DIVISION CHAINS SUMMARY (JOINT WITH P. DOYLE)

DARYL DEFORD

This project concerns generalizations of the Cantor–Bernstein Theorem to the case where we have injections $nA \hookrightarrow nB$ and $nB \hookrightarrow nA$. Using an early version of thethe Pan Galactic Solitaire approach[1], we have an algorithm that converges "eventually". The current algorithm leads to a natural notion of a "division chain", complementary to the notion of "addition chains" used in rapid exponentiation [2]. As in the addition case there seems to be simple bounds that can be applied, but actual asymptotics and values are much more difficult to compute.

There is an additional complication in the division case that sometimes it is more efficient to multiply a number by a small factor before reducing, this means that even knowing all of the previous best cases is not necessarily enough knowledge to construct the actual values inductively. For example, the straightforward procedure for constructing a strictly decreasing division chain for 683 gives [683, 682, 341, 340, 170, 85, 68, 34, 17, 16, 8, 4, 2, 1] but multiplying by 3 to get $2049 = 2^{11} + 1$ gives a 12 step chain.

References

[1] P. Doyle and C. Qiu: *Division by four*, preprint, https://math.dartmouth.edu/∼doyle/docs/four/four.pdf.
[2] D. Knuth: *Seminumerical Algorithms: The Art of Computer Programming, Vol. 2*, Addison Wesley, 1997.

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