LOGARITHMS AND WARMUPS

1. Inverse Functions

(1) Compute the following values:
  (a) $\log_5(6125)$
  (b) $3^{2\log_3(45)}$
  (c) $\log_8(2)$
  (d) $2^{\log_2(5)+\log_4(10)}$

(2) Express these as a single logarithm:
  (a) $2\log(4) - \log(2)$
  (b) $\log(12) + \frac{1}{2}\log(9) - 2\log(1)$
  (c) $\log_5(12) + \log_1 2(5)$

(3) Find the domains of these functions:
  (a) $\log(2x-1)$
  (b) $4^{\frac{1}{x}}$
  (c) $\dfrac{1}{1+2^x}$
  (d) $\sqrt{1-3^x}$

(4) Find inverse formulas for the functions:
  (a) $\sqrt{10-4x}$
  (b) 5^{x^5}
  (c) $\dfrac{4x-1}{2x+3}$
  (d) $2\log(x-1)$

(5) Solve for x:
  (a) $2^{-x} = 5$
  (b) $2\log(5-3x) = -1$
  (c) $2^{3^x} = 4$
  (d) $\log(x) + \log(x-1) = 4$
  (e) $7^{2x-1} = 3$
  (f) $\log(\log(x)) = 5$

(6) Find the values of:
  (a) $\sin^{-1}(\sqrt{3}/2)$
  (b) $\tan^{-1}(-1)$
  (c) $\cos^{-1}(1/2)$
  (d) $\sin^{-1}(x) + \cos^{-1}(x)$

(7) Find formulas with no trig functions for these expressions:
  (a) $\cos(\sin^{-1}(x))$
  (b) $\sin(\tan^{-1}(x))$
  (c) $\tan(\sin^{-1}(2x))^2$
  (d) $\sin(\cos^{-1}(\sin(\cos^{-1}(x))))$

Date: September 8, 2017.

2. Inequalities and Warmups

(1) Solve the inequalities for x:

(a) $5^{2x-4} < 125$
(b) $\log_2(x) > -2$
(c) $2 < \log_2(x) < 10$
(d) $\sqrt{4 - 2^{3x-5}} > 0$

(2) What is the sum of all possible solutions to $(x^2 - 5x + 5)^{x^2-3x+2}$

(3) If $\dfrac{1}{x + \dfrac{1}{y + \dfrac{1}{z}}} = \dfrac{3}{8}$ what is the product xyz?

(4) What is the sum $1000^2 - 999^2 + 998^2 - 997^2 + \cdots 1^2$?

(5) How many integers less than 1000 have exactly 5 positive divisors? What is the sum of these numbers?

(6) What is $\sqrt{7 + \sqrt{7 + \sqrt{7 + \sqrt{7 \cdots}}}}$?

(7) If the product of a set of numbers is 144 what is the smallest possible sum of the numbers? What is the greatest possible sum?

(8) What is the probability that three points selected from a 3×3 grid will form an isosceles triangle?

(9) The numbers 1 through 8 are written on some ping pong balls and placed in a bag. If they are drawn from the bag one at a time without replacement what is the probability that it never happens that number i is drawn ith? What if we allow replacement?

(10) A committee of 12 students has 6 mathematicians and 6 violinists. They need to form two different subcommittees each with two math students and two music students. If no student can serve on both committees how many combinations of subcommittees are possible?

Suppose that [image: p] and [image: q] are positive numbers for which [image:

$$\log_9 p = \log_{12} q = \log_{16}(p+q).$$

] Then [image: q/p] can be expressed in the form [image: $(x+\sqrt{y})/z$] , where [image: x], [image: y], and [image: z] are positive integers, and [image: y] is not divisible by the square of a prime. Find [image: $x+y+z$].

-The following problem: How many real solutions does [image: $\sin x = \log_{10} x$] have? (This problem is in radians.)

-The following problem: The sum of all [image: $x > 0$] such that [image: $x\lfloor x\lfloor x\rfloor\rfloor = 42$] can be expressed in the form [image: m/n], where [image: m] and [image: n] are relatively prime positive integers. Find [image: $m+n$].

-The following problem: Given [image: $0 \le x_0 < 1$], let [image:

$$x_n = \begin{cases} 2x_{n-1} & \text{if } 2x_{n-1} < 1, \\ 2x_{n-1} - 1 & \text{if } 2x_{n-1} \ge 1, \end{cases}$$

] for all integers [image: $n > 0$]. For how many [image: x_0] is it true that [image: $x_0 = x_5$]?