AIME PREPARATION

1. Warmup problems

(1) Four vertices of a regular octagon are chosen at random. What is the probability that they form a trapezoid?

(2) How many integer solutions are there to the inequality $x^2 - 4|x| + 3 \leq 0$.

(3) What is the minimum of $|x-1| + |x+2| + |x-3|$?

(4) If the mean of $\{13, 11, 3, 3, x\}$ is equal to its median what are the possible integer values of x?

(5) What is the 2017th digit in the decimal expansion of $1/7$?

(6) What is the units digit of 7^{2017}?

(7) What is the probability of rolling 2 die and having the product of the two rolls be prime?

(8) How many ways are there to rearrange the letters in rearrange so that the "a" letters are not next to each other?

(9) What is the probability that if you roll 6 dice you get all 6 possible values exactly once?

(10) What is the probability that if you roll 6 dice you get exactly one 1?

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2. Parametric problems

(1) Write the equation for $(2 - 3t, 4 + t)$ in point–slope form.

(2) What is the distance from the point $(2, 3)$ to the line described by $y = 2x + 3$?

(3) What is the distance from the point $(5, -2)$ to the line described by $2y - 4x = 1$?

(4) Consider two particles moving along the real number line, whose positions are given by $s(t) = 3t - 2$ and $r(t) = 4 - 2t$. At what point do the particles intersect?

(5) For the following pairs of lines, determine whether they are parallel or intersecting. If they intersect, determine the point of intersection
   (a) $(1 - t, 1 - t)$ and $(3 + t, 5 + t)$
   (b) $(t, 2t)$ and $(5 + 2t, 2 - t)$
   (c) $(-5 - 3t, 6 + 4t)$ and $(t - 3, t - 4)$
   (d) $(2t + 3, 6 - t)$ and $(11 - 4t, 2t + 2)$

(6) What shape is traced out by the function $f(t) = (\cos(t), \sin(t))$ for $0 \leq t \leq 2\pi$?

(7) Find two different equations for the line passing through the points $(-8, 1, 4)$ and $(3, -2, 4)$.

(8) Find the equation of a line that passes through $(1, 2, 3)$ and intersects the plane $5y + 6z = x - 6$.

(9) Find the intersection of the line $(4 - t, 2t - 1, -3t)$ and the line $(4t - 3, 1 - 2t, t + 1)$.

(10) Find the intersection of the line $(7t + 18, 243 - 105t, 25t + 42)$ and the plane $1096x + 487y - 222z = 345$.