MODULAR INVERSES

CROSSROADS ACADEMY
MATHCOUNTS PREPARATION

I) Find three inverses for $6 \pmod{11}$, $17 \pmod{12}$, and $18 \pmod{5}$.

II) Solve each equation for the variable:

$$\begin{aligned} 5x &\equiv 7 \pmod{12} \\ 4y &\equiv 3 \pmod{11} \\ 22z &\equiv 6 \pmod{87} \end{aligned}$$

III) Find the inverses of all the integers mod 7. Find the inverses of all the integers mod 6. Find the inverses of all the integers mod 7.

IV) For what integers n do all positive integers less than n have inverses mod n? For any positive integers a and b, when does a have an inverse mod b?

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a) Euler's φ function counts the number of integers less than n that are relatively prime to n. For example, $\varphi(10) = 4$ because $\{1, 3, 7, 9\}$ are all relatively prime to 10. Can you construct a formula for $\varphi(n)$?

b) Compute $\varphi(n)$ for each of $n = \{4, 10, 12, 75\}$.

c) For each positive integer, k less than 10, compute $k^{\varphi(10)}$. How can we use this to find inverses mod 10? What happens if we replace 10 with n?

d) Compute the inverse of 87 (mod 99) using the Euclidean algorithm. Why is this easier than using Fermat's Little Theorem?