complete and rigorous solution of the problems relative to some parts of the economic system, it were indispensable to take the entire system into consideration. But this would surpass the powers of mathematical analysis and of our practical methods of calculation, even if the values of all the constants could be assigned to them numerically. The object

Leon Walras (1834-1910)



ÉLÉMENTS

D'ÉCONOMIE POLITIQUE

PURE

OU

THÉORIE DE LA RICHESSE SOCIALE

PAR

LÉON WALRAS

Professeur d'Économie politique à l'Académie de Lausanne.

LAUSANNE

IMPRIMERIE L. CORBAZ & Cie, ÉDITEURS

PARIS
GUILLAUMIN & Cie, ÉDITEURS
14, rue Richelieu.

BALE
H. GEORG, LIBRAIRE-ÉDITEUR
Même maison à Genève.

1874

Elements of Pure Economics, or the theory of social wealth, 1874

Leon Walras (1834-1910)

- goal was to solve the problem that was left open by Cournot, i.e. characterize the price equilibrium of a market with many commodities;

- gave system of simultaneous equations for price equilibrium:

> *informal argument for the existence of an equilibrium based on the assumption that an equilibrium exists whenever the number of equations equals the number of unknowns*

- recognized the need for *dynamics leading to an equilibrium*

tâtonnement: price adjustment mechanism

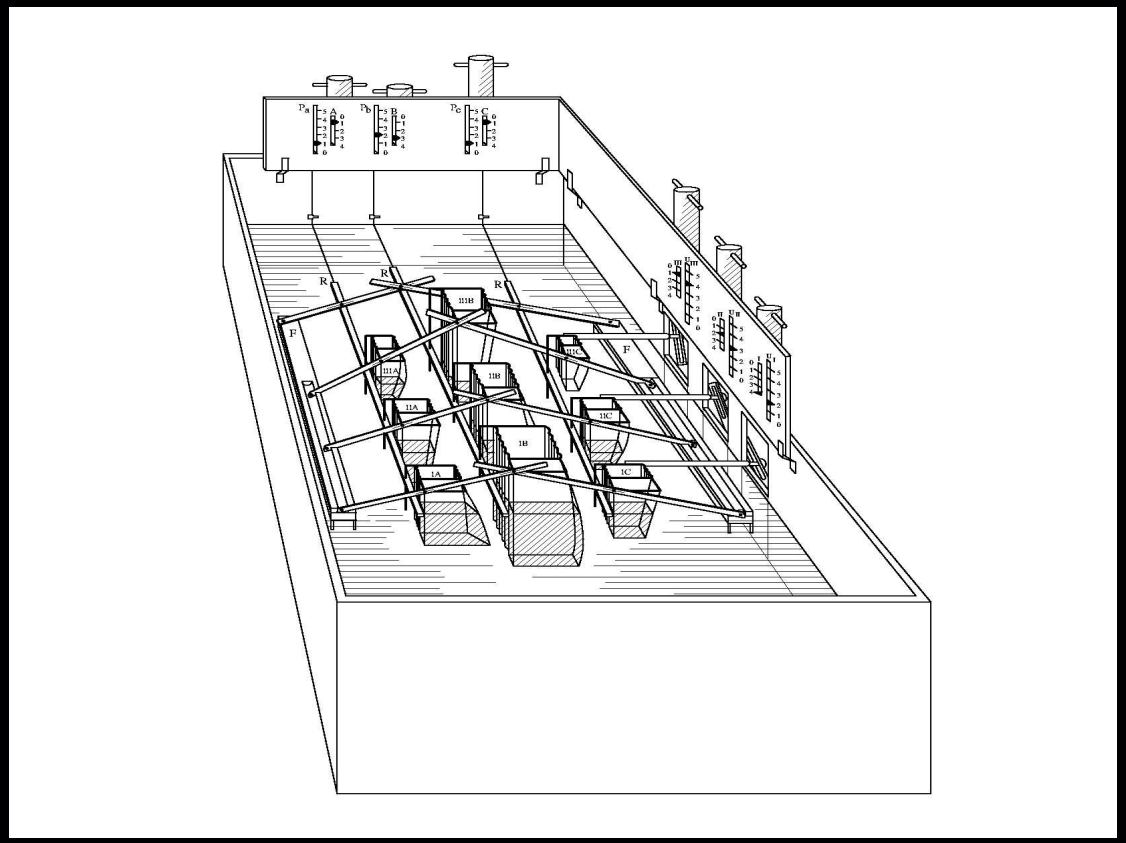
Irving Fisher (1867-1947)

hydraulic apparatus for solving markets with 3 traders with money and a producer of 3 commodities

Irving Fisher (1867-1947)

First computational approach!

1891



[Irving Fisher (1867-1947)

Stock Market Crash of 1929:

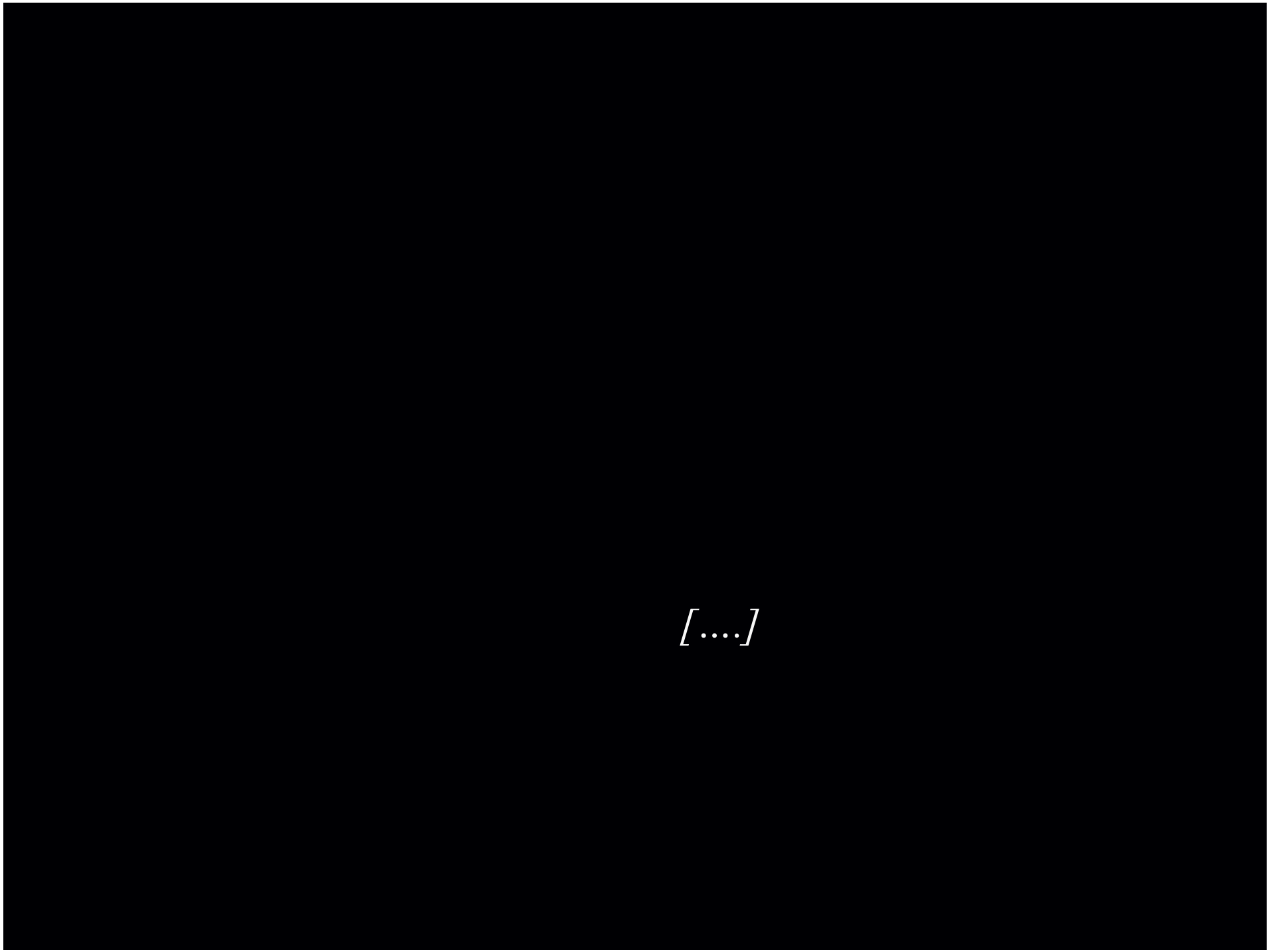
"Stock prices have reached what looks like a permanently high plateau."

[Shortly before the crisis]

Market is "only shaking out of the lunatic fringe"

[October 21, 1929 (8 days before black Tuesday)]

]



Arrow-Debreu-McKenzie Theorem

One of the most celebrated theorems in Mathematical Economics.

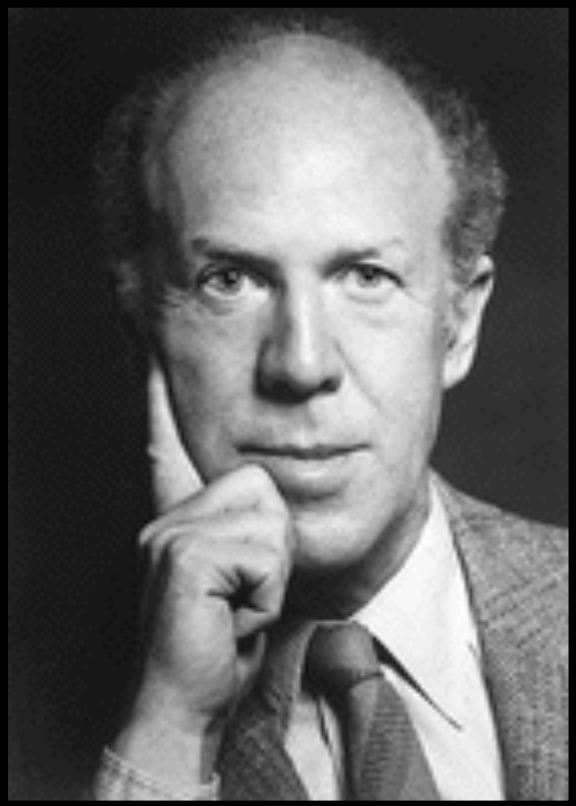
Established the existence of a market equilibrium under very general conditions using Brouwer/Kakutani's fixed point theorem.

it is hence highly non-constructive!

Kenneth Arrow

- Nobel Prize, 1972

Gerard Debreu



- Nobel Prize, 1983

Market Model (without production)

Consider a marketplace with:

n traders (or agents)

k goods (or commodities)
assumed to be *infinitely divisible*

Utility function of trader i:

$$u_i \ : \ \mathbb{R}_+^k \longrightarrow \mathbb{R}_+$$

non-negative reals

specifies trader's utility for bundles of goods

(common) assumption: u_i is continuous

Endowment of trader i:

$$e_i \ : \ \mathbb{R}_+^k$$

amount of goods trader comes to the marketplace with

Market Model (without production)

Suppose the goods in the market are priced according to some price vector $p \in \mathbb{R}^k_+$.

Under this price vector, each trader would like to sell some of his endowment and purchase an optimal bundle using her income from what s/he sold, solving the following program

$$\left.\begin{aligned} \max \;\; & u_i(x) \\ \text{s.t.} \;\; & p \cdot x \leq p \cdot e_i \\ & x \;\in\; \mathbb{R}^k_+ \end{aligned}\right\} \text{Program}_i$$

If u_i is continuous, then the above program has an optimum (since the constraint set is compact).

Competitive (or Walrasian) Market Equilibrium

A price vector $p \in \mathbb{R}^k_+$ is called a *competitive market equilibrium* iff there exists a collection of optimal solutions $\mathcal{X}_i(p)$ to Program$_i$, for all $i = 1,\ldots, n$, such that the total demand meets the total supply, i.e.

$$\underbrace{\sum_{i=1}^{n} \mathcal{X}_i(p)}_{\text{total demand}} \leq \underbrace{\sum_{i=1}^{n} e_i}_{\text{total supply}}$$

Arrow-Debreu Theorem 1954

Suppose:

(i) for all traders *i,* for all price vectors *p*, Program$_i$ has a unique optimum $\hat{\mathcal{X}}_i(p)$.

(ii) for all traders *i,* $\hat{\mathcal{X}}_i(p)$ is continuous.

e.g. when u_i is strictly concave

Then a competitive market equilibrium exists.