Price of Anarchy of Practical Auctions

Mechanism Design for Simple Auctions

Éva Tardos, Cornell

Joint work with Vasilis Syrgkanis

Games and Quality of Solutions

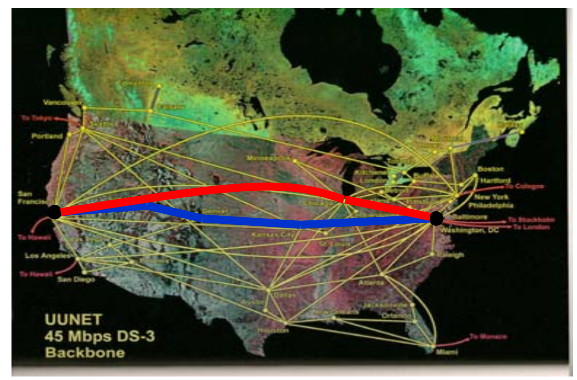
Tragedy of the Commons

- Rational selfish action can lead to outcome bad for everyone

Question:
How to design games that avoid such tragedies

Simple vs Optimal

- Simple practical mechanism, that lead to good outcome.
- optimal outcome is not practical



- Traffic subject to congestion delays

Congestion game =cost (delay) depends only on congestion on edges

Simple vs Optimal

- Simple practical mechanism, that lead to good outcome.
- optimal outcome is not practical

Also true in many other applications:

- Need distributed protocol that routers can implement
- Models a distributed process

e.g. Bandwidth Sharing, Load Balancing,

Games with good Price of Anarchy

- Routing:
- Cars or packets though the Internet
- Bandwidth Sharing:
- routers share limited bandwidth between processes
- Facility Location:
- Decide where to host certain Web applications
- Load Balancing
- Balancing load on servers (e.g. Web servers)
- Network Design:
- Independent service providers building the Internet

Today Auction "Games"

Basic Auction: single item Vickrey Auction

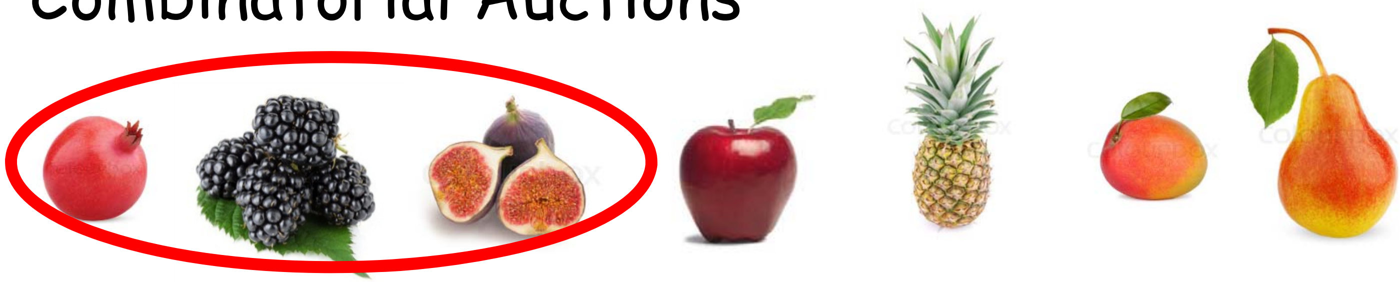
Player utility $v_i - p_i$ — item value -price paid

Vickrey Auction (second price)	- Truthful - Efficient - Simple

Extension VCG (truthful and efficient),
but not so simple

Vickrey, Clarke, Groves

Combinatorial Auctions



Buyers have values for any subset S: $v_i(S)$
user utility $v_i(S) - p_i$ — value –price paid

Efficient assignment: max $\sum_i v_i(S^*_i)$
over partitions S^*_i

- May be hard to compute
- Needs central coordination

Vickrey, Clarke, Groves

Combinatorial Auctions

Buyers have values for any subset S: $v_i(S)$
user utility $v_i(S) - p_i$ — value –price paid

Payment: welfare loss of others

$$p_i = \max \sum_{j \neq i} v_j(S_j) - \sum_{j \neq i} v_j(S^*_j)$$

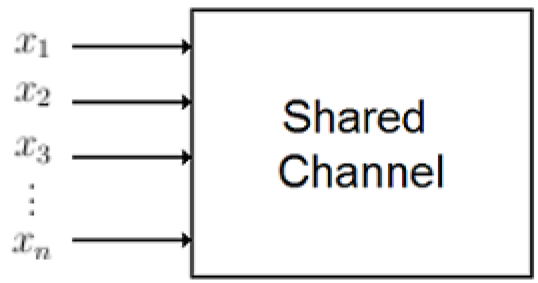
Truthful!

- Needs central coordination
- pricing unintuitive

Other games

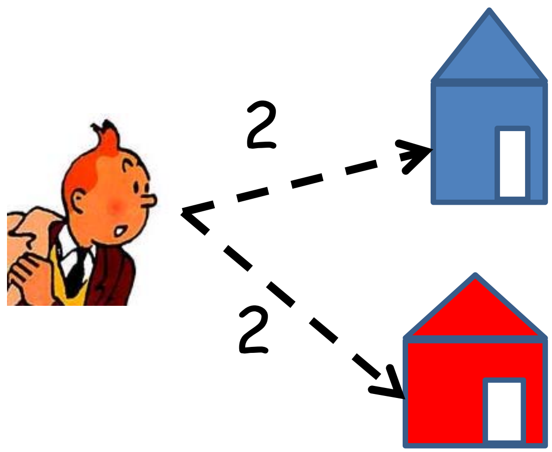
We will assume quasi-linear utility for money, value outcome x and price p has utility $v_i(x)-p$ for user i.

Public projects



Bandwidth Sharing

Truthful Auctions and Composition?



- Second Price Auction
  truthful and simple

Two simultaneous second price auctions?

No!

How about sequentially?

No!

Auctions as Games

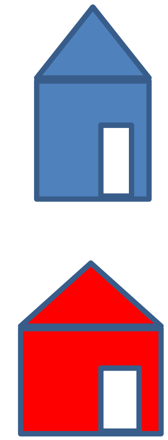
Simpler auction game are better in many settings.

Questions:

- Quality of Outcomes in Auctions
  - Which auctions have low Price of anarchy?
- What if stable solution is not found?
  - Is such a bound possible outside of Nash?
- What if other player's values are not known
  - Is such a bound possible for a Bayesian game?

- Each player plays in many games
  - How do games interact?

Auctions as Games



- Simultaneous second price?
  Christodoulou, Kovacs, Schapira ICALP'08
  Bhawalkar, Roughgarden SODA'10
- Greedy Algorithm as an Auction Game
  Lucier, Borodin, SODA'10
- AdAuctions (GSP)
  Paes-Leme, T FOCS'10, Lucier, Paes-Leme + CKKK EC'11
- First price?
  Hassidim, Kaplan, Mansour, Nisan EC'11
- Sequential auction?
  Paes Leme, Syrgkanis, T SODA'12, EC'12

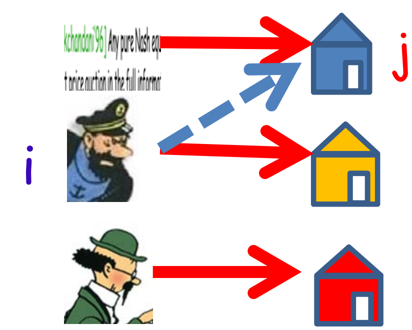
Question: how good outcome to expect?

Simultaneous 1st price

Theorem [Bikchandani'96] Any pure Nash equilibrium of simultaneous first price auction in the full information game has optimal welfare OPT= $\max \sum_i v_i(S_i^*)$

Proof item j sold at a price p_j

Claim: Prices p_j are market clearing:
If i gets some set S_i^* in optimum,
i can take each item $j \in S_i^*$ at price p_j



Market clearing prices imply max social welfare:

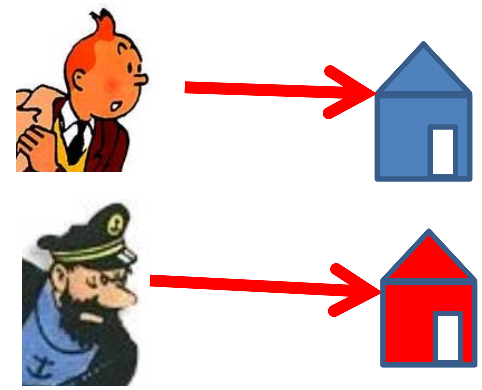
- Each player could claim her optimal set S_i^* to get value $v_i(S_i^*) - \sum_{\{j \in S_i^*\}} p_j$
- Current solution is no worse at the same prices

Simultaneous 1st price

Example:

 wants one house at value 1

 values one house at value 1



- Unique pure Nash: each his own house
- ∃ Mixed Nash: select house at random and bid in $[0,\frac{1}{2}]$ range. not optimal!
- Why?  won't know what to bid.

Smooth Auctions

Market clearing prices optimality proof:

player i has a bid b'_i, such that if current bids are b_i and item prices are p_j we get

$$\sum_i u_i(b'_i, b_{-i}) \geq OPT - \sum_j p_j$$

(λ,μ)-smooth auction game

$$\sum_i u_i(b'_i, b_{-i}) \geq \lambda OPT - \mu \sum_j p_j$$

b'_i may depend on valuations, b_i, but **not** on b_{-i}

Price of Anarchy

Theorem Auction game (λ,μ)-smooth game, then the price of anarchy is at most λ/max(1, μ).

Proof At Nash $u_i(b) \geq u_i(b_i', b_{-i})$

summing and using smoothness

$$\sum_i u_i(b) \geq \sum_i u_i(b_i', b_{-i}) \geq \lambda OPT - \mu \sum_j p_j$$

- If μ<1, use $u_i(b) \geq \mathrm{v_i(b)}$
- Else use total price paid $\leq \mathrm{v_i(b)}$

Smooth Games of Roughgarden

(λ,μ)-smooth auction game

$$\sum_i u_i(b'_i, b_{-i}) \geq \lambda OPT - \mu \sum_j p_j$$

b'$_i$ may depend on valuations, b$_i$, but **not** on b_{-i}

Roughgarden (λ,μ)-smooth game

$$\sum_i u_i(b'_i, b_{-i}) \geq \lambda OPT - \mu \sum_j v_i(b)$$

Connection:

- (λ,μ)-smooth auction ~ (λ,μ+1)-smooth game
- Add mechanism as a player

Examples of smooth auction games

- First price auction (1-1/e,1) smooth
  - See also Hassidim et al EC'12, Syrkhanis'12
- All pay auction ($\frac{1}{2}$,1)-smooth
- First price greedy combinatorial auction based on a c-approx algorithm is $(1-e^{-c}, c)$-smooth
  - See also Lucier-Borodin SODA'10
- First position auction (GFP) is ($\frac{1}{2}$,1)-smooth

Other applications include: public goods, bandwidth allocation (Joharu-Tsitsiklis), etc

Our questions

Simple Auctions as Games

✓ Quality of Outcomes in Auctions

Which auctions have low Price of anarchy?

- What if stable solution is not found?

Is such a bound possible outside of Nash?

- What if other player's values are not known

Is such a bound possible for a Bayesian game?

- Each player plays in many games

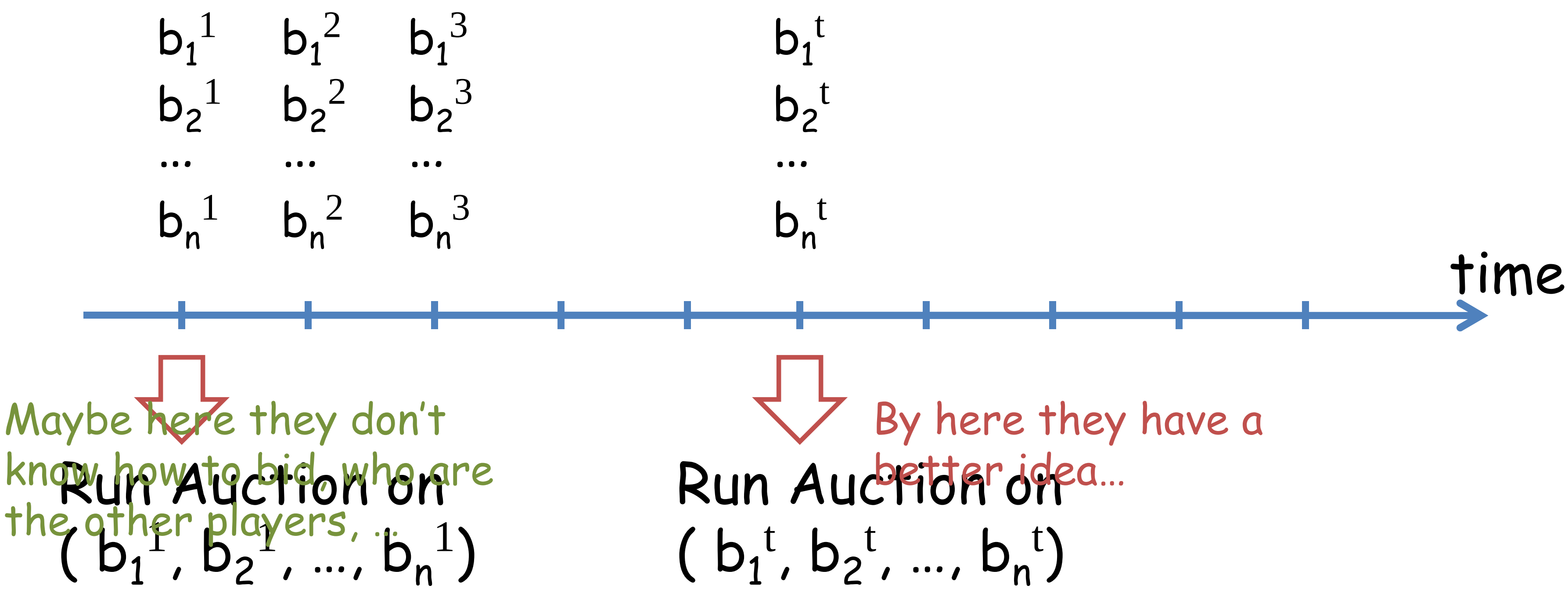
How do games interact?

Price of Anarchy

Theorem(Syrkganis-T'12) Auction game (λ,μ)-smooth game, then

- Price of anarchy is at most max(1, μ)/λ
- Also true for correlated equilibria (learning outcomes)

Learning outcome



Vanishingly small regret for any fixed strat x:

$$\sum_t u_i(b_i^t, b_{-i}^t) \geq \sum_t u_i(x, b_{-i}^t) - o(T)$$

including regret about swapping strat y to x

Price of Anarchy

Theorem(Syrkganis-T'12) Auction game (λ,μ)-smooth game, then

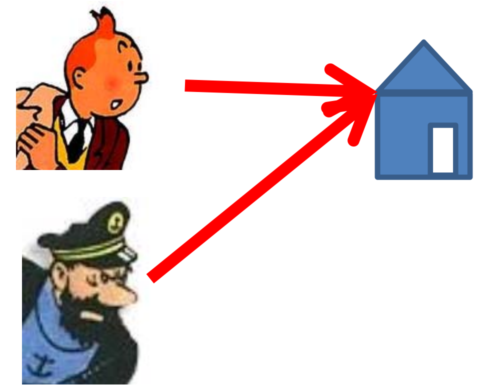
- Price of anarchy is at most $\max(1, \mu)/\lambda$
- Also true for correlated equilibria (learning outcomes)
- Also true for Bayesian game, assuming player types are independent
  - Roughgarden EC12 and Syrkganis'12 using universal smoothness

Bayesian game

Example:

 wants house at value 1

 values house at value [0,1] uniform



Nash isn't optimal!

Why?  won't know what to bid?

Bayesian extension theorem

Theorem(Syrkganis-T'12) Auction game (λ,μ)-smooth game, then Bayesian Price of anarchy is at most $\lambda/\max(1, \mu)$, assuming player types are independent

- Roughgarden EC12 and Syrkganis'12 using universal smoothness

Proof idea: consider random draw w, and take (λ,μ)-smooth deviation for valuations (v_i, w_{-i}) from strategy w_i. $b_i'((v_i, w_{-i}), w_i)$

- Bluffing technique: w_i

Our questions

Simple Auctions as Games

- ✓ Quality of Outcomes in Auctions
  - Which auctions have low Price of anarchy?
- ✓ What if stable solution is not found?
  - Is such a bound possible outside of Nash?
- ✓ What if other player's values are not known
  - Is such a bound possible for a Bayesian game?

- Each player plays in many games
  - How do games interact?

Simultaneous Composition

Multiple mechanisms M_j running independently

- Each one generates

  Outcomes x_{ij} and price p_{ij} for each player I

- Total payment $p_i = \sum_j p_{ij}$
- Value $v_i(x_{i1}, x_{i1}, \ldots, x_{im})$: value depends on all outcomes!

Utility: $v_i(x_{i1}, x_{i1}, \ldots, x_{im}) - \sum_j p_{ij}$

Simultaneous Composition

See next

Theorem(Syrkganis-T'12) simultaneous mechanisms M_j each (λ,μ)-smooth and players have no complements across mechanisms, then composition is also (λ,μ)-smooth

Corollary: Simultaneous first price auction has price of anarchy of e/(e-1) if player values are fractionally subadditive

- Simultaneous all-pay auction: price anarchy 2
- Mix of first price and all pay, PoA at most 2

Valuations: no complements across mechanisms

Fractionally subadditive: for all y^k and α^k such that $\sum_k \alpha^k y^k \geq x$ implies that $v(x) \leq \sum_k v(y^k)$.

Simult. mechanisms M_j outcome: $(x_1, x_2, \ldots, x_m)$

Fractionally subadditive, if for all x and all y^k and α^k such that $\sum_{\{k: y_j^k = x_j^k\}} \alpha^k \geq 1$ implies that

$$v(x) \leq \sum_k \alpha^k v(y^k)$$

no assumption within each mechanism

Valuations: no complements across mechanisms

Valuation XOS across mechanisms if

$$v(x) = \max_k \sum_j v_j^k(x_j)$$

for some valuations v_j^k

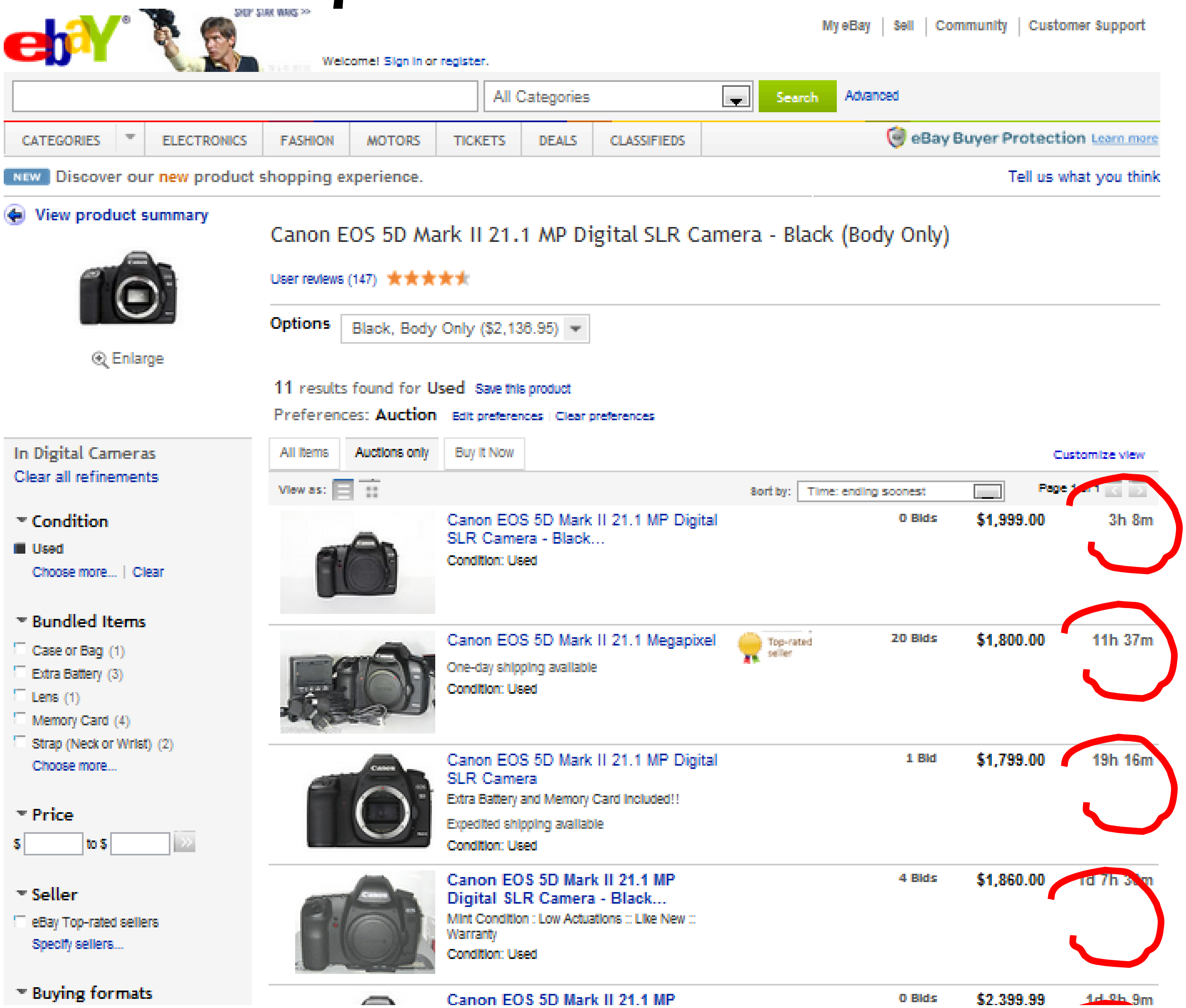
Theorem (Syrgkanis-T'12) XOS $\equiv$ Fractionally subadditive.

– Extending Feige STOC'06

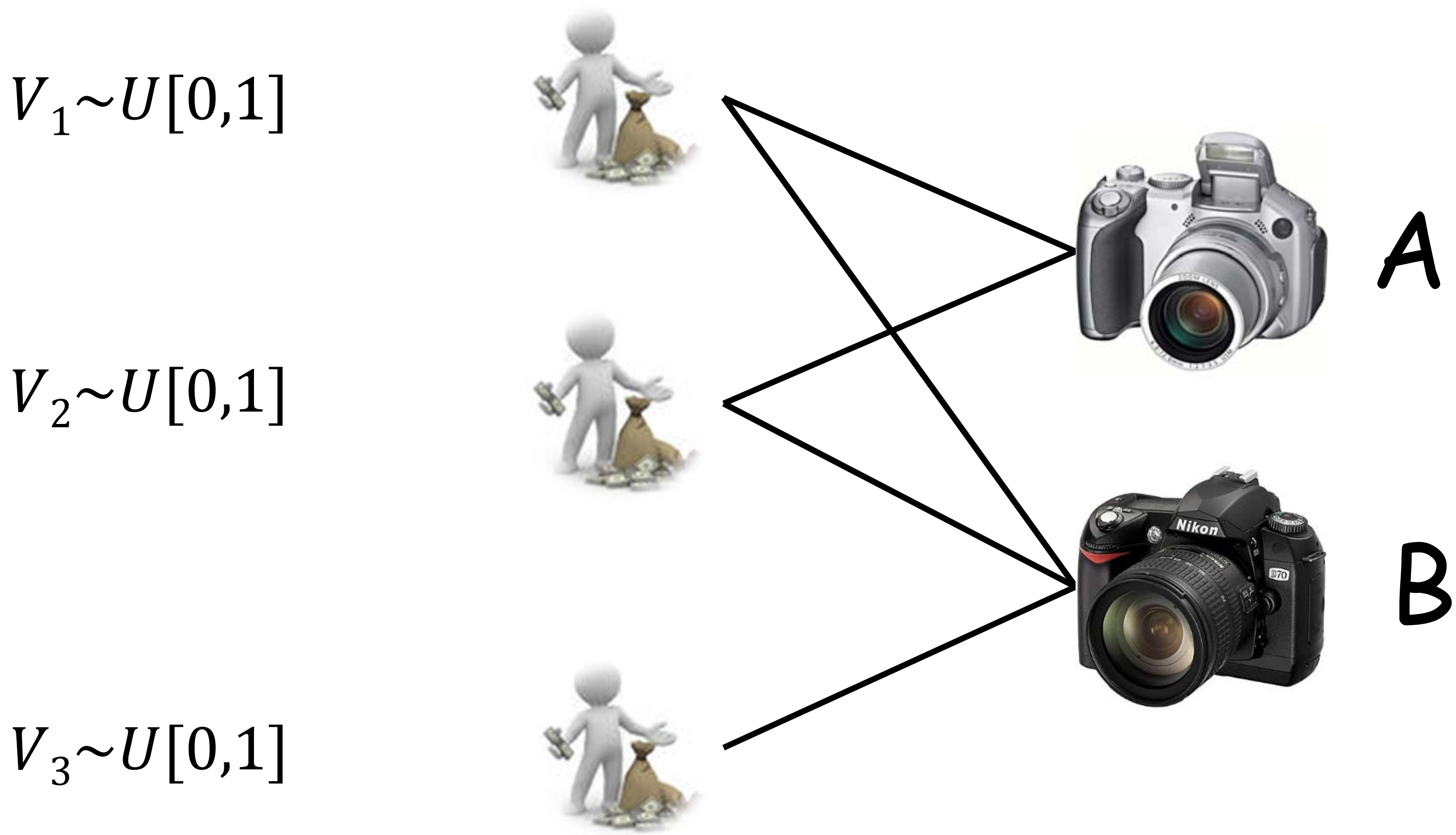
Fractionally Subadditive $\rightarrow$XOS

Theorem monotone valuation with diminishing marginal returns property $\Rightarrow$ can be expressed as XOS by capped marginal valuations:

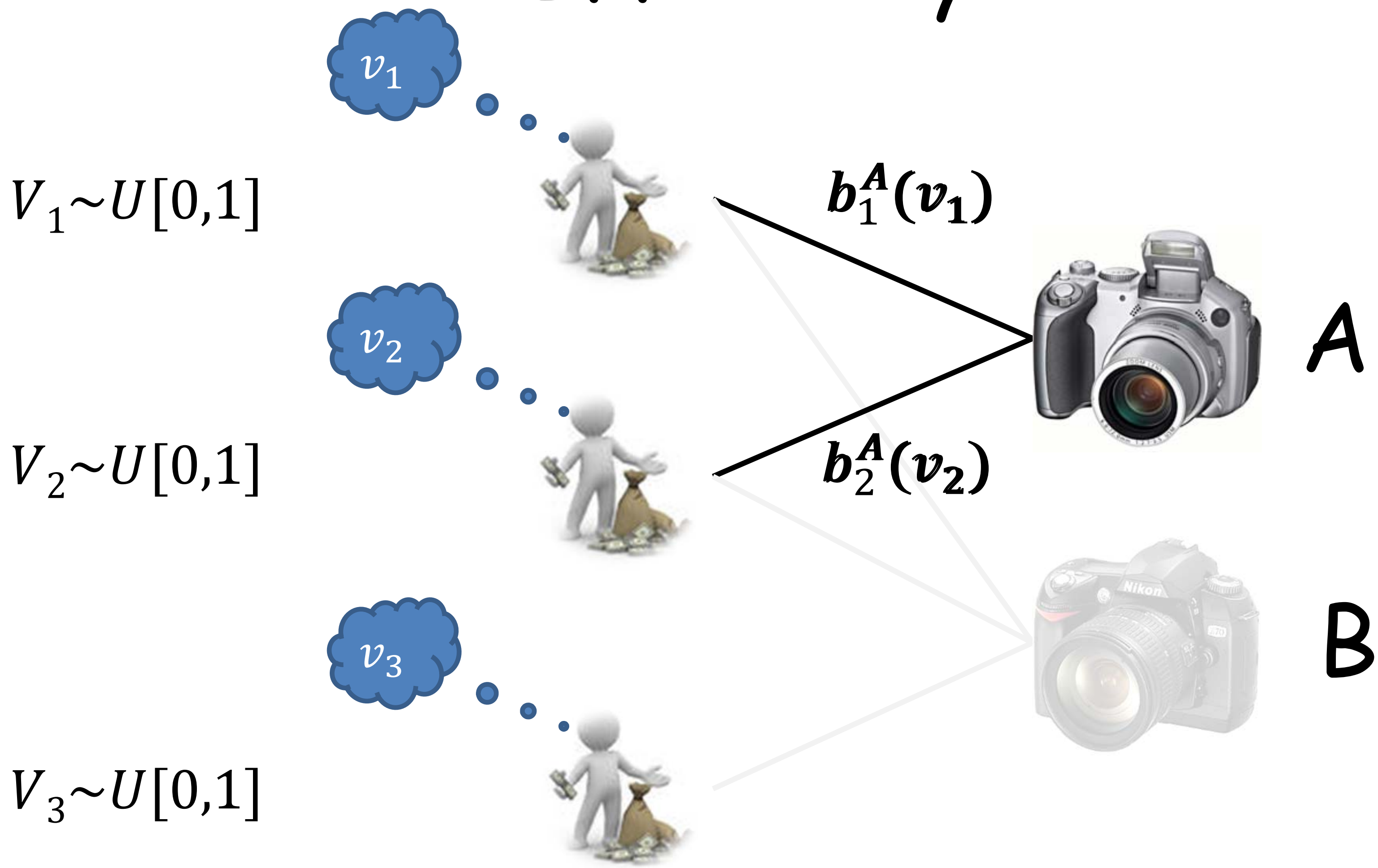
Sequential Auction



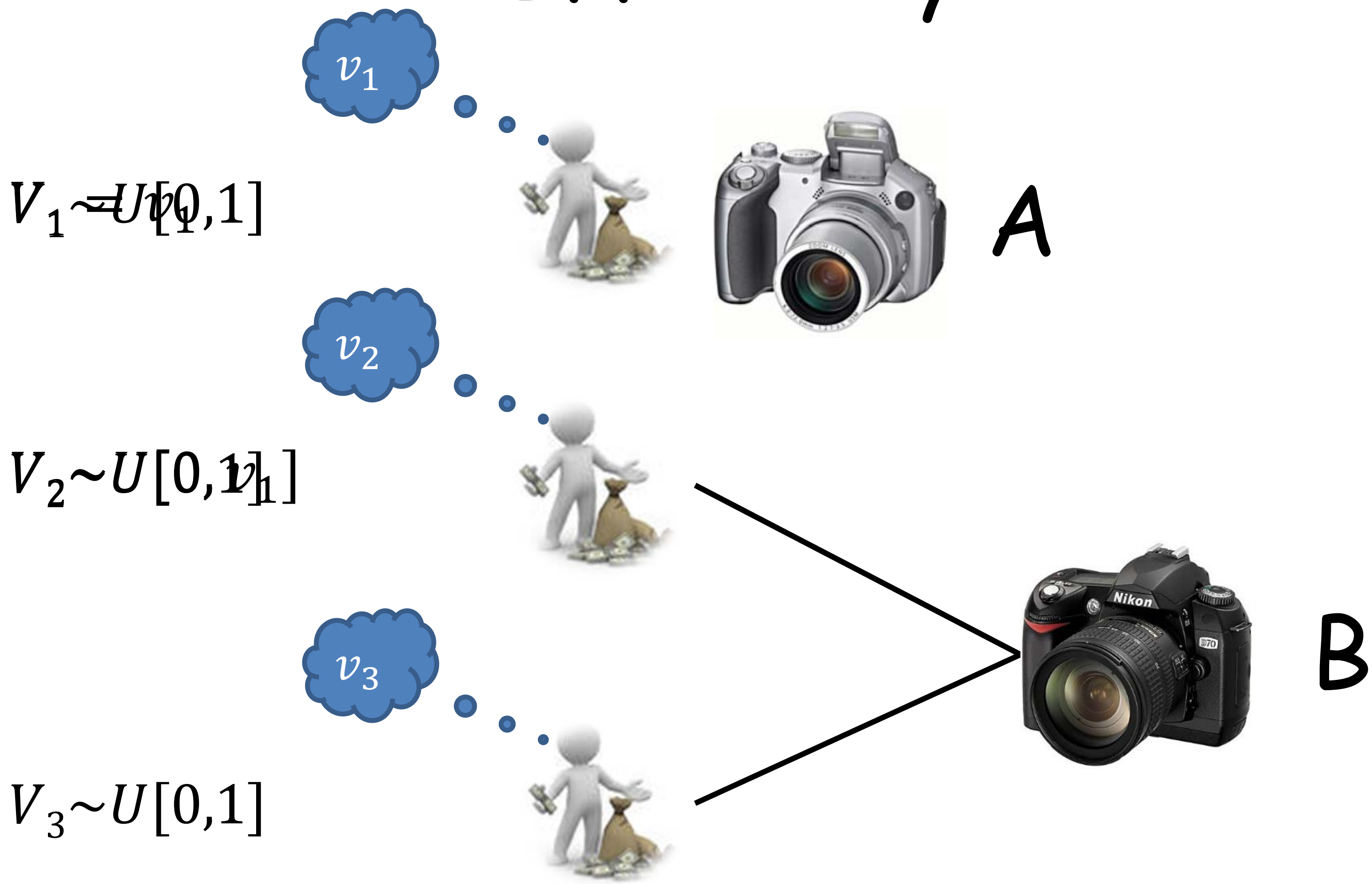
Incomplete Information and Efficiency



Incomplete Information and Efficiency

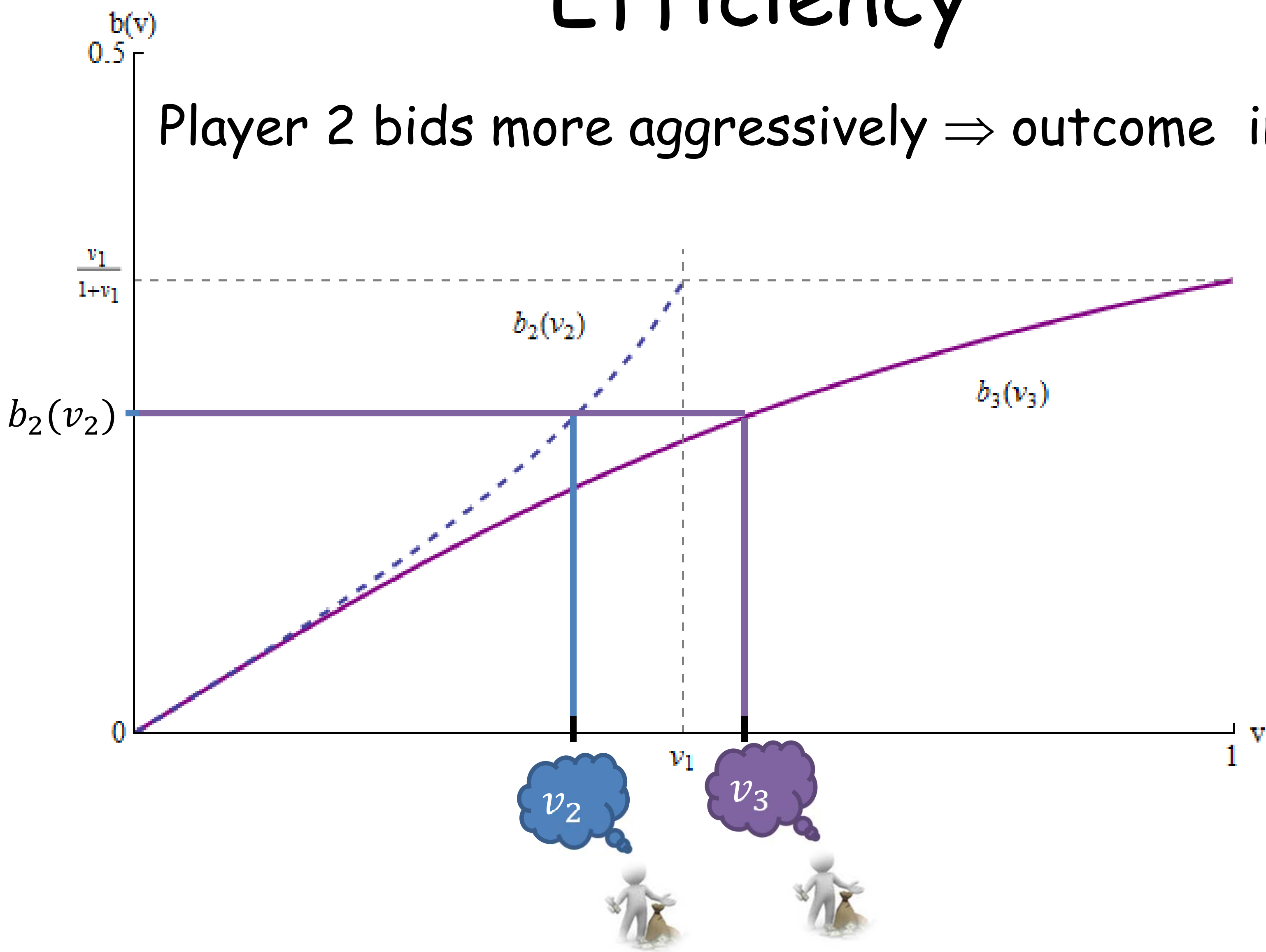


Incomplete Information and Efficiency

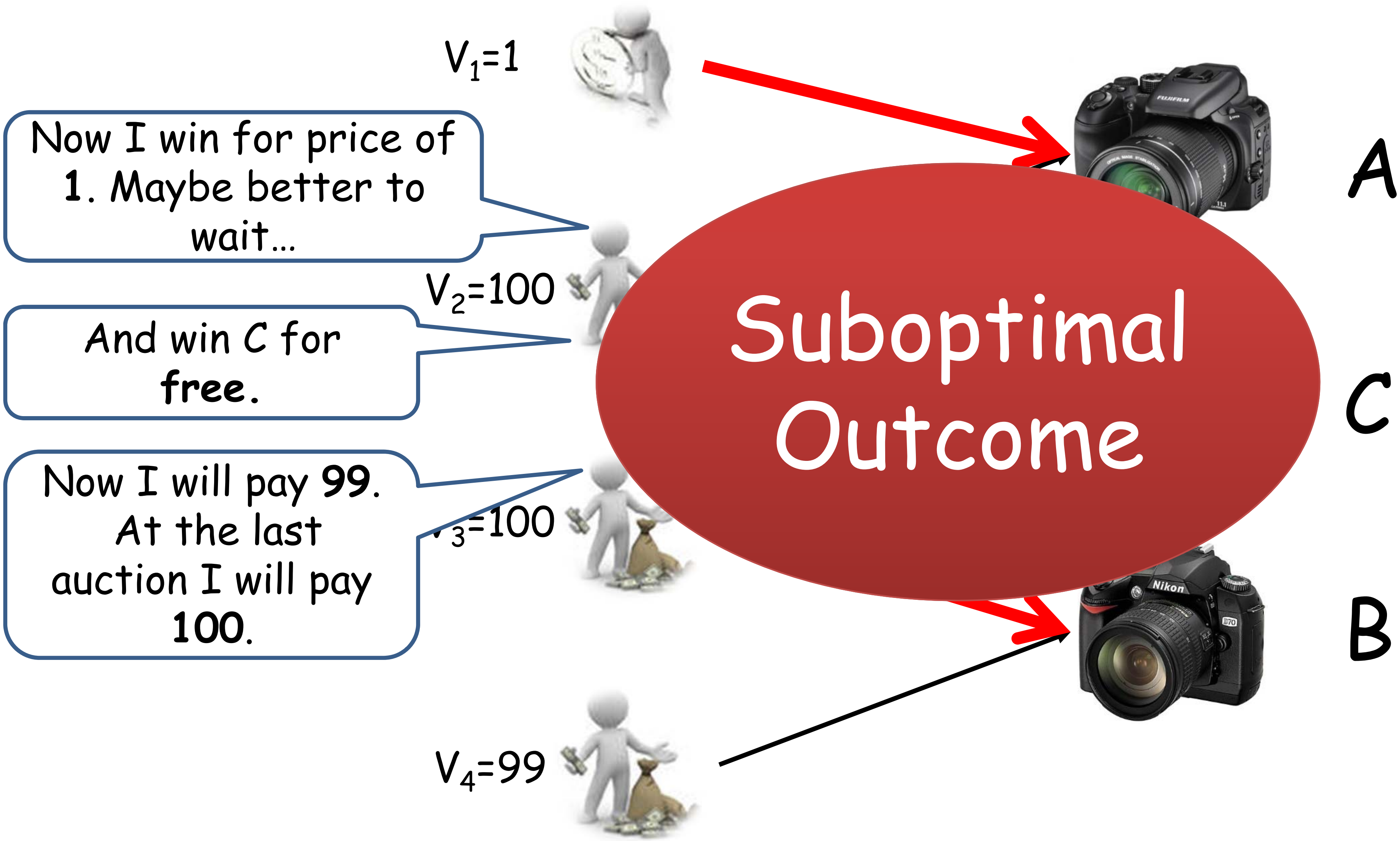


Incomplete Information and Efficiency

Player 2 bids more aggressively ⇒ outcome inefficient



Example



V1=1
Now I win for price of 1. Maybe better to wait…
V2=100
And win C for free.
Now I will pay 99. At the last auction I will pay 100.
3=100
V4=99
A
C
B
Suboptimal Outcome

Sequential Composition

Theorem (Syrkganis-T'12) sequential mechanisms M_j each (λ,μ)-smooth and player's value comes from best mechanism's outcome $v_i(x) = \max_j v_{ij}(x_{ij})$

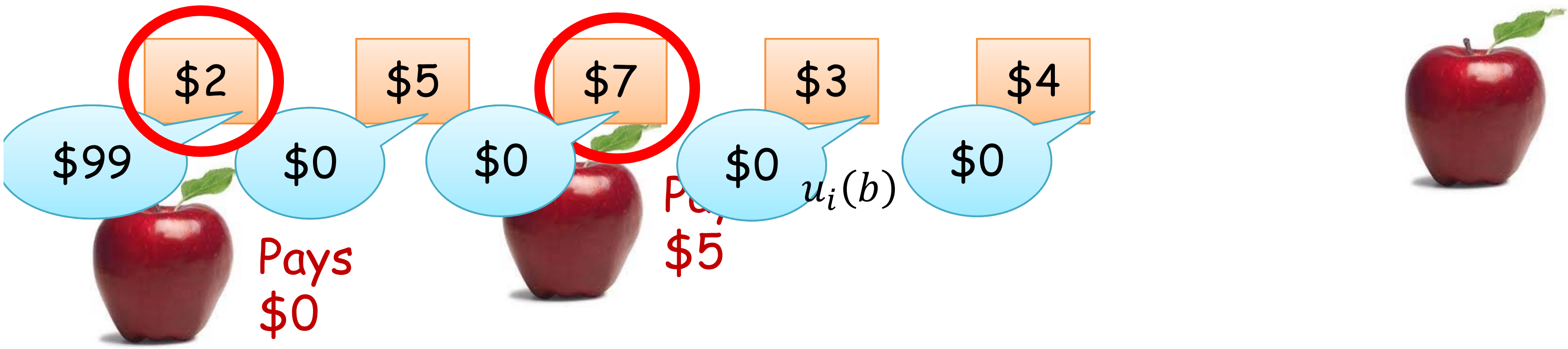
Then composition is $(\lambda,\mu+1)$-smooth

Corollary: Sequential first price auction has price of anarchy of 3.16 if player values comes from best mechanism outcome

- Simultaneous all-pay auction: price anarchy 4
- Mix of first price and all pay, PoA at most 4

Nash equilibria of bidding games

Vickrey Auction - Truthful, efficient, simple (second price)



but has many bad Nash equilibria

Assume bid ≤ value (higher bid is dominated)

Theorem: all Nash equilibria efficient: highest value winning

Price of Anarchy

Theorem [Christodoulou, Kovacs, Schapira ICALP'08]
Total value $v(N)=\sum_i v_{ij_i}$ at a Nash equilibrium of simultaneous second price auction is at least ½ of optimum OPT= $\max_{M^*} \sum_i v_{ij_i^*}$ (assuming $b_{ij} \leq v_{ij}$ $\forall$ i&j).

Extension of smoothness to weakly (λ, μ_1, μ_2)-smooth
Implies price of anarchy of $\lambda/(\max(1, \mu_1)+\mu_2)$, assuming no overbidding

Theorem(Syrkganis-T'12) simultaneous mechanisms M_j each (λ, μ_1, μ_2)-smooth and players have no complements across mechanisms, then composition is also (λ, μ_1, μ_2)-smooth

Simple Auction Games

- Smooth mechanism: natural generalization of market clearing prices
- Many simple games are smooth
- Smooth mechanisms remain smooth when composed (assuming no complements across mechanism)
- Good outcome quality (Nash, Bayesian Nash, learning outcomes)