

WARMUP PROBLEMS

CROSSROADS ACADEMY
MATHCOUNTS PREPARATION

1. MODULAR ARITHMETIC

- (1) What is $34 - 16 \pmod{14}$?
- (2) What is $12 \cdot 3 - 6 \pmod{24}$?
- (3) It is currently March. What month will it be in 1,000 months?
- (4) If it is currently 2:00pm what time will it be in 500 hours?
- (5) What is the last digit of 2^{95} ?
- (6) During penmanship class, Jasmine writes her name, letter by letter, over and over again for 20 minutes. If at the end of the time she has written 2018 letters, what is the last letter that she writes?
- (7) Solve $5x \equiv 7 \pmod{12}$ for x .
- (8) What is the remainder when $1234 \cdot 56 \cdot 789$ is divided by 6?
- (9) Solve for $5x - 1 \equiv 17 \pmod{9}$. Does $3x + 2 \equiv 17 \pmod{9}$ have a solution? How about $3x - 4 \equiv 17 \pmod{9}$?
- (10) How many solutions for x are there in $x^6 - 1 \equiv 0 \pmod{7}$?

2. FLOOR AND CEILING FUNCTIONS

- (11) Compute $\lfloor \pi + 7 \rfloor$ and $\lceil \pi + 7 \rceil$.
- (12) What is the biggest possible difference between $\lfloor x \rfloor$ and $\lceil x \rceil$?
- (13) How many positive integers x satisfy: $\lfloor \frac{x-1}{6} \rfloor < 7$?
- (14) How many integers between 1 and 30 satisfy $\lfloor \frac{x}{7} \rfloor = \lceil \frac{x}{7} \rceil$?
- (15) How many integers between 1 and 20 satisfy $\lfloor \frac{x+1}{5} \rfloor = \lceil \frac{x-2}{3} \rceil$?
- (16) What is the probability that a random x between 1 and 10 satisfies $\lceil \sqrt{\lfloor x \rfloor} \rceil = \lceil \sqrt{x} \rceil$?
- (17) If n is an integer, what is $\lfloor \frac{n}{2} \rfloor + \lceil \frac{n}{2} \rceil$ in terms of n ?
- (18) If n is an integer, what is $\lfloor \frac{n}{3} \rfloor + \lceil \frac{2n}{3} \rceil$ in terms of n ?
- (19) If n is an integer, what is $\lfloor -n \rfloor$ in terms of $\lceil n \rceil$?
- (20) Brian was born on March 27, 2001. What day of the week was this, given that June 1, 2014 was a Sunday?