

## AIME PREPARATION

### 1. WARMUP PROBLEMS

- (1) An dodecahedron has 12 faces and 30 edges. How many vertices does it have?
  
- (2) The faces of a standard soccer ball consist of 20 regular hexagons and 12 regular pentagons, each of which has a side length of 1 inch. This 3d figure is known as a truncated icosahedron.
  - (a) What is the total length of all of the seams on the ball?
  - (b) If no two pentagonal faces are adjacent, how many seams on the ball separate two hexagonal faces?
  - (c) How many vertices does this soccer ball have?
  - (d) What is the surface area of the ball?
  - (e) For the 2006 world cup in Germany, the soccer ball was similar to a truncated octahedron, formed by replacing each vertex of a regular octahedron with a square. Repeat parts (a)–(d) for this ball.
  
- (3) An ant starts on one (exterior) vertex of a cube. Can it travel to each other vertex exactly once before returning to its starting vertex if it is only allowed to travel on the edges of the cube and can travel along each edge at most once? Can it travel along each edge exactly once if it is allowed to visit each vertex any number of times? What if the ant is instead on an octahedron?
  
- (4) What is the radius of a hemisphere whose surface area in square inches is equal to its volume in cubic inches?
  
- (5) If the radius of a cylinder is halved but the height is double what is the percentage change in volume of the cylinder?
  
- (6) A cone and a hemisphere have equal surface areas and each have radius 3in. What is the height of the cone?

## 2. MORE PROBLEMS

- (1) A circle has diameter 26. What is the length of a cord whose midpoint is 12 from the center of the circle?
- (2) The surface area of a solid, metal hemisphere is 10 while a larger hemisphere made out of the same material has surface area of 40. If the smaller hemisphere weighs 2 pounds how heavy is the larger sphere?
- (3) You make a snowman out of three giant snow balls with diameters of 3, 6, and 12 feet respectively. If the largest ball weighs 100 pounds, what is the weight of the entire snowman?
- (4) Triangle  $ABD$  has  $AD = 17$ ,  $BD = 8$ , and  $C$  is a point along  $BD$  so that  $BC = 1$  and  $CD = 7$ . If angle  $B$  is an obtuse angle what are the possible integer lengths of  $AC$ ?
- (5) What is the maximum number of points of intersection of two circles and three straight lines in a plane?
- (6) A cube with side length 9 is circumscribed about a sphere and a smaller cube is inscribed inside the same sphere. What is the volume of the smaller cube?
- (7) A square with side length 9 is circumscribed about a circle and a smaller square is inscribed inside the same circle. What is the area of the smaller square?
- (8) A large cube is sliced into 125 smaller cubes and then the cubes that lie at the center of each face are removed along with the cubes directly beneath them, leading to a cube with 6 holes through the center. Then, the new object is dipped in a bucket of paint. For each integer  $0 \leq m \leq 6$  how many of the smaller cubes have exactly  $m$  painted faces?
- (9) Show that for any 5 points in the plane, such that no 3 of them are co-linear, have the property that there must exist a subset of 4 of the points that form a convex quadrilateral.