DATE METHODS

NEW HAMPSHIRE STATE TEAM
NATIONAL MATHCOUNTS PREPARATION

1. Introduction

Computing the day of the week based on the digits of the day–month–year has become a common part of "mental mathemagics" routines[1] in addition to competition problems. The mathematical tools used in these methods mostly just rely on modular arithmetic, which you are all familiar with. Here I will briefly outline some different approaches to solving the problem that Miriam sent:

Brian was born on March 27, 2001. What day of the week was this, given that June 1, 2014 was a Sunday?

2. Direct Approach

To figure this problem out directly we can try to figure out how many days the calendar shifts between the two dates.

When looking at how the day of the week changes for a fixed day between one month and then next the important thing is how many days are in the month. For 31 month days, the day of the week shifts forward 3 days. For example, if January 3rd is a Sunday then February 3 is a Wednesday. For 30 month days, the day of the week shifts forward 2 days. For example, if April 3rd is a Sunday then May 3 is a Tuesday. February is easy to figure out, since it is exactly 4 weeks long, so if February 3 is a Friday then March 3 is also a Friday.

When looking at date problems that span years there are two cases depending on whether or not there is a leap year. If not, then the day shifts forward by one. For example, January 1, 2017 is a Sunday so January 1, 2018 is a Monday. If it crosses leap year then the day shifts forward by two. For example, January 1, 2004 was a Thursday so January 1, 2005 was a Saturday.

To apply this to our problem we can start with March 27, 2001 and compute the shifts to March 27, 2014 and then compute the number of shifts to June 1, 2014. To move from 2001 to 2014 there are 13 years, 3 of which are leap years, so the day shifts forward by 16. We then shift forward 5 more days to get to April 1, 2014. Then, we shift to May 1 and then to June 1 up two and three days respectively. This makes our final shift $16+5+2+3 = 26 \equiv 5 \pmod 7$ which means that June 1, 2014 is five days later then March 27, 2001. Then, since we are given that June 1, 2014 is a Sunday we shift backwards five days to see that March 27, 2001 was a Tuesday.

Date: March 17, 2017.

[1] You can check out videos of Dr. Art Benjamin on Youtube for some incredible examples.

3. Doomsday Method

The next two methods simply compute the day of the week for March 27, 2001 directly without using the other date at all. The doomsday method makes use of the fact that the dates $\{4/4, 5/9, 6/6, 7/11, 8/8, 9/5, 10/10, 11/7, 12/12\}$ as well as the final day of February (either 28 or 29) all occur on the same day of the week every year. John Conway, who invented this method, called these doomsday[2] For simplicity I will just use the method that works in the 2000's, you have to change one of the constants for other centuries.

We start by defining some constants y is the last two digits of the year. Then, we define $a = \lfloor \frac{y}{12} \rfloor$, $b = y - 12a$, $c = \lfloor \frac{b}{4} \rfloor$, and finally $d = a + b + c$. To compute the day of the week for the doomsdays we count d days forward from Tuesday. In our example (which turns out not be very exciting), the year is 2001, so $y = 1$, $a = 0$, $b = 0$, $c = 1$, and $d = 1$. Thus, the doomsdays for 2001 all fall on Wednesday. The closest doomsday to our date is 4/4. Since 4/4 is a Wednesday and the March 27th is $8 \equiv 1 \pmod 7$ days earlier, our desired date is a Tuesday.

4. Zeller's Congruence

The final method is a formula that outputs the day of the week directly based on the digits of the month, day, and year. Let d be the day of the month, m be the number of the month (for this method January is 13 and February is 14 to take care of leap years), y the last two digits of the year, and z the floor of the year divided by 100. The, the day of the week is given by:

$$\left(d + \lfloor \frac{13(m+1)}{5} \rfloor + y + \lfloor \frac{y}{4} \rfloor + \lfloor \frac{z}{4} \rfloor - 2z\right) \pmod 7$$

The output of this uses one for Sunday, two for Monday,three for Tuesday, etc. For our problem we have that $d = 27$, $m = 3$, $y = 1$, and $z = 20$. Plugging this is gives:

$$\left(27 + \lfloor \frac{13(3+1)}{5} \rfloor + 1 + \lfloor \frac{1}{4} \rfloor + \lfloor \frac{20}{4} \rfloor - 2(20)\right) \pmod 7 =$$
$$(27 + 10 + 1 + 0 + 5 - 40) \pmod 7 = 3$$

which agrees with the previous answers.

[2] He is a famous mathematician who is known for giving creative names to mathematical objects.