COUNTING PROBLEMS

NEW HAMPSHIRE STATE TEAM
NATIONAL MATHCOUNTS PREPARATION

(1) How many even 4 digit squares are there?

(2) If a three digit number is selected randomly, what is the probability the digit 2 occurs at least once?

(3) You have two dice, one with faces $\{1, 2, 3, 4, 5, 6\}$ and one with faces $\{9, 10, 11, 12, 13, 14\}$. What is the probability that the sum of the dice is 12? What is the expected value of the sum if the two dice are rolled?

(4) How many triangles can be formed by connecting triples of the points (-3, -1), (-1,-1), (1,-1), (3,-1), (-2,0), (0,0), (2,0), (-3, 1), (-1,1), (1,1), and (3,1)? How many lines can be formed by connecting pairs of the points?

(5) How many ways are there to place four rooks on a 3×7 board so that there is at least one rook in each row? How many ways are there if we want there to be at least one row with no rooks?

(6) What is the number of positive divisors of a googol?

(7) What is the units digit of $1 + 2 + 3 + \cdots + 2017$?

(8) Three digits are drawn from the set $\{1, 2, 6, 7\}$ with replacement, and used to form a three digit number. What is the probability that the number is a multiple of 3?

(9) How many binary strings of length 5 have no consecutive 1's.

(10) Alice, Bob, Carol, Denise, and Erik are attending a movie together. How many ways can they sit together in a row if Bob doesn't want to sit next to Erik and Denise doesn't want to sit next to Carol?

(11) How many five letter permutations can be formed from the letters in "New Hampshire"?

(12) You have three fair coins and your friend has 1 fair coin and two weighted coins that show heads $\frac{3}{5}$ of the time. What is the probability that you get more heads than your friend if you both flip all of your coins?

(13) A point is chosen uniformly at random inside the unit square. What is the probability that the distance from the point to the nearest side is less than $\frac{1}{4}$?

(14) The cards in a standard deck are numbered from 1–52. You are playing a game where you draw the top three cards of the deck and lose if the sum of the lower two numbered cards is larger than the higher numbered card. If one of the cards you draw is numbered 51, what is the probability that you lose?