FUN PROBLEMS

CROSSROADS ACADEMY
MATHCOUNTS PREPARATION

1. Platonic Solids

The five platonic solids (Tetrahedron, Cube, Octahedron, Dodecahedron, and Icosahedron) are convex polyhedra where each face is a convex regular polygon and the same number of faces meet at each edge.

Polyhedron	Vertices (V)	Edges (E)	Faces (F)	Edges per vertex
Tetrahedron	4	6	4 Δs	3
Cube	8	12	6 $\square$s	3
Octahedron	6	12	8 Δs	4
Dodecahedron	20	30	12 Pentagons	3
Icosahedron	12	30	20 Δs	5

(1) Is it possible to the vertices of a cube with 2 colors so that no two adjacent vertices are the same color?

(2) What is the smallest number of colors necessary to color the faces of a cube so that no two adjacent faces are the same color?

(3) What is the smallest number of colors necessary to color the edges of a cube so that no two adjacent edges are the same color?

(4) If an ant starts at a vertex of an octahedron and only walks on the edges, is it possible for the ant to walk along each edge exactly once without repeating any? Which of the platonic solids is this possible on?

(5) If a sphere is circumscribed around a cube with side length 2, what is the surface area of the sphere?

(6) What is the volume of a sphere inscribed in a cube with surface area 64?

(7) Find the constants a, b, c, and d so that $aV + bE + cF = d$ for all of the platonic solids using the table above.

(8) Find the constants S and T so that the number of sides per face times F is equal to ET and the number of edges per vertex times V is equal to ES for each of the solids.

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2. 4 Colors and Euler's Formula

(1) Draw two planar graphs with a number of vertices that is a multiple of 4. Fill in each vertex with a 1 or a 2 however you want, so that there are the same number of 1s and 2s and pass this sheet to your neighbor.

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(2) How many edges, vertices, and faces are in each graph? Do these numbers satisfy Euler's formula?

(3) For each graph, what is the smallest number of colors needed to color the vertices so that no adjacent vertices are the same color?

(4) Split the vertices into 4 groups so that each group has the same number of vertices. For each group, count the number of 1s and 2s that appear. Is it possible to create the groups in such a way that three of the groups have more 1s than 2s? What if the groups have to be connected?