TRIANGLE PROBLEMS

CROSSROADS ACADEMY
AMC PREPARATION

1. Triangle Warm–up Problems

(1) What is the area of the triangle with integer side length and perimeter 8?

(2) For how many integers n is there a unique triangle with integer side length and perimeter n?

(3) Can you find a formula for the number of unique triangles with integer side length and perimeter n?[1]

(4) Can you find a formula for the number of unique scalene triangles with integer side length and perimeter n?[2]

(5) In an isosceles triangle ABC we know that $\angle A = 100^\circ$. What is $\angle B$?

(6) Prove that when $2a + 1$ is a perfect square, then there exists a Pythagorean triple with $\sqrt{2a+1}$ as its shortest side.

(7) Prove that for any two positive integers $a < b$ that $(2ab, a^2 - b^2, a^2 + b^2)$ is a Pythagorean triple.

(8) Three semicircles have their diameters on the sides of right triangle ABC. The area of the small semi circle is 5π and the area of the medium semicircle is 7π. What is the sum of the areas of all three semicircles?

(9) Consider two concentric circles with respective radii R and r and let $\overline{AB}$ be a chord of the larger circle tangent to the smaller circle with length 18. What is the area between the two circles?

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[1]Hint: It is probably easier to look at the odd and even cases separately.

[2]Hint: Perhaps in terms of the previous question?

2. Special Triangles

(1) A 20 foot ladder rests against a wall so the top of the ladder is 19 feet off the ground. If the top of the ladder drops 6 feet, how far did the bottom of the ladder move?

(2) What is the radius of the inscribed circle in a equilateral triangle of edge length 6?

(3) What is the height of a stop sign whose sides have length 1?

(4) An isosceles right triangle has hypoteneuse x. What is the area of the triangle in terms of x?

(5) Triangle ABC is a right triangle with $\angle ACB = 90°$, $\angle ABC = 60°$, and $AB = 10$. Let P be a randomly chosen point inside ABC and extend $\overline{BP}$ to meet $\overline{AC}$ at D What is the probability that $BD > 5\sqrt{2}$?

(6) A square and an equilateral triangle have the same perimeter. What is the ratio of the area of the circle circumscribed about the square to the area of the circle circumscribed about the triangle?

(7) What point on the line $y = x$ is equidistant from $(-2, 5)$ and $(3, -2)$?

(8) What is the area of square $ABCD$ with the point A is at $(1, -2)$ and the point C is at $(21, 19)$?

(9) What is the area of the 3–4–5 right triangle?[3]

[3]Hint: Heron's formula