AIME PREPARATION

1. Warmup problems

(1) How many subsets of $\{1, 2, 3, \ldots, 10\}$ are there?

(2) How many 4 element subsets of $\{1, 2, 3, \ldots, 10\}$ are there?

(3) How many 6 element subsets of $\{1, 2, 3, \ldots, 10\}$ have a prime number of even elements?

(4) Jane forms a subset of $\{1, 2, 3, \ldots, 10\}$ by flipping a coin for each element and putting it in the set if it is heads. What is the expected number of elements in the same? What is the expected value of the sum of the elements of her subset?

(5) John forms a subset of $\{1, 2, 3, \ldots, 10\}$ by adding n to his subset with probability $\frac{1}{n}$. What is the expected value of the sum of his subset?

(6) Is it possible to choose 7 integers out of $\{1, 2, 3, \ldots, 100\}$ so that none of the 21 possible sums (of distinct elements) or 42 possible differences (of distinct elements) are a multiple of 10?

(7) Is it possible to choose 10 integers out of $\{1, 2, 3, \ldots, 100\}$ so that there are no pairs of subsets that have the same sum?

(8) Is it possible to choose 51 integers out of $\{1, 2, 3, \ldots, 100\}$ so that there are no pairs of integers where one divides the other?

(9) Is it possible to choose 5 points in the plane with integer coordinates so that none of the midpoints of the pairs of points also have integer coordinates?

(10) Can you place 5 points on the interior of a 1×1 square in such a way that the distances between each pair of points are all more than $\frac{\sqrt{2}}{2}$?

(11) How many points must be selected inside of an equilateral triangle of side 10 in order to guarantee that there are two points whose distance is less than 5?

(12) 200 students take a 6-question exam, and each question is answered correctly by at least 120 students. Is it true that there must be some pair of students with the property that every question was answered correctly by at least one of them?

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