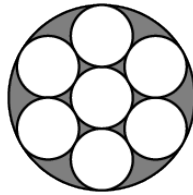# GEOMETRY PROBLEMS

CROSSROADS ACADEMY
MATHCOUNTS PREPARATION

1. GEOMETRY PRACTICE

(1) Each of the small circles in the figure has radius one. The innermost circle is tangent to the six circles that surround it, and each of those circles is tangent to the large circle and to its small-circle neighbors. Find the area of the shaded region.

(2) A 45° arc of circle A is equal in length to a 30° arc of circle B. What is the ratio of circle A's area and circle B's area?

(3) How many points with non–negative integer coordinates lie on the line $x + 2y = 100$?

(4) In the overlapping triangles $\triangle ABC$ and $\triangle ABE$ sharing common side AB, $\angle EAB$ and $\angle ABC$ are right angles, $AB = 4$, $BC = 6$, $AE = 8$, and $\overline{AC}$ and $\overline{BE}$ intersect at D. What is the difference between the areas of $\triangle ADE$ and $\triangle BDC$?

(5) A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars would increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

(6) Let S be the set of points (a, b) in the coordinate plane, where each of a and b may be -1, 0, or 1. How many distinct lines pass through at least two members of S?

(7) Let $A = (0, 9)$ and $B = (0, 12)$. Points A' and B' are on the line $y = x$, and $\overline{AA'}$ and $\overline{BB'}$ intersect at $C = (2, 8)$. What is the length of $\overline{A'B'}$?

(8) Square $ABCD$ has side length 2. A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicircle from C intersects side $\overline{AD}$ at E. What is the length of $\overline{CE}$?

(9) A rectangle with diagonal length 5 is twice as long as it is wide. What is the area of the rectangle?

(10) Square $EFGH$ is inside the square $ABCD$ so that each side of $EFGH$ can be extended to pass through a vertex of $ABCD$. Square $ABCD$ has side length $\sqrt{50}$ and $BE = 1$. What is the area of the inner square $EFGH$?