AIME PREPARATION

1. Warmup Problems

(1) Each day, Jenny ate 20% of the jellybeans that were in her jar at the beginning of that day. At the end of the second day, 32 remained. How many jellybeans were in the jar originally?

(2) Each integer 1 through 9 is written on a separate slip of paper and all nine slips are put into a hat. Jack picks one of these slips at random and puts it back. Then Jill picks a slip at random. Which digit is most likely to be the units digit of the sum of Jack's integer and Jill's integer?

(3) If $\sin(x) = 3\cos(x)$ then what is $\sin(x) \cdot \cos(x)$?

(4) If $|x| + x + y = 10$ and $x + |y| - y = 12$, find $x + y$

(5) The complex number z satisfies $z + |z| = 2 + 8i$. What is $|z|^2$? Note: if $z = a + bi$, then $|z| = \sqrt{a^2 + b^2}$.

(6) X, Y and Z are pairwise disjoint sets of people. The average ages of people in the sets $X, Y, Z, X \cup Y, X \cup Z$ and $Y \cup Z$ are $37, 23, 41, 29, 39.5$ and 33 respectively. Find the average age of the people in set $X \cup Y \cup Z$.

(7) Mrs. Walter gave an exam in a mathematics class of five students. She entered the scores in random order into a spreadsheet, which recalculated the class average after each score was entered. Mrs. Walter noticed that after each score was entered, the average was always an integer. The scores (listed in ascending order) were $71, 76, 80, 82$, and 91. What was the last score Mrs. Walters entered?

(8) If x, y, and z are positive numbers satisfying

$$x + 1/y = 4, \qquad y + 1/z = 1, \qquad \text{and} \qquad z + 1/x = 7/3$$

Then what is the value of xyz ?

(9) Two non-zero real numbers, a and b, satisfy $ab = a - b$. Which of the following is a possible value of $\frac{a}{b} + \frac{b}{a} - ab$?

(10) Which one of the ten digits is the last to appear in the units position of a number in the Fibonacci sequence?

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2. Old Problems

(1) For any positive integer k, let $f_1(k)$ denote the square of the sum of the digits of k. For $n \geq 2$, let $f_n(k) = f_1(f_{n-1}(k))$. Find $f_{2017}(11)$.

(2) Suppose that $|x_i| < 1$ for $i = 1, 2, \ldots, n$. Suppose further that $|x_1| + |x_2| + \cdots + |x_n| = 19 + |x_1 + x_2 + \cdots + x_n|$.

What is the smallest possible value of n?

(3) Let m/n, in lowest terms, be the probability that a randomly chosen positive divisor of 10^{99} is an integer multiple of 10^{88}. Find $m + n$.

(4) In triangle ABC, $\tan\angle CAB = 22/7$, and the altitude from A divides BC into segments of length 3 and 17. What is the area of triangle ABC?

(5) Find the smallest positive integer whose cube ends in 888.