PRACTICE TEAM ROUND

NEW HAMPSHIRE STATE TEAM
NATIONAL MATHCOUNTS PREPARATION

(1) Suppose $\langle ab \rangle$ is the arithmetic mean of a and b, (a, b, c) denotes the geometric mean of a, b, and c, and $[a, b, c, d]$ denotes the harmonic mean of a, b, c, and d. What is $(\langle\langle 0, 4\rangle, [3, 6, 12, 12]\rangle, (4, 1, 2^{-5}), 2)$?

(2) How many 2 digit integers have no 2 digit prime factors?

(3) The players on a basketball team made some three-point shots, some two-point shots, and some one-point free throws. They scored as many points with two-point shots as with three-point shots. Their number of successful free throws was one more than their number of successful two-point shots. The team's total score was 61 points. How many free throws did they make?

(4) A rectangular parking lot has a diagonal of 25 and an area of 168. What is the perimeter of the parking lot?

(5) In an eight term sequence the value of the third element is 5 and the sum of any three consecutive elements is 30. What is the sum of the first and last element?

Date: April 11, 2017.

(6) At a middle school students in 6th, 7th, and 8th grade do math problems for an average of $12, 15$, and 10 minutes per day, respectively. There are twice as many 6th graders and 7th graders and twice as many 7th graders as 8th graders. If a student from the school is selected at random what is the expected value for the number of minutes they study per day?

(7) The sum of the first 2011 terms of a geometric sequence is 200. The sum of the first 4022 terms is 380. Find the sum of the first 6033 terms.

(8) A point is selected at random from the triangle with vertices at $\{(-5,-5),(5,-5),(0,10)\}$. What is the probability that the product of the x and y coordinate is positive?

(9) Nine middle school students are seated randomly around a round table. If there are 3 6th graders, 3 7th graders, and 3 8th graders, what is the probability that each student is sitting next to at least one student from another grade?

(10) Circles A, B, and C with centers O_A, O_B, and O_C have radius 1. Circles A and B are tangent at a single point and circle C is tangent to the midpoint of $\overline{O_AO_B}$. What is the area inside C and outside of A and B?