LETTER REARRANGEMENTS

CROSSROADS ACADEMY
MATHCOUNTS PREPARATION

1. Alphabet Rearrangements

(1) How many ways are there to arrange all 26 letters of the alphabet, in any order?

(2) How many ways are there to arrange all 26 letters of the alphabet, if the arrangement must start with *abc*?

(3) How many ways are there to arrange all 26 letters of the alphabet, if the arrangement must start with *abc* **or** end with *xyz*?

(4) How many ways are there to arrange all 26 letters of the alphabet, if the arrangement must start with *abc* **1and** end with *xyz*?

(5) How many ways are there to arrange 14 letter strings (with no repeated letters) can be formed from the alphabet?

(6) How many ways are there to arrange all 26 letters of the alphabet, if all vowels occur in consecutive order *aeiou*?

(7) How many ways are there to arrange all 26 letters of the alphabet, if all vowels occur together but in any order between them?

(8) How many ways are there to arrange all 26 letters of the alphabet, if the vowels occur in order *aeiou* but not necessarily consecutively?

(9) With the substitution $a = 1$, $b = 2$, etc. what is the largest possible value of a permutation of all of the letters if we add the first letter, subtract the second letter, add the third letter, subtract the fourth letter...

(10) How many different arrangements of all 26 letters have the value that you got in the previous problem?

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2. More Letter Rearrangements

(1) How many ways are there to arrange the letters in Crossroads?

(2) How many ways are there to arrange the letters in Crossroads Academy?

(3) How many ways are there to arrange the letters in Crossroads Academy if all the vowels must be adjacent?

(4) How many ways are there to arrange the letters in Crossroads Academy into two "words"?

(5) How many ways are there to arrange the letters in Humuhumunukunukuapua'a?[1]

(6) How many dictionary words can be formed from *aeirst*?

(7) How many ways ways are there to arrange the letters $\{u, u, u, u, u, u, r, r, r, r, r, r, r, r, r, r, r\}$?

(8) How many paths are there from $(0, 0)$ to $(11, 6)$ taking only upwards or rightwards steps?

(9) How are problem 6 and problem 7 related?

(10) How many different rectangles can be found in an $m \times n$ grid?

Hard How many non–square rectangles can be found in an $m \times n$ grid?

Hard How many squares can be found in an $m \times n$ grid?

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[1]This is the name of the Hawaiian state fish. The alphabet of Hawaii is interesting because it only contains 13 letters.