HARDER PRACTICE TARGET PROBLEMS

CROSSROADS ACADEMY MATHCOUNTS PREPARATION

1. How many ways are there to write 2016 as the sum of two integers with no carrying in base 10?

2. What is the sum of the (real) solutions to
$$\sqrt[4]{x} = \frac{12}{7 - \sqrt[4]{x}}$$
?

3. What is the probability that a randomly chosen divisor of 10^{99} is an integer multiple of 10^{80} ?

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4. What is $(\log_2(x))^2$ if $\log_2(\log_8(x)) = \log_8(\log_2(x))$?

5. Yvonne walks down an upward-moving escalator and counts 150 steps. Her friend, Zack, walks up the same escalator and counts 75 steps. If Yvonne walks 3 times faster than Zack, how many steps of the escalator are visible?

6. What is $(\sqrt{5} + \sqrt{6} + \sqrt{7})(-\sqrt{5} + \sqrt{6} + \sqrt{7})(\sqrt{5} - \sqrt{6} + \sqrt{7})(\sqrt{5} + \sqrt{6} - \sqrt{7})?$

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¹think about robots and stagecoaches...

7. Xavier is adding up the page numbers from 1 to n in his favorite book but he accidentally added one page number twice. If his final sum is 1986 which page was added twice?

8. What is the product of the smallest 5 positive integers that are equal to the product of their proper divisors?

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9. FIND AN INTEGER n > 2 AND A SET OF k DISTINCT INTEGERS $\{s_1, s_2, \ldots, s_k\}$ LESS THAN n so that $k > \lceil \log_2(n) \rceil$ AND ALL OF THE POSSIBLE SUMS OF ELEMENTS ARE DISTINCT (I.E. THE 2^k POSSIBLE SUMS OF ELEMENTS OF S ARE ALL DIFFERENT.)

²As an example, $33 = 3 \cdot 11$ has this property but $12 \neq 2 \cdot 3 \cdot 4 \cdot 6$ does not.