

## MODULAR PROBLEMS

CROSSROADS ACADEMY  
MATHCOUNTS PREPARATION

- I) What is the smallest positive integer multiple of 31 that leaves a remainder of 1 when divided by 13? How about 41 and 14?
- II) What is the least number that leaves a remainder of 3 when divided by 7 and 11? What if we are dividing by 5 and 13 instead?
- III) For each of the numbers  $n \in \{1, 2, 3, 4, 5, 6\}$ , compute  $n^6 \pmod{7}$ .
- IV) For each of the numbers  $n \in \{1, 2, 3, 4, 5, 6, 7\}$ , compute  $n^4 \pmod{8}$ .

a) The school cafeteria serves chicken nuggets in packages of 4 and 7. What is the largest number of chicken nuggets that cannot be obtained by only taking whole packages of these sizes?

b) How many positive integers cannot be obtained by only taking whole packages of these sizes?

c) What happens if we replace 4 and 7 with 5 and 11?

d) Can you find the patterns when 4 and 7 are replaced with any two relatively prime integers  $m$  and  $n$ ?