EUCLIDEAN ALGORITHM, BEZOUT'S IDENTITY, AND THE CHINESE REMAINDER THEOREM

CROSSROADS ACADEMY
MATHCOUNTS PREPARATION

I) Find the GCDs of the following pairs of numbers: (10,75), (51,172), (2049, 54).

II) Find the GCD of (24541,9797).

III) Write a GCD problem that is easy to solve with factoring.

IV) Write a GCD problem that is easier to solve with the Euclidean Algorithm.

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a) Another definition of the GCD of (x,y) is the smallest positive integer that can be written as $ax + by$ when a and b are integers. For example, the GCD of (10,17) is 1 and $-5 \cdot 10 + 3 \cdot 17 = 1$.

b) Find integers a and b so that $a \cdot 84 + b \cdot 606 = \gcd(84, 606)$.

c) Find integers a and b so that $a \cdot 24541 + b \cdot 9797 = 97$.

d) Find integers a and b so that $a \cdot 52 + b \cdot 256 = 100$.

e) Find integers a and b so that $a \cdot 17 + b \cdot 697 = 1234567890$.

i) Find the smallest positive integer n so that

$$\begin{aligned} n &\equiv 1 \pmod{4} \\ n &\equiv 1 \pmod{12} \\ n &\equiv 1 \pmod{75} \end{aligned}$$

ii) Find the smallest positive integer n so that

$$\begin{aligned} n &\equiv 1 \pmod{6} \\ n &\equiv -1 \pmod{7} \\ n &\equiv 1 \pmod{2} \end{aligned}$$

iii) Find the smallest positive integer n so that

$$\begin{aligned} n &\equiv 3 \pmod{5} \\ n &\equiv 2 \pmod{7} \\ n &\equiv 4 \pmod{11} \end{aligned}$$

iv) Write a congruence problem that requires the Chinese Remainder Theorem to solve.

v) Write a congruence problem that can be solved without the Chinese Remainder Theorem.