

WARM UP PROBLEMS

NEW HAMPSHIRE STATE TEAM
NATIONAL MATHCOUNTS PREPARATION

- (1) What is the smallest integer factor of 2017?
- (2) How many perfect squares are factors of the product of the first 10 even integers?
- (3) How many 5 letter sequences with at least one S can be formed from the letters in the word "SIGNATURES"?
- (4) A cylindrical water tower is filling with water at a constant rate. Right now it is $\frac{1}{5}$ full but in 3 minutes it will be $\frac{1}{4}$ full. How long will it take to fill the entire tank?
- (5) If the length of the diagonal of a cube is increased by 20% what is the corresponding increase in volume?
- (6) Square ABCD has side length 1. If an equilateral triangle ABE is attached to side AB and a regular pentagon BCFGH is attached to side BC what is the measure of angle EAH?
- (7) Jackie and Phil have two fair coins and a third coin that comes up heads with probability $\frac{4}{7}$. Jackie flips the three coins, and then Phil flips the three coins. Find the probability that they get the same number of heads.
- (8) Equiangular hexagon $ABCDEF$ has side lengths $AB = CD = EF = 1$ and $BC = DE = FA = r$. The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r ?
- (9) Dave arrives at an airport which has twelve gates arranged in a straight line with exactly 100 feet between adjacent gates. His departure gate is assigned at random. After waiting at that gate, Dave is told the departure gate has been changed to a different gate, again at random. What is the probability that Dave has to walk more than 400 feet
- (10) A falling number, like 98,541, is an integer where each digit is smaller than the one on its left. How many 3 digit falling numbers exist?

- (11) 4 ants are standing on the vertices of a tetrahedron. In how many ways can the ants each move along a single edge of the tetrahedron to a new vertex so that each ant again has its own vertex?
- (12) A fly trapped inside a cubical box with side length 2 meters decides to relieve its boredom by visiting each corner of the box. It will begin and end in the same corner and visit each of the other corners exactly once. To get from a corner to any other corner, it will either fly or crawl in a straight line. What is the maximum possible length, in meters, of its path?
- (13) Carla and Dan both run a lap around a long track. It takes Carla 2 hours to finish her lap. If Carla runs 5 miles per hour faster than Dan, and it takes Dan 4 hours to finish, how long is the track?
- (14) 100 checkers are placed in a row. You can exchange any pair of checkers that has exactly one checker between them, for example you could exchange the 13th and 15th checker. Is it possible to reverse the order of the checkers by performing this operation less than 2017 times?
- (15) A robot paces back and forth along a desert trail for all eternity. Besides the robot, the trail is entirely empty except for an infinite stream of westward bound, evenly spaced stagecoaches each moving at a fixed constant rate of speed. When the robot is walking west it is passed by a stagecoach every 12 minutes. When it is walking east it is passed by a stagecoach every 4 minutes. Eventually, the robot's knee joints rust and it is forced to remain in place. How often do stagecoaches pass the robot after it gets stuck?