OLD PROBLEMS

1. 1990's Warmups

(1) Find the sum of all positive rational numbers that are less than 10 and that have denominator 30 when written in lowest terms.

(2) A positive integer is called ascending if, in its decimal representation, there are at least two digits and each digit is less than any digit to its right. How many ascending positive integers are there?

(3) A tennis player computes her win ratio by dividing the number of matches she has won by the total number of matches she has played. At the start of a weekend, her win ratio is exactly .500. During the weekend, she plays four matches, winning three and losing one. At the end of the weekend, her win ratio is greater than .503. What's the largest number of matches she could've won before the weekend began?

(4) In Pascal's Triangle, each entry is the sum of the two entries above it. In which row of Pascal's Triangle do three consecutive entries occur that are in the ratio $3 : 4 : 5$?

(5) For how many pairs of consecutive integers in $\{1000, 1001, 1002, \ldots, 2000\}$ is no carrying required when the two integers are added?

Date: October 6, 2017.

(6) Let S be a set with six elements. In how many different ways can one select two not necessarily distinct subsets of S so that the union of the two subsets is S? The order of selection does not matter; for example, the pair of subsets $\{a, c\}$, $\{b, c, d, e, f\}$ represents the same selection as the pair $\{b, c, d, e, f\}$, $\{a, c\}$.

(7) How many ordered four-tuples of integers (a, b, c, d) with $0 < a < b < c < d < 500$ satisfy $a + d = b + c$ and $bc - ad = 93$?

(8) What is the smallest positive integer than can be expressed as the sum of nine consecutive integers, the sum of ten consecutive integers, and the sum of eleven consecutive integers?

(9) Find $x^2 + y^2$ if x and y are positive integers such that $xy + x + y = 71$ and $x^2y + xy^2 = 880$.

(10) Given a rational number, write it as a fraction in lowest terms and calculate the product of the resulting numerator and denominator. For how many rational numbers between 0 and 1 will 20! be the resulting product?