MORE GEOMETRY PROBLEMS

CROSSROADS ACADEMY
AMC PREPARATION

1. Area Warm–up Problems

Compute the areas of the following shapes:

(1) 3–4–5 right triangle

(2) 2–2–$\sqrt{8}$ right triangle[1]

(3) Equilateral triangle with $s = 4$

(4) 9–25–30 triangle[2]

(5) Parallelogram with base 10 and height 11

(6) Trapezoid with bases 6 and 4 and height of 7

(7) Rhombus with diagonal 12 and 4

(8) Octagon with horizontal and vertical sides 2 and diagonal sides $\sqrt{2}$[3]

(9) Circle with circumference 3

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[1]Hint: What are the angles of this triangle?

[2]Hint: Heron's Formula

[3]Hint: Pick's Theorem might be helpful here.

2. Geometry Practice

(1) Let ABCD be a square with E the midpoint of AB and F the midpoint of BC. What is the ratio of the area of EBF to ABCD?

(2) If the sum of the lengths of diagonals in a square is 10 what is the area?

(3) If the length of the diagonal of a rectangle is 10 what is the maximum area?

(4) A horse is hitched to the outside corner of a $30' \times 40'$ barn with a $50'$ rope. How many square feet of grass can the horse reach?

(5) What is the area of a circle centered at the centroid of an equilateral triangle of side length $2\sqrt{3}$ if the circle is tangent to the triangle.

(6) How many prime side length triangles are there if the longest edge is shorter than 30?

(7) What is the sum of the internal angles of an octagon?

(8) Find the area of the triangle formed by connecting every other vertex of a regular hexagon of side length 3.

(9) Find the area of the quadrilateral formed by connecting every other vertex of a regular octagon of side length 4.

(10) Find the area of the pentagon formed by connecting every other vertex of a regular decagon of side length 5.

3. Distance Formula

(1) Find the distance between the points (2,7) and (-4,3).

(2) Find the slope of the line between (7,12) and (13,-2)

(3) Find the equation of the line connecting the points (1,2) and (4,9)

(4) Find the center and radius of the circle described by $x(x-4)+y(y+6)-36=0$.

(5) Where does the line $-7y = 3x + 15$ intersect the circle from the previous question?

(6) What is the length of the longest diagonal of a cube with $s = 2$?

(7) If we connect all of the vertices of a cube to each other how many triangles are formed? How many of those are right triangles? Isosceles? Equilateral?

(8) Find the length of the altitude to the hypoteneuse of the 3–4–5 right triangle[4].

[4]Hint: Use the area formula.

4. Challenge Problems

(1) Suppose that two line segments are drawn: first from (-2,4) to (-4,-2) and then from (5,6) to (7,-28). What is the shortest distance between the midpoints of the line segments?

(2) What is the area of a pentagon with vertices at (0,0), (0,3), (4,5), (5,3), (5,0)?

(3) If the center of a circle of radius 2 is placed at random in a circle of radius 4, what is the probability that the smaller circle will be fully contained either on or in the larger circle? What if we use squares of side lengths of 2 and 4? Equilateral triangles?

(4) What is the greatest number of intersection points of two circles and one triangle?

(5) Suppose that 64 cubes each with side length 2 are used to form a larger cube. If the 8 smaller cubes at the corners are removed and the remaining figure is dipped in blue paint, what is the painted surface area of the unit cubes that have at least three faces painted?

(6) If a sphere is inscribed in a cube and the cube's surface area is 216 square units, what is the volume of the sphere?

(7) On an analog clock, what is the first time after 1:00 when the minute hand and the hour hand form a 90 degree angle?