Discrete Probability Practice

JHU – CTY Science and Technology Series

Dartmouth College Department of Mathematics

Basic Probabilities

(1) What is the probability that the top card of a standard deck is the eight of hearts?

(2) What is the probability that the top card of a standard deck is a spade?

(3) What is the probability that the top card of a standard deck is an ace?

(4) If we roll two standard six sided dice, what is the probability that the sum of the rolls is seven?

Sum Rule

(1) What is the probability that the top card of a standard deck is a heart or a three?

(2) If we flip a coin, what is the probability that it is heads or tails?

(3) What is the probability that the top card of a standard deck is a prime number?

(4) If we roll a standard six sided die, what is the probability that the roll has an odd prime factor?

Product Rule

(1) If we flip two coins, what is the probability that there is one heads and one tails?

(2) If we flip three coins, what is the probability that they are all heads?

(3) What is the probability that the top card of a standard deck is a red face card?

(4) If we roll two standard six sided dice, what is the probability that the sum of the rolls is even and greater than 4?

Random Integers

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(1) Write down a random integer between one and twenty: _____

(2) Write the prime factorization of your number: _____

(3) Is your number ...

- even? _____
- odd? _____
- a multiple of 5? _____
- a perfect square? _____
- a perfect cube? _____
- prime? _____
- divisible by a square? _____

(4) For each of the properties above, what is the probability that a random integer between one and twenty has that property?

- even: _____
- odd: _____
- a multiple of 5: _____
- a perfect square: _____
- a perfect cube: _____
- prime: _____
- divisible by a square: _____

(5) What number do you think is most commonly selected by people "at random"?

(6) Why do you think people prefer that number?

Coin Flipping Game

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(1) Flip a coin 10 times and record the results below:

____ ____ ____ ____ ____ ____ ____ ____ ____ ____

(2) How many heads did you record? ____________

(3) Look at each pair of adjacent flips. How many of each of the following types are there?

- HH ____________
- TT ____________
- HT ____________
- TH ____________

(4) Do you expect the two–flip distributions to occur with equal probability?

(5) Play the following game with the person sitting next to you. Each player chooses one of the four two–coin sequences { HH, HT, TH, TT }. Flip a coin, recording the result on the lines below. Continue flipping until one player's sequence occurs consecutively. That player gets one point. Restart the game on the next line, best two out of three rounds wins.

____ ____ ____ ____ ____ ____ ____ ____ ____ ____

____ ____ ____ ____ ____ ____ ____ ____ ____ ____

____ ____ ____ ____ ____ ____ ____ ____ ____ ____

____ ____ ____ ____ ____ ____ ____ ____ ____ ____

____ ____ ____ ____ ____ ____ ____ ____ ____ ____

(6) Which sequence do you think is the best choice? Why?

Penny Flipping Challenge

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In the penny flipping game, we played with two–coin sequences. The same game can also be played with longer sequences. In the three–coin sequence version there are eight possible choices: { HHH, HHT, HTH, THH, HTT, THT, TTH, TTT }. Assuming that the other player announces their choice before you have selected your sequence, can you always choose another sequence that will give you a better than 50% chance of winning? What should your strategy be?

Monty Hall Game

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Imagine that you are a contestant on a game show, facing three closed doors numbered one, two, and three. You can choose exactly one of the doors and receive the prize behind that door. Two of the doors are hiding goats, while the remaining door is hiding 1,000,000 dollars. After you choose a door, the host opens one of the other doors and reveals a goat. You are now offered the opportunity to switch your choice to the other unopened door.

(1) What is the probability that the money is behind the door that you originally selected?

(2) What is the probability that the money is behind the door that you could switch to?

(3) What should you do?

Try this game out with the person sitting next to you. Take turns being the host and the contestant. Do the results of your games match the probabilities that you computed above?

More Random Integers

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Write 20 random numbers on the blank spaces below:

(1) _____ (11) _____

(2) _____ (12) _____

(3) _____ (13) _____

(4) _____ (14) _____

(5) _____ (15) _____

(6) _____ (16) _____

(7) _____ (17) _____

(8) _____ (18) _____

(9) _____ (19) _____

(10) _____ (20) _____

First Digit	Tally Marks	Final Count
1		
2		
3		
4		
5		
6		
7		
8		
9		

All Digits	Tally Marks	Final Count
1		
2		
3		
4		
5		
6		
7		
8		
9		

Random Words and Letters

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Write five random words on the blanks below. Compute the distributions for the individual letters in the words that you selected in the table below. How does your distribution compare to the overall English distribution on the next page?

(1) _____

(2) _____

(3) _____

(4) _____

(5) _____

Letter	Tally Marks	Final Count
A		
B		
C		
D		
E		
F		
G		
H		
I		
J		
K		
L		
M		
N		
O		
P		
Q		
R		
S		
T		
U		
V		
W		
X		
Y		
Z		

English Letter Distribution

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Letter	Percentage
e	12.7%
t	9.1%
a	8.1%
o	7.5%
i	7.0%
n	6.7%
s	6.3%
h	6.1%
r	6.0%
d	4.2%
l	4.0%
c	2.8%
u	2.8%
m	2.4%
w	2.4%
f	2.2%
g	2.0%
y	2.0%
p	1.9%
b	1.5%
v	1.0%
k	0.8%
j	0.2%
x	0.2%
q	0.1%
z	0.1%