TRIG PREPARATION PART 1

1. Helpful Formulas

This page describes some of the basic formulas and identities for relating sine and cosine functions. Together with knowledge of a few of the standard angles from the special right triangles, these equations can be used to derive and solve a wide variety of problems.

1.1. Pythagorean Identity.

$$\sin(x)^2 + \cos(x)^2 = 1$$

1.2. Parity and Shifts.

$$\begin{aligned}
\sin(-x) &= -\sin(x) \\
\cos(-x) &= \cos(x) \\
\sin(90 - x) &= \cos(x) \\
\cos(90 - x) &= \sin(x) \\
\sin(180 - x) &= \sin(x) \\
\cos(180 - x) &= \cos(x)
\end{aligned}$$

1.3. Angle Sums and Differences.

$$\begin{aligned}
\sin(x + y) &= \sin(x)\cos(y) + \cos(x)\sin(y) \\
\sin(x - y) &= \sin(x)\cos(y) - \sin(y)\cos(x) \\
\cos(x + y) &= \cos(x)\cos(y) - \sin(x)\sin(y) \\
\cos(x - y) &= \cos(x)\cos(y) + \sin(x)\sin(y)
\end{aligned}$$

1.4. Doubled and Halved Angles.

$$\begin{aligned}
\sin(2x) &= 2\sin(x)\cos(x) \\
\cos(2x) &= \cos^2(x) - \sin^2(x) \\
\sin(\frac{x}{2}) &= \pm\sqrt{\frac{1-\cos(x)}{2}} \\
\cos(\frac{x}{2}) &= \pm\sqrt{\frac{1+\cos(x)}{2}}
\end{aligned}$$

1.5. Product to Sum Identities.

$$\begin{aligned}
\cos(x)\cos(y) &= \frac{1}{2}(\cos(x+y) + \cos(x-y)) \\
\sin(x)\sin(y) &= \frac{1}{2}(\cos(x-y) - \cos(x+y)) \\
\sin(x)\cos(y) &= \frac{1}{2}(\sin(x+y) + \sin(x-y))
\end{aligned}$$

1.6. Sum to Product Identities.

$$\begin{aligned}
\cos(x) + \cos(y) &= 2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})) \\
\cos(x) - \cos(y) &= -2\sin(\frac{x+y}{2})\sin(\frac{x-y}{2})) \\
\sin(x) + \sin(y) &= 2\sin(\frac{x+y}{2})\cos(\frac{x-y}{2})) \\
\sin(x) - \sin(y) &= 2\sin(\frac{x-y}{2})\cos(\frac{x+y}{2}))
\end{aligned}$$

1.7. Law of Cosines.

$$\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\cos(A) \\
b^2 &= a^2 + c^2 - 2ac\cos(B) \\
c^2 &= a^2 + b^2 - 2ab\cos(C)
\end{aligned}$$

1.8. Law of Sines.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

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2. Triangle Warmups

(1) Three semicircles have their diameters on the sides of right triangle ABC. The area of the small semi circle is 5π and the area of the medium semicircle is 7π. What is the sum of the areas of all three semicircles?

(2) A 20 foot ladder rests against a wall so the top of the ladder is 19 feet off the ground. If the top of the ladder drops 6 feet, how far did the bottom of the ladder move?

(3) A square and an equilateral triangle have the same perimeter. What is the ratio of the area of the circle circumscribed about the square to the area of the circle circumscribed about the triangle?

(4) A square and an equilateral triangle have the same area. What is the ratio of the area of the circle circumscribed about the square to the area of the circle circumscribed about the triangle?

(5) Use your favorite 30–60–90 and 45–45–90 triangles to determine the 6 trig function values for $\frac{\pi}{3}$, $\frac{\pi}{4}$, and $\frac{\pi}{6}$.

(6) Sketch the function $5\cos(\frac{x}{2}) - 1$ below.

(7) From the function $2\sin(\pi x + 3) + 4$ determine the amplitude, period, phase shift, and vertical shift.

3. Trig Identities

(1) How many solutions for x are there of $\sin(x)\tan(x) + 2x\sec(x) = -\cos(x)$?

(2) Use the angle sum identities to compute $\sin(105)$, $\sin(15)$, $\cos(75)$, and $\cos(15)$.

(3) What is $\cos(20) \cdot \cos(40) \cdot \cos(80)$?

(4) Let w, x, y, z satisfy $w^2 + x^2 = y^2 + z^2 = 1$ and $wy - xz = \frac{1}{2}$. What is $wz + xy$?

(5) Compute $\sin(1)$ times the average of $2\sin(2), 4\sin(4), \ldots, 178\sin(178), 180\sin(180)$.

(6) In triangle ABC we know that $AB = 3$, $BC = 5$ and $AC = 7$. What in the measure of angle ABC?

(7) Consider two triangles, ABC and DEF with $AB = DE = 7$, $BC = EF = 5$, and $\angle CAB = \angle FDE = 30$. Use the law of sines to show that $\sin(\angle ACB) = \sin(\angle DFE)$. Must the two triangles by congruent?